ROBOT NAVIGATION USING THE VECTOR POTENTIAL APPROACH

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ABSTRACT

In this paper the Artificial Vector Potential is used for constructing a navigation control that can drive an arm manipulator to a target set while avoiding undesired regions in the workspace. It is shown that a Vector Potential Field can better navigate a robot than a Scalar Potential Field. The strategy that is suggested for constructing the navigation control is very flexible in the sense that it allows the addition or deletion of obstacles with minimal adjustment to the control. An efficient technique to generate the navigation field in the N-D space is proposed. Simulation results are also provided.

I. INTRODUCTION

The Artificial potential Field approach was simultaneously proposed by Khatib [1], Miyazaki and Arimoto [2], as well as Pavlov and Voronin [3]. It gained popularity because of its ability to transform path planning into a task that can be performed by the low level controller. This meant a great reduction in computational complexity. In general, a planning technique navigates the system

$$D(q)\ddot{q} + c(q,\dot{q}) + g(q) = u \quad (1)$$

where q is the position in the natural coordinates of the robot, D is the inertial matrix which is symmetric and positive definite, c is a vector containing the coriolos and centripetal torques, g is a vector containing the gravity torques, and u is the torque needed to drive the position to a target set {T}, while avoiding the obstacles {A}.

To perform such task, Kahtib augmented the system potential with an artificial potential (V_{art}) that is the sum of an attractive potential field surrounding the target (V_a), and a repulsive potential field (V_r) fencing the obstacles. u is taken as the gradient flow of that field

$$u = \nabla V_a + \nabla V_r. \quad (2)$$

This approach faced two major problems. The first has to do with the formation of local equilibrium zones that can trap the robot before reaching its target. The second problem is caused by using an inverse quadratic function to generate V_r, which require an infinite control effort. Khosla and Volpe [4] alleviated these problems by generating the potential using superquadrics formulation; their approach was found to be effective in an uncluttered environment. Warren [5] reduced the minima problem by first choosing a trial path, then modifying it under the influence of the potential field. Krogh [6] formulated the problem as one of transferring the state of a dynamical system from an initial to a final state. He introduced the idea of the Generalized Potential Field that is a function of both position and speed, and further derived the necessary conditions for convergence [7]. However, no formal way was supplied to generate the control action. Tilove [8] compared the classical potential approach with the generalized one, and found that the generalized potential produces a smoother path. Koditshchek and Rimon [9,10,11] constructed a potential function that encodes the desired behavior in the sense that it has a global minima at the target and takes lower value away from the obstacle than at its boundary. The gradient flow was shown to drive a robot to its target with zero velocity, while avoiding obstacle along the way [12]. The mechanical system for which the field is derived uses a Lagrangian that is specified by a

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kinetic energy cost functional where the gravity term is not present. Such a term or residuals of it will cause a steady state error and could lead to collision with the obstacles. Several techniques utilizing, Harmonic and Newtonian Potential Fields [13]-[16] were suggested for navigation. Unfortunately, the gradient flow from these fields only marks an obstacle free path to the target, and can't be used to provide the control input. In a recent paper, Utkin et al [17] supplied an interesting technique that enables the use of the field from these methods for steering the robot. They used sliding mode theory to make the system trajectory coincide with those of the gradient flow of the navigating potential field.

The objective of this paper is to construct a realizable, smooth, bounded control that would prevent a robot from entering undesired regions and guarantee convergence to a target. To achieve this goal,we found it necessary to use Vector Potential Fields (VPF) which are shown to have a better ability for navigation than Scalar Potential Fields (SPF). This paper is organized as follows. Section II provide a qualitative evaluation of SPFs and points out the advantage of using VPFs. Section III provide a strategy for constructing the navigation control. Sections IV, and V, tackle the implementation of VPF's. Section VI contains some examples, and conclusions are placed in section VII.

