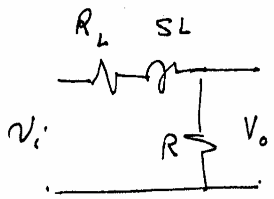
HW8/14-3, 14-4, 14-8, 14-15, 14-20

<u>14.3</u>



a)
$$v_o = \frac{v_i \cdot R}{(R_L+R)+sL}$$

$$\therefore H(s) = \frac{R}{(R+R_L)+sL}$$

$$= \frac{R/L}{s + \left(\frac{R+R_L}{L}\right)}.$$

b) $H(s)\Big|_{max}$ at $s=0$

c) $H(s)_{max} = \dfrac{R}{R+R_L}$

d)
$$H(j\omega) = \frac{R/L}{\left(\frac{R+R_L}{L}\right)+j\omega}$$

$$|H(j\omega)| = \frac{\cancel{R/L}}{\left[\left(\frac{R+R_L}{L}\right)^2+\omega^2\right]^{1/2}} = \frac{1}{\sqrt{2}}\frac{\cancel{R/L}}{(R+R_L)/L}$$

$$2\left(\frac{R+R_L}{L}\right)^2 = \left(\frac{R+R_L}{L}\right)^2 + \omega^2$$

$$\left(\frac{R+R_L}{L}\right)^2 = \omega^2$$

$$\therefore \quad \omega = (R+R_L)/L$$

e) numerical substitution.

<u>14.4</u>

$$\omega_c = 1/RC = \frac{10^9}{10^3 \times 100} = 10^4 \text{ rad/s}$$

$$H(j\omega) = \frac{1/RC}{\sqrt{\omega^2+(1/RC)^2}} = \frac{10^4}{\sqrt{10^8+10^8}}\Bigg|_{\text{at } \omega_c}$$

$$= \frac{1}{\sqrt{2}}$$

$$H(j\omega)\Big|_{\text{at } 0.1\omega_c} = \frac{10^4}{\sqrt{10^6+10^8}} = \frac{1}{\sqrt{1.01}} \simeq 1$$

$$H(j\omega)\Big|_{\text{at } \omega = 10\omega_c} = \frac{10^4}{\sqrt{10^{10}+10^8}} = \frac{1}{\sqrt{100+1}}$$

$$\simeq 0.1$$

c)
$$H(j\omega) = \frac{\omega_c}{\sqrt{\omega^2+\omega_c^2}} \angle -\tan^{-1}(\omega/\omega_c)$$

at $\omega = \omega_c$ $\quad H(j\omega_c) = \frac{1}{\sqrt{2}} \angle -45^\circ$

at $\omega = 0.1\omega_c$ $\quad H(j\omega) = 1 \angle -\tan^{-1}(0.1) \simeq 1 \angle -5.7^\circ$

at $\omega = 10\omega_c$ $\quad H(j\omega) = 0.1 \angle -\tan^{-1} 10 = 0.1 \angle -84.29^\circ$

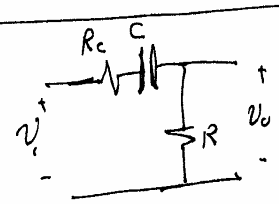
$\therefore$ output is:

$\omega = \omega_c \rightarrow \quad v_o = \frac{200}{\sqrt{2}} \cos(\omega_c t - 45^\circ)$

at $\omega = 0.1\omega_c \quad v_o = 200 \cos(0.1\omega_c t - 5.7^\circ)$

$\omega = 10\omega_c \quad v_o = 20 \cos(10\omega_c t - 84.29^\circ)$.

<u>14.8</u>



a)
$$v_o = \frac{v_i \cdot R}{(R+R_c)+\frac{1}{sC}}$$

$$\therefore H(s) = \frac{RsC}{1+sC(R+R_c)} = \left(\frac{s\,[R/(R+R_c)]}{\frac{1}{C(R+R_c)}+s}\right)$$

b) at $j\omega \to \infty$

c)
$$H(j\omega) = \left(\frac{R}{R+R_c}\right)\left\{\frac{j\omega}{\frac{1}{C(R+R_c)}+j\omega}\right\}$$

$$H(j\omega)\Big|_{max} = \left(R/(R+R_c)\right)$$

d)
$$\frac{\cancel{R}}{(\cancel{R}+R_c)}\cdot\frac{1}{\sqrt{2}} = \frac{\cancel{R}}{\cancel{R}+R_c}\cdot\frac{\omega_c}{\sqrt{\left(\frac{1}{C(R+R_c)}\right)^2+\omega_c^2}}$$

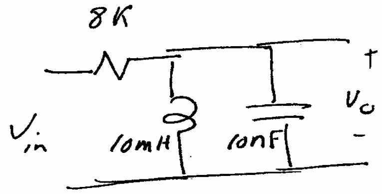
$$2\omega_c^2 = \left(\frac{1}{C(R+R_c)}\right)^2 + \omega_c^2$$

$$\omega_c = \frac{1}{C(R+R_c)}$$

e)
$$H(j\omega) = \frac{R}{R+R_c}\,\frac{\omega}{\sqrt{\left(\frac{1}{C(R+R_c)}\right)^2+\omega^2}} \angle 90^\circ - \tan^{-1}\omega C(R+R_c)$$

