# A Hybrid, Self-organizing Controller for Multi-agent Motion Planning in a Stationary Environment

by

Ahmad A. Masoud
Suite #1, 94 Bagot St., Kingston, Ontario
Canada K7L-3E5
Email: masoud@king.igs.net

#### ABSTRACT

This work explores the construction of a decentralized traffic controller for more than one agents sharing a workspace with stationary forbidden regions. The suggested controller is required to be complete and have a computational effort that linearly increase with the number of agents. The controller is also required to be self-organizing; therefore, able to generate the needed information that is required to execute a successful action. In addition to the above, the controller is required to be open to enable any agent to join or leave the group without the remaining agents having to adjust the manner in which they function. To meet these requirements a new definition of decentralization is suggested. This definition, in effect, equates decentralization to self-organization. systems, as well as the potential field approach for control action synthesis are used to convert the above definition into a workable framework for the generation of decentralized control. Theoretical development, as well as simulation results are provided.

## 1. INTRODUCTION

The utilization of common resources by agents seeking different goals is almost always an unavoidable operational necessity. While resources is desirable, let alone necessary, it is nevertheless a potential source of problems. These problems arise when two or more agents attempt to utilize a resource in conflicting manners. To prevent conflict, a controller ( a planner) must be utilized to guarantee the proper multiplexing of resources among agents. There are two approaches for constructing these controllers: a centralized approach, and a decentralized approach. In a centralized approach a supervisory control simultaneously observes the states of all the agents. A search is then initiated in the Hyper Action Space of the group (the HAS houses the spaces of possible, admissible actions that individual agents can project) to generate and fan synchronized sequences of instructions (i.e. the solution) to all the agents in order for them to progressively modify their behavior and reach their respective goals. Due to the above, centralized controllers are:

1- Inflexible, in the sense that any change in the agents or the environment immediately translates into a change in the HAS which in turn necessitates repeating the search of the HAS for a new solution. 2- Has crippling complexity that usually grows exponentially with the number of agents.

3- Prone to problems in sensory communication and action synchronization.

Knowing that there are more than one interpretation of decentralization, the author chooses to consider a multi-agent system decentralized if

every agent in the group is independent from the others in sensory data acquisition, information processing, and action projection. It is not hard to see that these requirements, in effect, makes decentralization equivalent to self-organization. Unlike centralized top-down approaches, selforganization is a bottom-up approach [1] where the system designer is only required to supply the individual agents with basic "self-control" capabilities (Geno-type, or G-type). The overall control action that shapes the behavior of the whole group (Pheno-type, or P-type) evolves in space and time as a result of the interpretation of the G-type in the context of a particular environment [2]. It ought to be cautioned against the general misconception that the above is merely equivalent to breaking down the overall control scheme into control components that are executed by the agents in a parallel-distributed manner. In a decentralized control system it is impossible to a priori know the whole control scheme which in turn makes it impossible to carry out the above decomposition. Instead, the designer must focus on constructing the proper G-type that leads to the satisfaction of the goals, and the upholding of the constraints. In order to construct a proper G-type control:

- 1- Each agent must individually develop a control action to drive it towards its goal. Such a control need not take into consideration the control actions of the other agents.
- 2- Each agent must have the ability to generate a control that can resolve any conflict with other agents through bilateral interaction.

Some of the properties of decentralized control are:

- 1- There is no need to construct or search a HAS in order to generate a solution. For decentralized control, the solution emerges as a result of the agents interacting among themselves and with the environment.
- 2- No interagent communication is required.
- 3- Synchronous behavior is an emergent phenomenon that results from asynchronous interaction.
- 4- The controller's complexity linearly grows with the number of the agents.
- 5- Decentralized systems form "open" systems that enables any agent that is willing to interact to join or leave the group without the other agents having to adjust the manner in which they process information or project actions.

