The material covered in this class will be as follows:

- ⇒Boolean Algebra
- ⇒Postulates
- ⇒Two-valued Boolean Algebra
- ⇒Basic Theorems and Properties
- ⇒Venn Diagrams

Boolean Algebra:

It is defined with a set of elements, set of operators, and a number of unproved axioms or postulates. We are interested in two-valued Boolean Algebra.

Axiomatic Definition of Boolean Algebra:

Boole in 1854 introduced the treatment of logic and Shannon in 1938 introduced the 2-valued Boolean Algebra (Switching Theory).

We define Boolean Algebra by using the following *Huntington's postulates* defined on a set of two elements *B* and two binary operators: +, .

1. Closure **a.** with respect to + **b.** with respect to .

2. Identity **a.** wrt + called 0 such that x + 0 = 0 + x = x
b. wrt . called 1 such that x.1 = 1.x = x

3. Commutative **a.** x + y = y + x **b.** x.y = y.x

4. Distributive **a.** x.(y+z) = x.y + x.z **b.** x + (y.z) = (x + y).(x + z)

5. For $x \in B$ there is $x' \in B$ such that $x + x' = 1, \; and \; x.x' = 0$

6. There exists at least two elements $x, y \in B$ *such that* $x \neq y$.

The two-valued Boolean algebra:

Defined on a set of two elements B = {0, 1} and two binary operators . and +

X	Y	X.Y
0	0	0
0	1	0
1	0	0
1	1	1

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

X	X'
0	1
1	0

According to the definition of the two binary operators given in the tables, they become the AND and the OR logic operators of the Binary Logic.

LAWS AND THEOREMS OF BOOLEAN ALGEBRA

Duality:

If binary operators and identity elements are interchanged, then the dual is obtained.

Examples:

1. $(X + Y + Z + \ldots)^D = XYZ\ldots$
2. $(XYZ\ldots)^D = X + Y + Z + \ldots$
3. $\{f(X_1,X_2,\ldots,X_n,0,1,+,.)\}^D = f(X_1,X_2,\ldots,X_n,1,0,.,+)$

Theorems and Postulates:

	Identity	Dual
Postulate 2	$X + 0 = X$	$X.1 = X$
Postulate 5	$X + X' = 1$	$X.X' = 0$
Theorem 1	$X + X = X$	$X.X = X$
Theorem 2	$X + 1 = 1$	$X.0 = 0$
Theorem 3	$(X')' = X$	
Cummutative law (Postulate 3)	$X + Y = Y + X$	$X.Y = Y.X$
Associative law (Theorem 4)	$(X + Y) + Z = X + (Y + Z) = X + Y + Z$	$(XY)Z = X(YZ) = XYZ$
Distributive law (Postulate 4)	$X(Y + Z) = XY + XZ$	$X + (YZ) = (X + Y)(X + Z)$
DeMorgan's Theorem (Theorem 5)	$(X + Y + Z + \ldots)' = X'Y'Z'\ldots$	$(XYZ\ldots)' = X'+Y'+Z'+\ldots$
Absorption Theorem (Theorem 6)	$X + X.Y = X$	$X(X + Y) = X$

Proof of some Theorems:

__Theorem 1__

$X + X = X$

Proof:

$= (X + X).1 = (X + X).(X + X')$

$= X + X.X' = X + 0 = X$

__Dual of Theorem 1__

$X.X = X$

Proof:

$= X.X + 0 = X.X + X.X'$

$= X(X + X') = X.1 = X$

__DeMorgan's Theorem__

$(X + Y)' = X'Y'$

Using the definition of the complement X and X' such that X + X' = 1 and X.X' = 0
(X + Y) and X'Y' are complements if (X + Y) + X'Y' = 1 and (X + Y).X'Y' = 0
Therefore, this proves the theorem.

X + Y + X'Y' = X + (Y + X')(Y + Y') = X + (Y + X').1 = X + Y + X' = 1 + Y = 1
(X + Y)X'Y' = XX'Y' + YX'Y' = 0.Y' + 0.X' = 0 + 0 = 0

Absorption Theorem

X + XY = X → X.1 + XY = X(1+Y) = X.1 = X

Proving Distributive Law by Truth Table

X(Y + Z) = XY + XZ

X	Y	Z	Y + Z	X(Y + Z)	X.Y	X.Z	X.Y + X.Z
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

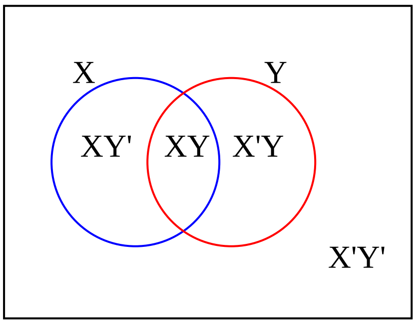
Operator Precedence:

For evaluating Boolean Expressions:
1. Parentheses 2. NOT 3. AND 4. OR

Venn Diagrams:

Venn diagrams may be used to prove Boolean algebra theorems and logic expressions.

2 Variable Venn Diagrams



X + XY = X