Number Systems

The material covered in this class will be as follows:

- Number systems.
- Binary number system.
- Conversion from any number system to decimal.
- Conversion from decimal to any number system.
- Relations between binary, octal, and hexadecimal numbers.
- Representation of hexadecimal digits in binary, octal, and decimal.

After finishing this class, you should be able to:

- Represent any number in any number system.
- Convert between the different number systems.
- Appreciate the ease of conversion between binary, octal and hexadecimal numbers.

Any number N can be represented in any base (r). As →

$$\ldots\ldots\ldots\ldots\ldots\; a_3\, a_2\, a_1\, a_0\, .\, a_{-1}\, a_{-2}\; \ldots\ldots\ldots\ldots\ldots$$

$$\therefore \quad N = \ldots\ldots + a_3 r^3 + a_2 r^2 + a_1 r + a_0 r^0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \ldots\ldots\ldots$$

r is the base (or the radix) of the number system.

Examples:

- Decimal number → $49.75_{10} \equiv 4\times10^1 + 9\times10^0 + 7\times10^{-1} + 5\times10^{-2}$
- Octal number → $236.05_8 \equiv 2\times8^2 + 3\times8^1 + 6\times8^0 + 0\times8^{-1} + 5\times8^{-2}$
- Binary number → $1101.01 = 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 + 0\times2^{-1} + 1\times2^{-2}$
- Hexadecimal number→ r = 16, Digits are 0 - 9,A,B,C,D,E & F

  $A6F.8 \equiv A\times16^2 + 6\times16^1 + F\times16^0 + 8\times16^{-1}$

Conversion From Base *r* To Decimal:

Use the positional notation used in the previous examples to convert from any base to decimal.

Examples:

- $236.05_8 \equiv 2\times8^2 + 3\times8^1 + 6\times8^0 + 0\times8^{-1} + 5\times8^{-2} \equiv 128 + 24 + 6 + (5/64) \equiv 158.078125_{10}$
- $A6F.8 \equiv A\times16^2 + 6\times16^1 + F\times16^0 + 8\times16^{-1} \equiv 10\times256 + 96 + 15 + 0.5 = 2671.5_{10}$
- $1101.01 \equiv 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 + 0\times2^{-1} + 1\times2^{-2} \equiv 8 + 4 + 1 + 0.25 \equiv 13.25_{10}$

Conversion From Decimal To Base r:

The number is separated into integer part and a fraction. The integer is successively divided by the new base, keeping track of the remainder, until the quotient is zero. The fraction is multiplied by the base, keeping track of the generated integer part, until the required accuracy is reached.

Example: Convert 41.25_{10} to binary.

	0	1	2	5	10	20	41(/2)
Remainder →	1	0	1	0	0	1	

$$\therefore 41_{10} \equiv 101001_2$$

0.25	0.5	.0					
.	0	1	← Integer part				

$$\therefore 0.25_{10} \equiv 0.01_2$$

$$\therefore\ \mathbf{41.25_{10} \equiv 101001.01_2}$$

The 16 Hexadecimal digits expressed in decimal, octal and binary are as follows:

Hex.	Decimal	Octal	Binary
0	00	00	0000
1	01	01	0001
2	02	02	0010
3	03	03	0011
4	04	04	0100
5	05	05	0101
6	06	06	0110
7	07	07	0111
8	08	10	1000
9	09	11	1001
A	10	12	1010
B	11	13	1011
C	12	14	1100
D	13	15	1101
E	14	16	1110
F	15	17	1111