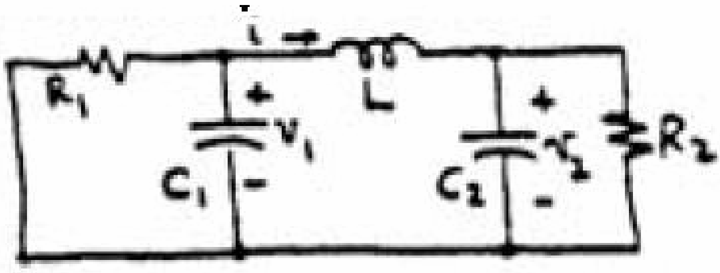
7.6



By KVL,

$$L\frac{di}{dt} + v_2 - v_1 = 0$$

$$\frac{di}{dt} = \frac{1}{L}v_1 - \frac{1}{L}v_2$$

By KCL,

$$C_1\frac{dv_1}{dt} + \frac{v_1}{R} + i = 0$$

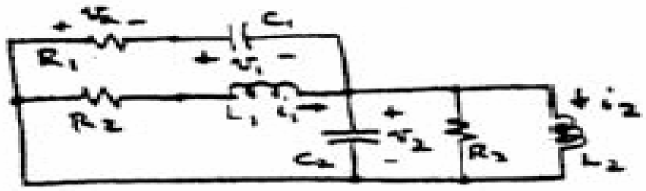
$$\frac{dv_1}{dt} = -\frac{1}{C_1}i - \frac{1}{RC_1}v_1$$

By KCL,

$$C_2\frac{dv_2}{dt} + \frac{v_2}{R_2} = i$$

$$\frac{dv_2}{dt} = +\frac{1}{C_2}i - \frac{1}{R_2C_2}v_2$$

7.9



By KVL,

$$L_1\frac{di_1}{dt} + v_2 + R_2 i_1 = 0$$

$$\frac{di_1}{dt} = -\frac{R_2}{L_1}i_1 - \frac{1}{L_1}v_2$$

By KVL,

$$L_2\frac{di_2}{dt} = v_2$$

$$\frac{di_2}{dt} = \frac{1}{L_2}v_2$$

By KCL,

$$C_2\frac{dv_2}{dt} + \frac{v_2}{R_3} + i_2 = i_1 + \frac{v_a}{R_1} = i_1 + \frac{-v_2 - v_1}{R_1}$$

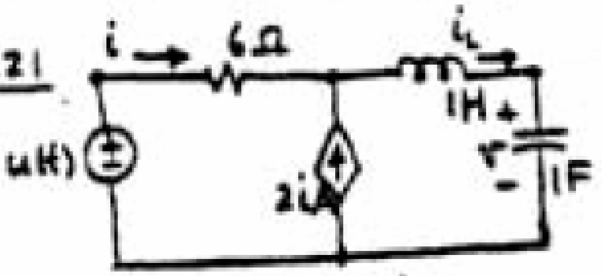
$$C_2\frac{dv_2}{dt} = i_1 - \frac{1}{R_1}v_1 - \left(\frac{1}{R_1} + \frac{1}{R_3}\right)v_2 - i_2$$

$$\frac{dv_2}{dt} = \frac{1}{C_2}i_1 - \frac{1}{C_2}i_2 - \frac{1}{R_1C_2}v_1 - \left(\frac{1}{R_1C_2} + \frac{1}{R_3C_2}\right)v_2$$

By KCL, $C_1\frac{dv_1}{dt} = \frac{v_a}{R_1} = \frac{-v_1 - v_2}{R_1} \Rightarrow \frac{dv_1}{dt} = -\frac{1}{R_1C_1}v_1 - \frac{1}{R_1C_1}v_2$

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -\frac{R_2}{L_1} & 0 & 0 & -\frac{1}{L_1} \\ 0 & 0 & 0 & \frac{1}{L_2} \\ 0 & 0 & -\frac{1}{R_1C_1} & -\frac{1}{R_1C_1} \\ \frac{1}{C_2} & -\frac{1}{C_2} & -\frac{1}{R_1C_2} & \frac{-1}{R_1C_2} - \frac{1}{R_3C_2} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix}$$