In this paper a controller is developed for the special case where "space" is the resource that is being shared among the agents in order for each to lay a safe path to its respective destination. The controlling process is required to provide the agents with the ability to simultaneously participate in reaching an accommodating arrangement that

enables all of them to safely reach their targets. The suggested controller is Hybrid in nature [3] (for an extensive survey of Hybrid systems see the paper by Labinaz et. al. [4]). This is a direct consequence of decentralization where the agents are allowed to pursue their subjectively constructed, continuous control (in the sequel this component is called the Purpose Field (PRF)) while dealing with the discrete event of a conflict when and where it arises (the continuous control component that is used to resolve conflicts is called the Conflict Resolving Field (CRF)). To the best of the author's knowledge, the suggested hybrid control differs from the available ones in the sense that no discrete automation is built on top (interfaced with) the continuous process. Instead, discrete events manifest their presence as "patterns" induced on a substrate of continuous action field. In order for the continuous control to possess the flexibility needed to realize complex pattern configurations, a potential field approach is adopted to generate the control field [5]. In this approach the control field is induced by operating on a potential field surface (a manifold) with the proper vector differential operator. The manifold is conditioned to become capable of inducing a control field by forcing its differential properties to conform to a proper partial differential governing relation. The manifold must also be subjected to the proper boundary conditions. The aim of the above is to generate a multi-agent control that is at least :

1- Complete (i.e. if a solution exist, the controller will find it; otherwise, the controller will provide an indicator that no solution exist).
2- Flexible (i.e. the addition or removal of agents from the group will not disturb the operation of the planner.

3- Computationally feasible to implement for a large size group.

This paper is organized as follows: in section 2 the problem is formulated, in section 3 the construction of the controller is outlined. Section 4 contains results, and section 5 conclusions. On going work is briefly outlined in section 6.

### 2. PROBLEM FORMULATION

Here, an agent is assumed to be a massless, M-D hyper sphere (Di) that has a radius di and a center xi  $(xi\in R^M)$ ,

$$Di(x) = \{x : |x-xi| \le di\}$$
  $i=1,...,L$  (1)

where L is the number of agents in the workspace. The workspace  $(\Omega)$  is assumed to contain stationary forbidden regions (0) with boundary  $\Gamma \in \mathbb{R}^N$   $(\Gamma = \partial 0, M \ge N)$ . Each agent has a target zone Ti which is also an M-D hyper sphere with a fixed center ci, where

$$Di(x) \subset Ti(x)$$
  $xi=ci$ . (2)

The target zones are assumed not to intersect each other or the obstacles,

$$Ti(x) \cap Tj(x) \equiv \phi, \quad i \neq j$$
 (3)  
 $Ti(x) \cap O \equiv \phi \quad i = 1,...L.$ 

The suggested controller assumes the form of the first order nonlinear dynamical system  $\,$ 

$$\begin{aligned} \mathbf{X} &= \mathbf{H}(\mathbf{X}, \Gamma, \mathbf{C}) = \mathbf{U}\mathbf{1}(\mathbf{X}, \Gamma, \mathbf{C}) + \mathbf{U}\mathbf{2}(\mathbf{X}, \Gamma, \mathbf{U}\mathbf{1}), \quad (4) \\ \text{where } \mathbf{X}\mathbf{C}\mathbf{R}^{ML}(\mathbf{X}^t = (\mathbf{x}\mathbf{1}^t \dots \mathbf{x}\mathbf{L}^t)^t), \mathbf{C}\mathbf{C}\mathbf{R}^{ML}(\mathbf{C}^t = (\mathbf{x}\mathbf{1}^t \dots \mathbf{x}\mathbf{L}^t)^t), \\ \text{also } \mathbf{H}: \mathbf{R}^{ML}\mathbf{x}\mathbf{R}^{ML}\mathbf{x}\mathbf{R}^N & \rightarrow \mathbf{R}^M. \quad \text{The U1 component of H} \end{aligned}$$