II. A Qualitative Evaluation of SPF's

To the best of our knowledge, current potential-based planning techniques construct the navigation control from the gradient flow of a SPF

$$u = -\nabla V \qquad (3)$$

This flow is known to be normal to the associated family of equipotential surfaces, along which ∇V vanishes, making it impossible to direct motion along these contours. This can also be deduced form the vector identity

$$\nabla \times \nabla V = 0 \qquad (4)$$

where the curl operator ($\nabla\times$) detect a vector field circulating the closed equipotential contours. As a result total controllability in the sense of being able to arbitrarily direct motion in the space can not be achieved. Moreover, with the loss of controllability over the tangent subspace which spans N-1 degrees of freedom of an N-D space, controllability is expected to deteriorate with an increase in dimensionality. To remove this deficiency and to synthesize a complete set of force fields that are able to freely project a force in any direction in the space, an underlying VPF ($\mathbf{A}$) can be used to generate the navigation control

$$u = -(\nabla V + \nabla \times \mathbf{A}) \qquad \nabla \cdot \mathbf{A} = 0. \qquad (5)$$

$\mathbf{A}$ is made to have a purely circulating nature by setting its divergence to zero. Figure-1 qualitatively illustrates the behavior of flow fields from both a SPF and a VPF.

III. The Proposed Navigation Strategy

A unifying factor among the diverse potential-based planning techniques is the requirement that the navigation control tie the internal environment of the robot represented by its dynamics to the external environment represented by the workspace and the obstacles in it. Such a function necessitates that the control accommodate the needs of both environments by being sufficiently smooth and bounded to meet the limitations on the dynamics of the system, and flexible in the sense that the effort needed to adjust the control is proportional to the changes in the workspace. One way to achieve such a goal is to divide the navigation control into two parts

$$u = u_g + u_l \qquad (6)$$

where u_g properly controls the robot in the obstacle-free space, and drives the motion to the desired target. u_g can be a simple Proportional-Derivative (PD) control [18]. u_l is called the steering control, and is strictly localized to the vicinity of the obstacles such that the controls corresponding to different obstacles do not intersect. u_l is designed in the local coordinates of the obstacles then transformed to the natural coordinates of the robot. It functions to smoothly deflect the motion away from the obstacles in a manner that prevent collision and allow u_g to sweep the robot to the target (Figure-2). By totally shifting the task of managing the obstacles to u_l (u_g is independent of u_l; while, u_l is dependent on u_g and the geometry of the corresponding obstacle) great flexibility is achieved in the sense that the addition or deletion of an obstacle affects only the particular corresponding steering control.

u_l is divided into two functionally distinct components. The first component is radial to the obstacle surface. It acts to prevent the robot from penetrating the region occupied by the obstacle (u_{ln}). It is called the Penetration Prevention Control (PPC). The function of the other component is to align the robot on the right part of the obstacle surface to allow u_g to sweep the robot to the target (u_{lt}). This component acts tangentially to the obstacle surface, and is called the Local Alignment Control (LAC). For a smooth diversion of the motion away from the obstacle, the steering control is made to occupy a surrounding finite region ($A\delta$). The proper control on the inner boundary of $A\delta$ (Γ) is derived in the following sections, while the control on the outer boundary ($\Gamma\delta$) is set to zero. The generation of u_l inside $A\delta$ is treated as a Vector Boundary Value Problem which, in this paper, is reduced to solving four scalar boundary value problems. The main focus of the rest of this paper is on designing u_l.

IV. The Boundary Steering Control

In this section the control at the inner boundary of $A\delta$ ($u\Gamma_l$) is derived in terms of the normal ($e_n(\Gamma)$), and tangent ($e_t(\Gamma)$) to Γ. Similar to u_l, $u\Gamma_l$ has two components, the boundary penetration prevention control (BPPC), and the boundary local alignment control (BLAC).