7.21



By KCL, $3i = i_L \Rightarrow i_L/3 = i$

By KVL,

$$u(t) = 6i + 1\frac{di_L}{dt} + v$$

$$u(t) = 2i_L + \frac{di_L}{dt} + v$$

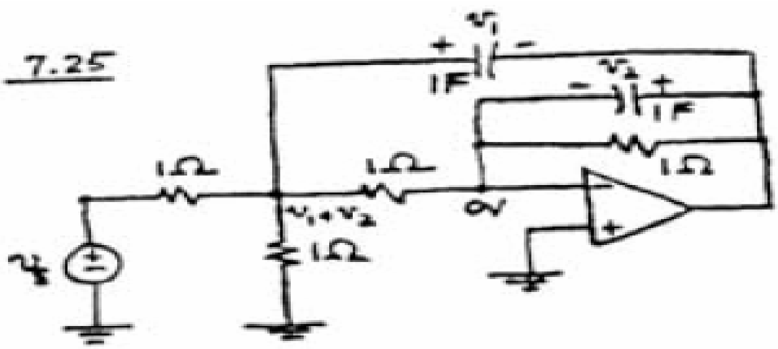
$$\frac{di_L}{dt} = -2i_L - v + u(t)$$

By KCL, $i_L = 1\frac{dv}{dt}$

$$\frac{dv}{dt} = i_L$$

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_L \\ v \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

7.25



By KCL, $\frac{(v_1+v_2)-v_s}{1} + \frac{v_1+v_2}{1} + \frac{v_1+v_2}{1} + 1\frac{dv_1}{dt} = 0$

$$\frac{dv_1}{dt} = -3v_1 - 3v_2 + v_s$$

By KCL,

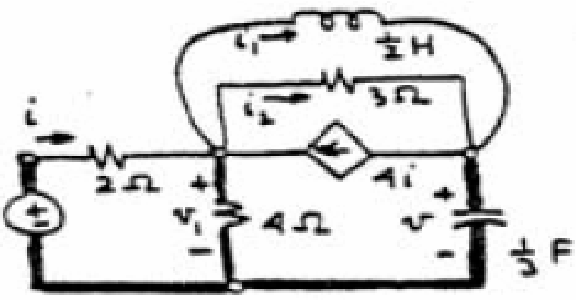
$$\frac{v_1+v_2}{1} + \frac{v_2}{1} + 1\frac{dv_2}{dt} = 0$$

$$\frac{dv_2}{dt} = -v_1 - 2v_2$$

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_s$$

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ v \end{bmatrix} = \begin{bmatrix} -\frac{24}{37} & -\frac{66}{37} \\ \frac{27}{37} & -\frac{9}{37} \end{bmatrix}\begin{bmatrix} i_1 \\ v \end{bmatrix} + \begin{bmatrix} \frac{60}{37} \\ -\frac{12}{37} \end{bmatrix} v_s$$

7.38



1. 4Ω resistor chosen for tree

2. i_1 and v are the state variables. v_1 and i_2 are assigned.

3. KVL: $\frac{1}{2}\frac{di_1}{dt} + v - v_1 = 0 \Rightarrow \frac{di_1}{dt} = -2v + 2v_1$

4. KCL: $\frac{1}{3}\frac{dv}{dt} + 4i = i_1 + i_2 \Rightarrow \frac{dv}{dt} = -12i + 3i_1 + 3i_2$

5. KVL: $2i + v_1 - v_s = 0 \Rightarrow 2i + v_1 = v_s$

$3i_2 + v - v_1 = 0 \Rightarrow 3i_2 - v_1 = -v$

6. KCL: $i + 4i = i_1 + i_2 + \frac{v_1}{4} \Rightarrow 20i - 4i_2 - v_1 = 4i_1$

7.

$$\Delta = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 3 & -1 \\ 20 & -4 & -1 \end{vmatrix} = -6 - 60 - 8 = -74 \qquad \Delta_i = \begin{vmatrix} v_s & 0 & 1 \\ -v & 3 & -1 \\ 4i_1 & -4 & -1 \end{vmatrix} = -3v_s + 4v - 12i_1 - 4v_s = -12i_1 + 4v - 7v_s$$

$$\Delta_{i_2} = \begin{vmatrix} 2 & v_s & 1 \\ 0 & -v & -1 \\ 20 & 4i_1 & -1 \end{vmatrix} = 2v - 20v_s + 20v + 8i_1 = 8i_1 + 22v - 20v_s \qquad \Delta_{v_1} = \begin{vmatrix} 2 & 0 & v_s \\ 0 & 3 & -v \\ 20 & -4 & 4i_1 \end{vmatrix} = 24i_1 - 60v_s - 8v$$

$$i = \frac{\Delta_i}{\Delta} = \frac{12}{74}i_1 - \frac{4}{74}v + \frac{7}{74}v_s \qquad i_2 = \frac{\Delta_{i_2}}{\Delta} = -\frac{8}{74}i_1 - \frac{22}{74}v + \frac{20}{74}v_s$$

$$v_1 = \frac{\Delta_{v_1}}{\Delta} = -\frac{24}{74}i_1 + \frac{8}{74}v + \frac{60}{74}v_s$$

8.