continuously act to drive the state X to C. On the other hand, U2 remains inactive till a constraint is about to be violated or a deadlock situation is about to form. Once such a situation transpire, U2 acts as a "behavior modifier", to U1 in order to prevent a constraint from being violated, and/or to resolve a deadlock situation. The activities of U2 dissipates once the conflict is resolved. As can be seen, U1 is independent of U2, while U2 is dependent, among other things, on U1. In the sequel this dependence is made implicit, and U2 is referred to as U2(X, $\Gamma$ ). In general the controller is required to:

$$\lim_{t \to \infty} xi \longrightarrow ci \qquad i=1,..,L \qquad (5)$$

$$Di(x) \cap Dj(x) \equiv \phi \qquad i\neq j, \quad \forall t,$$

$$Di(x) \cap O \equiv \phi \qquad i=1,..,L.$$

#### 3. THE CONTROLLER

In conformity with decentralized planning, only the self-controllers for each agent need to be synthesized. A self-controller for agent-i assists in laying a trajectory to ci by progressively modifying its state (xi) under the influence of uli and u2i, where the vectors u1i and u2i (both  $\in$  R) are the i'th elements in U1 and U2 respectively,

$$\dot{x}_i = u_{1i}(x_i, \Gamma, c_i) + u_{2i}(X, \Gamma). \tag{6}$$

The uni component is a vector field that encodes the agent's plan to reach ci. As for u2i it consists of two components

$$u2i(X,\Gamma) = u2oi(xi,\Gamma) + u2di(X),$$

wher

$$u2di(X) = \sum_{j=1}^{L} u2ij(xi,xj) \qquad i \neq j. \quad (7)$$

u2ij is a local vector field that encodes a plan for the corrective action released by agent-i to avoid a conflict with agent-j. As for u2o, this component provides extra guard against colliding with the static obstacles; its presence is convenient not necessary. In the following the construction of these components is outlined.

## 3.1 the PRF Component (uli)

It is required that  ${\tt uli}$  be constructed so that for the dynamical system

$$\dot{x}_i = u_{1i}(x_i, \Gamma, c_i),$$
 (8)  
 $\lim_{x \to \infty} x_i \to c_i,$ 

1

$$xi \cap \Gamma \equiv \phi$$
  $\forall t$ 

The vector field that uni symbolizes is induced by a vector differential operator that is acting on a potential field (Vi). This field has one unique stable equilibrium point at ci and a structure that is designed to steer xi away from  $\Gamma$ . Figure-1 show the block diagram describing the field generation process; more details may be found in [5]. Two examples of synthesis techniques are the Harmonic [6] and Biharmonic field approaches [7]. In the first the differential properties of the potential manifold are constrained by solving Laplace equation

$$\nabla^2 \text{Vi}(\mathbf{x}i) = 0\,, \tag{9}$$
 subject to the proper boundary conditions on  $\Gamma$  and ci. The PRF is generated as

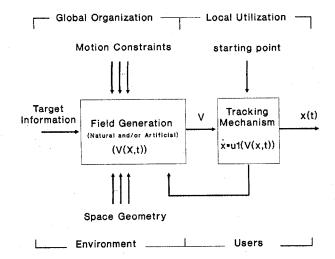


Figure-1: Basic Structure of reflexive potential-based Path planning techniques.

uii = 
$$-\nabla Vi$$
.

As for the second method, the potential manifold is constrained by solving

$$\nabla^4 \text{Vi}(\textbf{xi}) = 0, \tag{10}$$

subject to the proper conditions on  $\Gamma$  and ci. The PRF is generated as

 $uli = -\nabla \nabla^2 Vi.$  Figure-2 show an example of a PRF that is generated by the second method. Success was also encountered in generating PRF's for the moving target case [8].

