The material covered in this class will be as follows:

⇒Canonical forms
- Sum of minterms
- Product of maxterms

⇒Conversion between the different forms

⇒Standard Forms
- Sum of products (SOP)
- Product of sums (POS)

⇒Two level implementation

⇒Other logic operations

Canonical and Standard Forms

Minterms and Maxterms for three binary values

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	1	0	xyz'	m_6	$x'+y'+z$	M_6
1	1	1	xyz	m_7	$x'+y'+z'$	M_7

A function is said to be in Canonical Form if it is expressed either in Sum of Minterms form or Product of Maxterms form.
Note: A term that is a Minterm or a Maxterm must contain all the input variables.

Functions of Three Variables

x	y	z	F_1	F_2	F_1'
0	0	0	0	0	1
0	0	1	1	0	0
0	1	0	0	0	1
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	1	0

$$F_1 = x'y'z + xy'z' + xyz$$
$$F_2 = x'yz + xy'z + xyz' + xyz$$

$$F_1 = m_1 + m_4 + m_7$$
$$= \sum m(1,4,7) = \sum(1,4,7)$$
$$F_2 = m_3 + m_5 + m_6 + m_7$$
$$= \sum(3,5,6,7)$$

$$F_1' = m_0 + m_2 + m_3 + m_5 + m_6$$
$$= \sum(0,2,3,5,6)$$
$$= x'y'z' + x'yz' + x'yz + xy'z + xyz'$$

$$F_1 = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$$
$$= M_0.M_2.M_3.M_5.M_6$$
$$= \prod(0,2,3,5,6)$$
$$Similarly, F_2 = \prod(0,1,2,4)$$

Example 1: Express $F = A + B'C$ in Sum of Minterms or SOP form.

A	**B**	**C**	**A+B'C**
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$F = A(B+B') + B'C = AB + AB' + B'C$$
$$= AB(C+C') + AB'(C+C') + (A+A')B'C$$
$$= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C$$
$$= ABC + ABC' + AB'C + AB'C' + A'B'C$$
$$= m7 + m6 + m5 + m4 + m1$$
$$= \sum(1,4,5,6,7)$$

Example 2: Express F = *xy* + *x*'z in Product of Maxterms or POS form.

x	**y**	**z**	**F**
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$F = (xy + x')(xy + z)$$
$$= (x + x')(y + x')(x + z)(y + z)$$
$$= (x'+y)(x + z)(y + z)$$
$$= (x'+y + zz')(x + yy'+z)(xx'+y + z)$$
$$= (x'+y + z)(x'+y + z')(x + y + z)(x + y'+z)(x + y + z)(x'+y + z)$$
$$= (x + y + z)(x + y'+z)(x'+y + z)(x'+y + z')$$
$$= M_0.M_2.M_4.M_5$$
$$= \prod(0,2,4,5)$$

Conversion between Canonical Forms

Example:

$$F(A, B, C) = \sum(1,4,5,6,7)$$
$$F'(A, B, C) = \sum(0,2,3)$$
$$\therefore F(A, B, C) = (m_0 + m_2 + m_3)'$$
$$= M_0.M_2.M_3 = \prod(0,2,3)$$

Standard Forms

A Standard form can be obtained upon simplifying a Sum of Minterms or Product of Maxterms expression. A term in a SOP or POS form need not contain all the variables of a function.

S O P → Sum Of Products

$$F_1 = y'+xy + x'yz'$$

P O S → Product Of Sums

$$F_2 = x(x'+z)(x'+y + z'+w)$$

A non-standard form can be changed to SOP form.

$$F3 = (AB + CD)(A'B'+C'D')$$
$$= A'B'CD + ABC'D'$$

Truth Tables of 16 Functions of Two Binary Variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Operator Symbol			.	/		/		$\oplus$	+	$\downarrow$	Θ	'	$\subset$	'	$\supset$	$\uparrow$	

Boolean Expressions for the 16 Functions of Two Variables

Boolean Functions	Operator Symbols	Name	Comments
$F_0 = 0$		Null	Binary Constant 0
$F_1 = xy$	$x.y$	AND	x and y
$F_2 = xy'$	x/y	Inhibition	x but not y
$F_3 = x$		Transfer	x
$F_4 = x'y$	y/x	Inhibition	y but not x
$F_5 = y$		Transfer	y
$F_6 = xy' + xy'$	$x \oplus y$	Exclusive-OR	x or y but not both
$F_7 = x + y$	$x + y$	OR	x or y
$F_8 = (x + y)'$	$x \downarrow y$	NOR	Not-OR
$F_9 = xy + x'y'$	$x \Theta y$	Equivalence	x equals y
$F_{10} = y'$	y'	Complement	Not y
$F_{11} = x + y'$	$x \subset y$	Implication	If y then x
$F_{12} = x'$	x'	Complement	Not x
$F_{13} = x' + y$	$x \supset y$	Implication	If x then y
$F_{14} = (xy)'$	$x \uparrow y$	NAND	Not-AND
$F_{15} = 1$		Identity	Binary constant 1