Correspondence

Comments on "Frequency Determination of Weak Sinusoids Buried in Noise"[1]

AHMAD A.A. MASOUD

In the above paper,[1] the author wishes to make the following remarks. The paper did not give any theoretical explanation for the excellent performance of the proposed scheme. The smoother, integrator, and extremum counter are incapable on their own to produce at extremely low signal-to-noise ratio (SNR) the high performance reported in the paper. There are also many redundant stages in the proposed method which add unnecessary cost, complexity, reduce the accuracy, and even make the method practically difficult to implement.

For example, the presence of the smoother and integrator are unnecessary in both the presence of the window and at such a high number of samples per cycle. This can be theoretically proven, but for the sake of brevity, only simulation results which are carried out on the VAX 11/780 are given here in Fig. 1. The data consist for four cycles of a sinusoid with 256 samples per cycle and a SNR of -13 dB, using only a 50-stage triangular window. As can be seen the output is highly smoothed without the need for the previously mentioned stages.

It is known that the integration of a white Gaussian noise process yields a bias with a trend that is linearly increasing with time. Since a huge number of samples are used, the bias will attain a high value which will most probably cause the electronics circuitry to malfunction. This can easily be seen by considering an arbitrary small bias ϵ associated with each sample. Since L cycles are processed (L is usually large) with each cycle containing Ms samples (proposed Ms is very large) and since these samples will also pass through a window of length K, the bias will most probably ultimately reach the value

$$B \approx \epsilon L M s K$$

which is definitely very large even if ϵ is very small.

In contrast, if only the window is used, the bias will not exceed:

$$B \approx \epsilon K.$$

In the case of the integrator the bias continues to grow with time, making its removal difficult to accomplish, while in the case of the window, the bias will grow for the first K samples and then will settle somewhere near the value given above. The shape of the bias in both cases is given in Fig. 2.

Another reason that makes the proposed method extremely difficult to realize is the huge demand on the sampling rate.

For example, a sinusoid occurring in the kilohertz range requires a sampler in the megahertz range (if such a large number of samples is to be taken). Since it is known that most commercial samplers are incapable of such performance, and since no realization of the processor is given, the ability to implement the processor and, therefore, its practical significance are questioned.

It can be proven that the ability of the sample reduction to smooth the signal is a function of the frequency of the sinusoid, the sampling rate, and the length of the window. It can be seen from Fig. 1 that at such a high rate and with the length of the window, the output is so smooth that the sample reduction will be of no use in obtaining a smoother signal. In fact the use of sample reduction under the given conditions reduces the accuracy of the measurement. If a simpler, less costly method than the extremum counter is used for frequency measurements, such as counting the number of samples per cycle (the cycle can be easily detected from zero crossings), one can obtain an estimate of the period lying between

$$T(1 - 1/Ms) \leq \hat{T} \leq T$$

where T is the true period and $\hat{T}$ is the estimated period. It is easy to see that if Ms is large, the estimated period will lie close to the actual one. Even more accuracy can be obtained if the averaging takes place over L cycles, where the estimate will lie within

$$T(1 - 1/(L\,Ms)) \leq \hat{T} \leq T$$

which allows the method to discriminate between very close sinusoids and obtain very accurate measurements.

The conclusion drawn from the previous discussion is that in its present form the processor will most probably not work. Many redundant stages have been introduced, which will add more cost, cause deterioration in the performance, and even make the method practically difficult to implement. No explanation is given on how to force the proposed stages to work in coordination.

Reply[2] *by Abid M. Elabdalla and Ahmad I. Abu-El-Haija*[3]

Masoud states that "the smoother, integrator, and extremum counter are incapable on their own to produce at extremely low signal-to-noise ratio (SNR) the high performance reported in the paper," and then he states that "there are many redundant stages in the proposed method $\cdots$ ". These two statements obviously contradict each other since the first one questions the performance of the proposed method, while the other one states that many stages are redundant (hence, can be removed without affecting performance). He then claims through an example relating to his Fig. 1, that accurate measurement can be obtained without the smoother and integrator and, therefore, he shows that part of the steps used in the original algorithm (weighted averaging window) give good results. This is not consistent with his claim that the proposed steps are incapable of producing good performance.

Masoud claims that the frequency can be measured by just passing the signal through a triangular window and by counting zero crossings. Instead of giving a proof, he presents an example with a sinusoid having a SNR of -13 dB. The authors have tried smoothing the above sinusoid (at -13 dB and many other SNR's) using just a triangular window, and have found that the results reported by Masoud are totally incorrect. In particular, there is *no way* to obtain Fig. 1(c) by applying a triangular window to the signal given in Fig. 1(b).

Masoud states in the above paper,[1] that "the integration of a white Gaussian noise process yields a bias with a trend that is linearly increasing with time." This is in general not true, and it depends upon the noise statistics. Since the mean of the noise process

Manuscript received January 19, 1988; revised January 30, 1988.
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IEEE Log Number 8822220.

[1] A. I. Abu-El-Haija and A. M. Elabdalla, *IEEE Trans. Instrum. Meas.*, vol. IM-36, pp. 971–974, Dec. 1987.