#### 3.2 The CRF component (u2i)

This component is strictly localized to the vicinity of the robots and the obstacles. u2di is restricted to Di, where Di is a hyper sphere with a center xi and a radius di such that:

 $Di(x) \in Di(x) \in Ti(x)$  xi=ci. (11) On the other hand u2oi is restricted to a thin strip O that is surrounding O. Each subcomponent of u2di (u2ij) has two components, a radial one (u2ij), and a tangent one (u2ij)

$$u2ij = u2ij_r + u2ij_t$$
 (12)

The radial component is generated from the gradient flow of a spherically symmetric scalar potential field (vij) [9]

$$u2ij_r = \nabla vij(xi,xj), \qquad (13)$$

$$vij(xi,xj) = \begin{bmatrix} \xi(|xi-xj|) & |xi-xj| \le di \\ 0 & |xi-xj| \le di \end{bmatrix}$$

where  $\xi$  is a monotonically decreasing function with  $d\xi/d\rho=0$  for r≥di, and r is the radial distance from xj. As for u2ij, its flowlines run tangent to

Di. This component requires a vector potential (Aij) to generate [10,11],

$$u_{2ij} = \nabla x A_{ij}(x_i, x_j) \quad \nabla \cdot A_{ij} \equiv \phi, \quad (14)$$

where  $\nabla x$  is the curl operator and  $\nabla \cdot$  is the divergence operator. All the tangent components has to circulate the agents in the same direction (Figure-3). This is a necessary condition to prevent a deadlock situation from forming [12]. As for u2oi it is constructed from the gradient of a scalar potential field (Vo(xi, $\Gamma$ )) that is confined to 0

$$u_{20i} = \nabla V_0(x_i, \Gamma)$$
. (15)

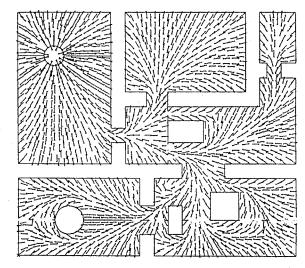


Figure-2: A PRF (biharmonic potential)

#### 3.3 The Overall Controller:

While the behavior of a single agents is generated by the system in equation-6, the group behavior is described by the hyper system

$$\begin{bmatrix} \dot{\mathbf{x}} \mathbf{1} \\ \dot{\mathbf{x}} \mathbf{L} \end{bmatrix} = \begin{bmatrix} \mathbf{u} \mathbf{1} \mathbf{1} (\mathbf{x} \mathbf{1}, \mathbf{c} \mathbf{1}, \mathbf{\Gamma}) \\ \mathbf{u} \mathbf{1} \mathbf{L} (\mathbf{x} \mathbf{L}, \mathbf{c} \mathbf{L}, \mathbf{\Gamma}) \end{bmatrix} + \begin{bmatrix} \mathbf{u} \mathbf{2} \mathbf{0} \mathbf{1} (\mathbf{x} \mathbf{1}, \mathbf{\Gamma}) \\ \mathbf{u} \mathbf{2} \mathbf{0} \mathbf{L} (\mathbf{x} \mathbf{L}, \mathbf{\Gamma}) \end{bmatrix} + \sum_{\substack{j=1 \\ \mathbf{u} \neq j}}^{\mathbf{L}} \begin{bmatrix} \mathbf{u} \mathbf{2} \mathbf{1} \mathbf{j} (\mathbf{x} \mathbf{1}, \mathbf{x} \mathbf{j}) \\ \mathbf{u} \mathbf{2} \mathbf{L} \mathbf{j} (\mathbf{x} \mathbf{L}, \mathbf{x} \mathbf{j}) \\ \mathbf{u} \mathbf{2} \mathbf{L} \mathbf{j} (\mathbf{x} \mathbf{L}, \mathbf{x} \mathbf{j}) \end{bmatrix}$$

$$(16)$$

This system evolves in dimensionality as agents join or leave the group; hence it satisfies the openness property. Also the structure of the control field keeps mutating under the influence of the G-type controllers and the environment to guarantee that the whole control field reach a sufficient level of in-formation that would enable the projection of a successful control action.

