KFUPM-EE DEPT.
EE570- Stochastic Processes
Dr. Ali Muqaibel

Assignment # 5   Due: Week 11

Analysis and Processing of Random Signals
Version 2.0

1. State whether or not each of the following functions can be a valid power density spectrum with justification:

    a. $\dfrac{|\omega|\exp(-4\omega^2)}{1+j\omega}$

    b. $\cos(3\omega)\exp(-\omega^2 + j2\omega)$

    c. $\dfrac{\omega^5}{(12+\omega^2)^6}$

    d. $\cos^2(\omega)\exp(-8\omega^2)$

2. Let $V(t) = X(t) + Y(t)$. Under what conditions does
$$S_{VV}(\omega) = S_{XX}(\omega) + S_{YY}(\omega)$$

3. Let $Y(t) = X(t) - X(t - t_0)$

    a. Find $R_{XY}(\tau)$ and $S_{XY}(\omega)$

    b. Find $R_{YY}(\tau)$ and $S_{YY}(\omega)$

4. Let $Y_n = \dfrac{(X_{n+1}+X_n+X_{n-1})}{3}$ which is a smoothed version of X_n.

    a. Find $R_{YY}(n), S_{YY}(\omega), and\ E[Y_n^2]$.

    b. Suppose that X_n is a white Gaussian noise process. Find the joint pmf for Y_n and Y_{n+1}

5. Assume a random process has a power spectrum

$$S_{XX}(\omega) = \begin{cases} 4 - \left(\dfrac{\omega^2}{9}\right) & |\omega| \le 6 \\ 0 & elsewhere \end{cases}$$

Find (a) the average power (b) the rms bandwidth and (c) the autocorrelation (d) the fraction of power lies in the frequency band $|\omega| < 1$, (e) the fraction of power lies in the frequency band $|\omega| < \omega_{rms}$.   Comment!

6. Suppose that a random telegraph signal with transition rate α is the input signal in an amplitude modulation system. Plot the power spectral density of the modulated signal assuming $f_c = \alpha/\pi$ and $f_c = \dfrac{10\alpha}{pi}$. Support your hand analysis with Matlab Monte Carlo simulation.

7. A random noise $X(t)$, having a power spectrum
$$S_{XX}(\omega) = \frac{3}{49 + \omega^2}$$

is applied to a differentiator that has a transfer function $H_1(\omega) = j\omega$. The differentiator's output is applied to a network for which

$$h_2 = u(t)t^2 \exp(-7t).$$

The network's response is a noise denoted by $Y(t)$

(a) What is the average power in $X(t)$?
(b) Find the power spectrum of $Y(t)$.
(c) Find the average power in $Y(t)$.
(d) Support your answers to parts (a) ,(b) , and (c) ..using Matlab
Simulation (part (d) is a <u>bonus</u> optional open problem)

"Seek knowledge from the cradle to the grave"   Translation of an Arabic saying

Good luck , **Dr. Ali H. Muqaibel**