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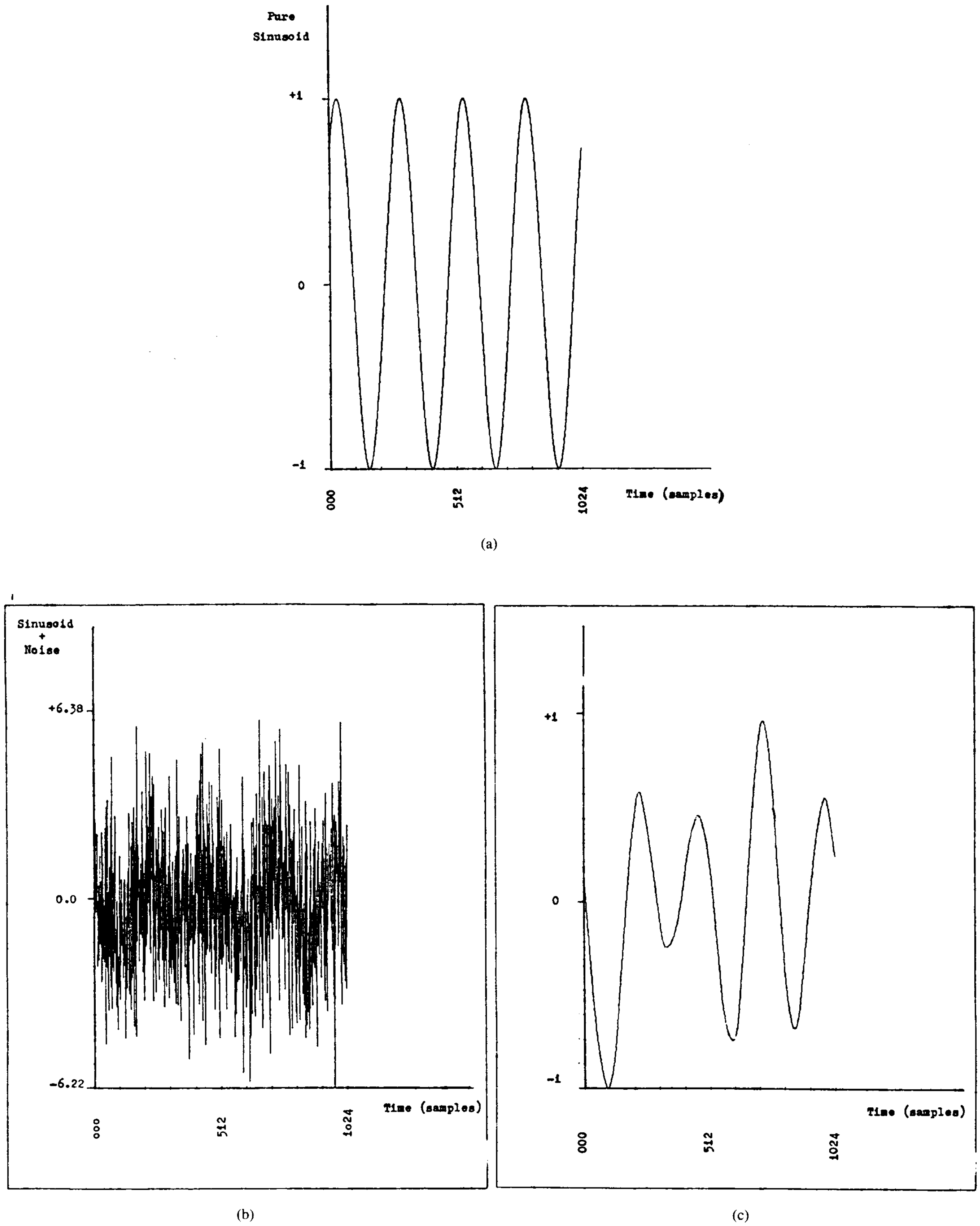
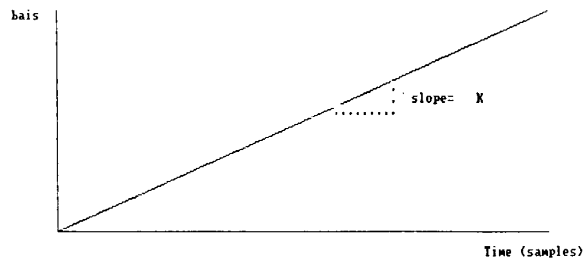
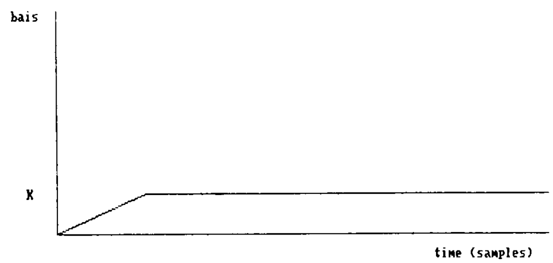


Fig. 1. (a) The clean sinusoid: no. of samples = 1024, no. of cycles = 4. (b) The noisy sinusoid noise = AWGN: no. of samples = 1024, no. of cycles = 4, SNR = −13 dB. (c) Recovered sinusoid: no. of samples = 1024, no. of cycles = 4, $K = 50$.



(a)



(b)

Fig. 2. (a) Bias trend as a function of time when using (a) the smoother, integrator, and window; and (b) only window.

is computed by dividing its integral (i.e., the sum of the noise samples) in a finite time interval over this interval, integrating a zero-mean white Gaussian noise will give a result close to zero. On the other hand, integrating a noise process with a nonzero mean gives an output that is linearly increasing (or decreasing) with time.

Masoud states that the method is extremely difficult to realize because of the high sampling rate required. This is irrelevant because it is stated in the abstract of the paper[1] that this method is attractive for sinusoidal signals which *can be* sampled at a high rate relative to the Nyquist limit. However, if sampling in the megahertz range is required, commercial samplers are available on the market (e.g., from LeCroy), and the algorithm can certainly be realized.

The suggestions of Masoud to measure the frequency of a sinusoid by counting the number of samples between two or more consecutive zeros are known and published in the literature (see reference [2] of the original paper[1]).