It can be shown that the above system can be made globally, asymptotically stable [12] if the following conditions are satisfied:

1- for 
$$\dot{x}_i = u_{1i}(x_i, c_i, \Gamma)$$
  $i=1,...,L$  (17)  
 $\lim_{r \to \infty} x_i \to c_i$ ,

 $d \qquad xi \cap \Gamma \equiv \phi ,$ 

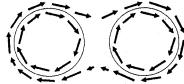


Figure-3a: Rotation in the same direction produces a singularity-free rotating global field.

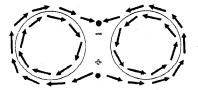


Figure-3b: Opposite circulation produces singularties in the circulating field.

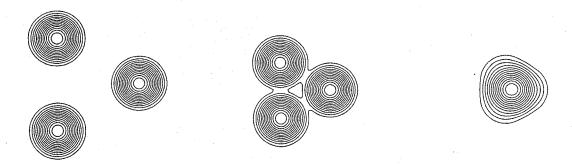


Figure-4 An example of the evolutionary nature of conflict resolving fields where the global fields keeps mutating under the influence of the environments and the local interaction rules to create a sophisticated-enough CRF that can resolve the conflict facing a group of robots.

- 2~  $Di(x) \cap Dj(x) \equiv \phi$ , and  $O \cap Di(x) \equiv \phi$   $i \neq j$ ,
- 3-  $\forall x \in \Omega$ ,  $\exists xk : x \in \{x : |x-xk| \le \rho\} \subset \Omega$ ,

where  $\rho$  is the sum of the diameters of the two largest agents. This condition simply requires the smallest passage in  $\Omega$  to be wide enough to allow the largest two agents to pass at the same time (i.e. a two-way street must have enough room for two cars to pass at the same time).

Since the uli's are used to satisfy condition 1, u2ijr's and u2oi's are used to satisfy condition 2, the only components left free for the designer to prevent deadlock are the circulating field components uijt's. These components are the key to enabling the agents to sense, reason, and act locally, yet achieve a purposive, coordinated global behavior. Figure-4 show the fields belonging to the agents in Figure-5a. As can be seen as the agents began to interact, the individual circulating fields amalgamated forming one global circulating field.

## 4. RESULTS

In this section several simulation experiments are conducted to explore the decentralized, self-organizing, nature of the controller. Each example is presented as a sequence of frames with each frame depicting the state of the robots at different instants of the solution.

In Figure-5a, three robots operating in an obstacle-free environment are initially positioned on the vertices of an equilateral triangle and are required to proceed to their respective destinations. The robots construct the, obviously, conflicting plans that require each to move along a straight line to its target. The agents independently execute their plans till a conflict is detected. Once this situation arise, the agents respond by reducing the system's degrees of freedom where they act as one rotating body to position themselves where they can proceed unimpeded along straight lines to their respective targets. Although the agents are acting individually in an asynchronous manner, the overall behavior of the group is synchronized (global synchronization without a synchronizer) and coordinated. Moreover, the group was able to augment the informational content of its behavior from the impoverished level that is initially contained in the individual PRF's