14-15

This is exactly example 14.6



Find: ω_o, f_o, Q, ω_{c_1}, f_{c_1}, ω_{c_2}, f_{c_2}, β

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad f_o = \frac{\omega_o}{2\pi}$$

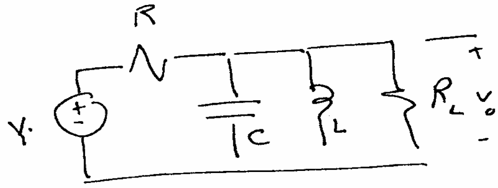
$$Q = \omega_o RC = \frac{R}{\omega_o L}$$

$$\beta = \omega_o / Q$$

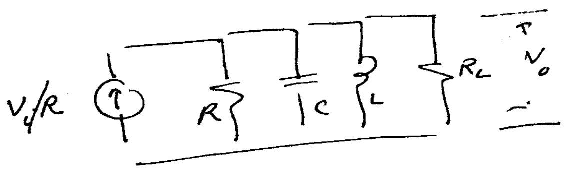
$$\omega_{c_{1,2}} = \pm \beta/2 + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

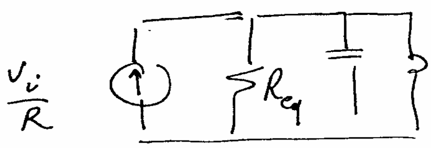
14-20

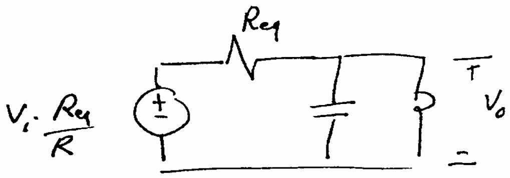
The problem



may be transformed into:







Thus, let

$$v_i' = V_i (R_{eq}/R)$$

$$R_{eq} = R R_L / (R + R_L)$$

$$\therefore \frac{v_o}{v_i'} = H(s) = \frac{s/R_{eq}C}{s^2 + \frac{s}{R_{eq}C} + \frac{1}{LC}}$$

$$\therefore \frac{v_o}{v_i} = \frac{(R_{eq}/R) \cdot \frac{s}{R_{eq}C}}{s^2 + \frac{s}{R_{eq}C} + \frac{1}{LC}} = \frac{s/RC}{s^2 + \frac{s}{R_{eq}C} + 1/LC}$$

$$H(j\omega) = \frac{1}{RC} \frac{j\omega}{\left(-\omega^2 + \frac{1}{LC}\right) + j\frac{\omega}{R_{eq}C}}$$

$$|H(j\omega)| = \frac{1}{RC} \frac{\omega}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \frac{\omega^2}{R_{eq}^2 C^2}}}$$

$$\therefore \omega_o = \frac{1}{\sqrt{LC}}$$

$$H_{max}\Big|_{\text{at } \omega_o} = (R_{eq}/R)$$

at cut-off:

$$\frac{R_{eq}}{R} \cdot \frac{1}{\sqrt{2}} = \frac{\omega_c / RC}{\sqrt{\left(\frac{1}{LC} - \omega_c^2\right)^2 + \frac{\omega_c^2}{R_{eq}^2 C^2}}}$$

$$\frac{2\omega_c^2}{R_{eq}^2 C^2} = \left(\frac{1}{LC} - \omega_c^2\right)^2 + \frac{\omega_c^2}{R_{eq}^2 C^2}$$

14-20 Continue:

$$\frac{\omega_c}{R_{eq}C} = \pm\left(\frac{1}{LC} - \omega_c^2\right)$$

$$\therefore \omega_c^2 \pm \frac{1}{R_{eq}C}\omega_c - \frac{1}{LC} = 0$$

which gives:

$$\omega_{c_{1,2}} = \pm\frac{1}{2R_{eq}C} + \sqrt{\left(\frac{1}{2R_{eq}C}\right)^2 + \frac{1}{LC}}$$

$$\therefore \beta_L = \frac{1}{R_{eq}C}$$

$$Q_L = \frac{\omega_0}{\beta} = \frac{1}{\sqrt{LC}} \cdot R_{eq}C$$

$$= \sqrt{\frac{C}{L}} \cdot R_{eq}.$$

$$\beta_u = \frac{1}{RC}$$

$$\therefore \beta_L = \frac{1}{\frac{RR_L}{R+R_L}C} = \frac{(R+R_L)/R_L}{R\ C}$$

$$\beta_L = (1 + R_L/R) \cdot \beta_u.$$

Also

$$Q_L = \sqrt{C/L}\, R_{eq} \qquad Q_u = \sqrt{\frac{C}{L}}\, R.$$

$$\therefore Q_L = \sqrt{C/L} \cdot \left(\frac{RR_L}{R+R_L}\right) = \sqrt{\frac{C}{L}}\, R_L \left(\frac{R}{R+R_L}\right)$$

$$Q_L = Q_u \cdot \left(\frac{1}{1 + R_L/R}\right).$$