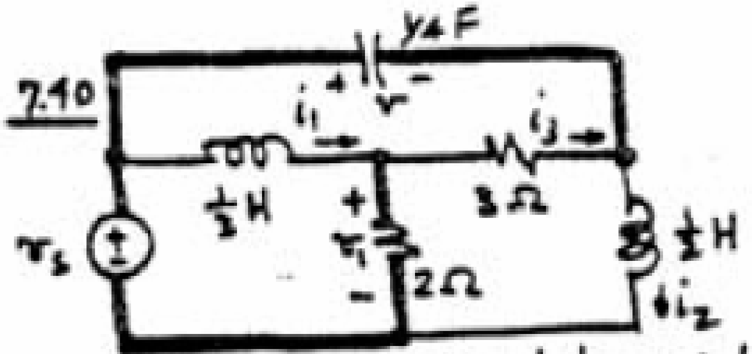
$$\frac{di_1}{dt} = -2v + 2\left(-\frac{24}{74}i_1 + \frac{8}{74}v + \frac{60}{74}v_s\right) = \underline{-\frac{24}{37}i_1 - \frac{66}{37}v + \frac{60}{37}v_s = \frac{di_1}{dt}}$$

$$\frac{dv}{dt} = -12\left(\frac{12}{74}i_1 - \frac{4}{74}v + \frac{7}{74}v_s\right) + 3i_1 + 3\left(-\frac{8}{74}i_1 - \frac{22}{74}v + \frac{20}{74}v_s\right)$$

$$= -\frac{144}{74}i_1 + \frac{48}{74}v - \frac{84}{74}v_s + \frac{222}{74}i_1 - \frac{24}{74}i_1 - \frac{66}{74}v + \frac{60}{74}v_s$$

$$\frac{dv}{dt} = \frac{54}{74}i_1 - \frac{18}{74}v - \frac{24}{74}v_s = \underline{\frac{27}{37}i_1 - \frac{9}{37}v - \frac{12}{37}v_s = \frac{dv}{dt}}$$

7.40



1. 2Ω resistor chosen for tree

2. i_1, i_2 and v are the state variables. i_3 and v_1 are assigned.

3. KVL: $\frac{1}{3}\frac{di_1}{dt} + v_1 - v_s = 0 \Rightarrow \frac{di_1}{dt} = -3v_1 + 3v_s$

$\frac{1}{2}\frac{di_2}{dt} - v_s + v = 0 \Rightarrow \frac{di_2}{dt} = -2v + 2v_s$

4. KCL: $\frac{1}{4}\frac{dv}{dt} + i_3 = i_2 \Rightarrow \frac{dv}{dt} = 4i_2 - 4i_3$

5. KVL: $3i_3 - v + v_s - v_1 = 0 \Rightarrow 3i_3 - v_1 = v - v_s$

6. KCL: $i_1 = \frac{v_1}{2} + i_3 \Rightarrow 2i_3 + v_1 = 2i_1$

$5i_3 = 2i_1 + v - v_s$

$v_1 = 2i_1 - 2i_3$ $\Leftarrow i_3 = \frac{2}{5}i_1 + \frac{1}{5}v - \frac{1}{5}v_s$

$= 2i_1 - 2\left(\frac{2}{5}i_1 + \frac{1}{5}v - \frac{1}{5}v_s\right)$

$v_1 = \frac{6}{5}i_1 - \frac{2}{5}v + \frac{2}{5}v_s$

8. $\frac{di_1}{dt} = -3\left(\frac{6}{5}i_1 - \frac{2}{5}v + \frac{2}{5}v_s\right) + 3v_s = \underline{-\frac{18}{5}i_1 + \frac{6}{5}v + \frac{9}{5}v_s = \frac{di_1}{dt}}$

$\frac{dv}{dt} = 4i_2 - 4\left(\frac{2}{5}i_1 + \frac{1}{5}v - \frac{1}{5}v_s\right) = \underline{-\frac{8}{5}i_1 + 4i_2 - \frac{4}{5}v + \frac{4}{5}v_s = \frac{dv}{dt}}$

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix} = \begin{bmatrix} -\frac{18}{5} & 0 & \frac{6}{5} \\ 0 & 0 & -2 \\ -\frac{8}{5} & 4 & -\frac{4}{5} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix} + \begin{bmatrix} \frac{9}{5} \\ 2 \\ \frac{4}{5} \end{bmatrix} v_s$$