to a level that enables them to successfully reach their goals. In Figure-5b another interesting property of decentralized controllers is investigated. This property is related to the role individual agents play in assuring that conflict is evaded and purpose is satisfied. In a centralized approach the supervisory control determine these roles by decomposing the overall plan for reaching the target into subplans that are executable in a parallel- distributed manner by the agents. These roles remain fixed after the assignment. Failure of one agent to fulfill its obligation to the group by properly carrying out it role, most likely results in the failure of all the agents in reaching their targets. Decentralized systems behave in a different manner. The conflict resolving effort needed for all the agents to reach their targets has a lucid nature in the sense that it dynamically gets reallocated to the agents that are willing and able to participate in the conflict resolution effort. Here, an agent's role keeps adapting to the situation in a manner that would, to the best of the agent's ability, enable all the agents to safely reach their targets. This is clearly demonstrated in Figure-5b. Here a setting similar to the one in Figure-5a is used, except for D2 which refuses to participate in conflict resolution and instead follow the plan encoded by its PRF which requires it to move along a straight line towards its target. As can be seen the remaining agents adjusted their behavior to compensate for the intransigence of D2. In Figure-6 the evolutionary, self-organizing nature of the controller is clearly demonstrated. Figure-6 shows two groups of four robots each and moving in opposite directions along a confined path. They are blocking each other's way with the goal being the left group to go to the right and vice versa. It is interesting to notice that the groups collectively resolve the conflict by forming right and left lanes and confining the motion of the members of the groups along their respective lane. It ought to be mentioned that lane formation is a high-level, holistic, organizational activity that fundamentally differs from the local, simple, problem-solving activities which the individuals are equipped with. In Figure-7a importance of the circulating fields for conflict resolution is examined. Here, a group of eight agents, each required to hold its position, except for D8 where it is required to move to C8. No circulating fields were used. As can be seen,

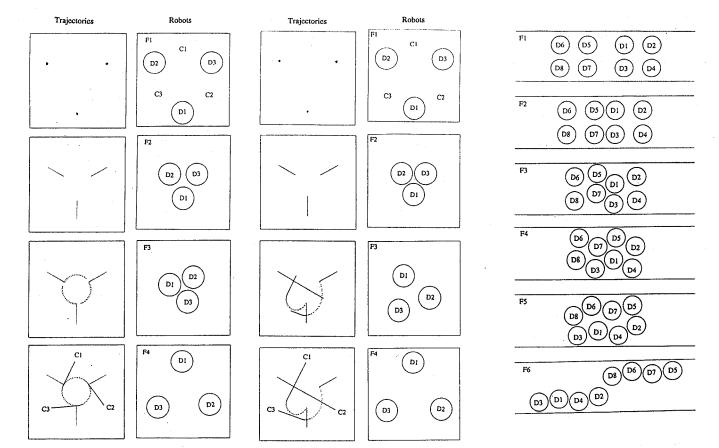


Figure-5a: All Robots
Functional.

Figure-5b: D2 malfunction.

Figure-6: Two groups blocking each other's way.

while D8 managed to pass the first group of agents, it got trapped in a deadlock situation when it attempted to pass the second group. In Figure-7b, the circulating fields were added. As can be seen D8 was able to successfully reach its target and the remaining agents maintain their original positions.

### 5. CONCLUSIONS

This work describes the construction of a complete, decentralized motion planner for agents sharing a workspace with stationary forbidden regions. Major emphasis throughout the work is placed on, what the author believe to be, a new definition of decentralization. This definition emphasizes the autonomy of the individual agents in terms of acquiring data, processing information, and actuating motion. Decentralized controllers that are based on the suggested definition are found to exhibit several attractive properties. May be the most important one is the ability of such controllers to on-line generate the additional information that is needed to execute a successful action. It is also noticed that the controller exhibits intelligent dispatching capabilities that enables it to redistribute the task of conflict evasion on the properly functioning agents. This is the major cause of the robust performance which the controller exhibits in case of sensor or actuator failure. The work also employs an idea that is of central importance for the controller to achieve the above capabilities. That is the ability to construct global useful activities through local interacting activities without the agents, necessarily, being aware of the generated global behavior. The local G-type control that is used by the individual agents does support such a mode of behavior and is the backbone for building effective decentralized controllers. The work also presents the potential field approach as a powerful tool for generating control fields that are particularly suited for constructing intelligent, decentralized controllers.

## 6. FUTURE WORK

While the suggested controller relief the agents from the need to know their dynamic environment (the other agents), it requires them to accurately know the location of the forbidden regions. Ongoing work [13,14] by the author is geared towards removing this requirement. In addition to not having to know all the agents sharing the workspace, the modified G-type controller does not need to a priori know the location of the forbidden regions. A PRF for a modified G-type controller is constructed so that for the system

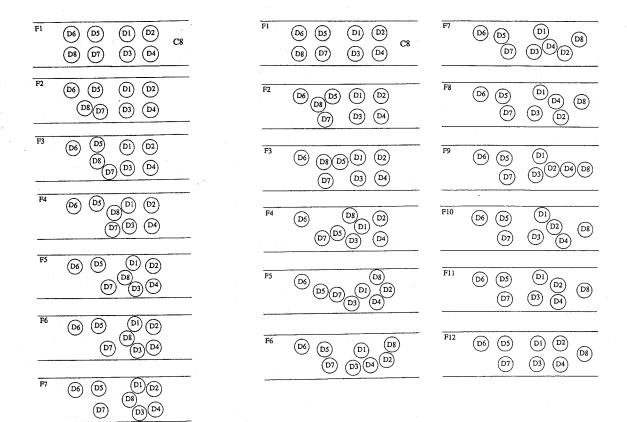


Figure-7a: D1-D7 hold position, D8 move to C8, No circulating fields.

and

Figure-7b: D1-D7 hold position, D8 move to C8, Circulating fields added.

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 \dot{\mathbf{x}} = \mathbf{f}_{\mathbf{j}}(\mathbf{x}_{\mathbf{i}}, \mathbf{c}_{\mathbf{i}}, \mathbf{Q}_{\mathbf{m}}, \mathbf{f}_{\mathbf{j}-1}) \qquad \mathbf{x}_{\mathbf{i}}(0) = \mathbf{x}_{\mathbf{i}}_{\mathbf{0}} \qquad (19) 
 \begin{aligned}
 & \mathbf{f}_{\mathbf{0}} = \mathbf{f}_{\mathbf{i}}(\mathbf{x}_{\mathbf{i}}, \mathbf{c}_{\mathbf{i}}, \mathbf{\Gamma}) \\
 & \mathbf{f}_{\mathbf{0}} = \mathbf{f}_{\mathbf{i}}(\mathbf{x}_{\mathbf{i}}, \mathbf{c}_{\mathbf{i}}, \mathbf{\Gamma}) \\
 & \mathbf{f}_{\mathbf{0}} = \mathbf{f}_{\mathbf{i}}(\mathbf{x}_{\mathbf{i}}, \mathbf{c}_{\mathbf{i}}, \mathbf{\Gamma}) \\
 & \mathbf{f}_{\mathbf{0}} = \mathbf{f}_{\mathbf{0}}(\mathbf{x}_{\mathbf{i}}, \mathbf{c}_{\mathbf{i}}, \mathbf{C}_{
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where, k is a finite positive integer,  $Q \in \mathbb{Z}^2$ , and fy maps the hybrid space to the agent's continuous M-D action space  $\mathbb{R}^M \times \mathbb{R}^2 \times \mathbb{R}^M \to \mathbb{R}^M$ . All what is needed to construct the PRF's is an impoverished estimate of  $\Gamma$  ( $\Gamma$ ).  $\Gamma$  can even be the empty set  $\phi$  ( $\Gamma \subseteq \Gamma \subseteq \Phi$ ). Figure-8a,b,c show the first, second, and third attempts by the agent at reaching its target in a totally unknown environment. As can be seen from

zero knowledge of the environment, the agent was able to :

1- Successfully reach the target at the end of each attempt.  $\ensuremath{\,^{\sim}}$ 

2- Improve its performance till it optimized it by the third attempt.

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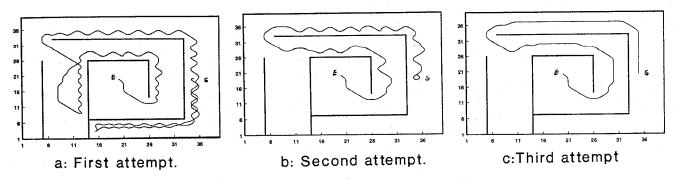


Figure-8: An agent navigating an unknown environment.

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