The BPPC ($u\Gamma_{ln}$)

Here, it is shown that the control

$$u\Gamma_{ln}(q,\dot q) = \alpha_l(q,\dot q)e_n(\Gamma) \qquad (7)$$

can prevent the robot from entering $\Gamma\delta_d$ ($\partial A\delta_d$). α_l is a scalar positive function

$$\alpha_l(q,\dot q) = \alpha_l'(\dot q,q) + \alpha_l''(\dot q_n,q_n). \quad (8)$$

where $\alpha_l''(\dot q_n,q_n)=C_{\dot q}|\dot q_n|\cdot\alpha_{ql}''(q_n)$, (9)

and q_n is the radial distance form Γ to the robot position

$$q_n = q^t\cdot e_n(\Gamma) \qquad (10)$$

For convenience the system equation is placed in the following form

$$\dot q = \int f(q,\dot q)+D^{-1}(q)u_l \, dt \qquad (11)$$

where $f(q,\dot q) = -D^{-1}(q)[c(q,\dot q)-g(q)+u_g]$.
Let $G(q_n)$ be a measure of that distance

$$G(q_n) = \frac{1}{2}q_n^2 \qquad (12)$$

To guarantee that the robot will not enter Γ, the time derivative of G must always be positive. It can be shown that this condition reduces to guaranteeing that

$$\int_{t^-}^{t} e_n^t(\Gamma)[f(q,\dot q)+\alpha_l'(q,q)D^{-1}(q)e_n(\Gamma)]dt\geq 0$$
$$\int_{t^-}^{t} \alpha_l''(q,q)e_n^t(\Gamma)D^{-1}(q)e_n(\Gamma)dt \geq |\dot q_n^-| \qquad (13)$$

where t^- is the time at which the robot enters $A\delta$, and $\dot q^-$ is the speed at which it enters that region. It can further be shown that the following conditions can satisfy (12)

$$\operatorname*{Inf}_{q,\dot q} \alpha_l'(q,\dot q) \geq \frac{\operatorname*{Sup}_{q,\dot q}|e_n^t(\Gamma)f(q,\dot q)|}{\min_q[e_n^t(\Gamma)D^{-1}(q)e_n(\Gamma)]}$$

and $\qquad \alpha_{ql}''(q_n) > 0 \qquad (14)$

$$C_{\dot q}\geq\frac{|\dot q_n^-|}{\delta_d \min_{q_n}(\alpha_{ql}''(q_n))\min_q[e_n^t(\Gamma)D^{-1}(q)e_n(\Gamma)]}$$

where δ_d is the minimum width of $A\delta_d$.

The BLAC ($u\Gamma_{lt}$)

The first step in designing the BLAC is to partition Γ into two parts Γ_T, and

Γ_o, such that when $u_1 = u\Gamma_{1n}$, q satisfies the following: at $t=t_1$

$$q(t_1) \in \Gamma_T \quad \text{then} \quad \lim_{t\to\infty} q(t) \in T$$

and

$$q(t_1) \in \Gamma_o \quad \text{then} \quad \lim_{t\to\infty} q(t) \in \Gamma_o \tag{15}$$

The second step is to clamp the motion to the obstacle in the Γ_o regions. This may be accomplished by the control

$$u\Gamma^c_{1n} = u\Gamma_{1n} - \eta(q)u\Gamma_{1n}(q + \delta\cdot e_n(\Gamma), \dot{q}) \tag{16}$$

where δ is arbitrarily small and positive and $\eta(q)$ has the form

$$\eta(q) = \begin{cases} 1 & q \in \Gamma_o \\ 0 & q \in \Gamma_T \end{cases}$$

The final step is to construct the BLAC on Γ to drive the motion toward Γ_T. First, we need to define the vector ξ

$$\xi = [\ \xi_1 .. \ \xi_{N-1}\ ] \qquad \xi \in Q$$

where Γ is the image of Q, and ξ is a parametric representation of Γ. The control is constructed by first choosing a point $\xi_r \in \Gamma_T$. Ensuring convergence to ξ_r ensures that the robot will enter Γ_T. Using the passivity property of robotics manipulators [18], it can be shown that the system forces that determines convergence to a point in the position space of a manipulator has the form

$$f_q(q) = \lim_{\dot{q}\to 0} f(q,\dot{q})$$

Therefore, the proposed control can be selected as follows

$$u\Gamma_{1t}(\xi) = -\alpha_2(q)\cdot\frac{\xi - \xi_r}{\|\xi - \xi_r\|} \tag{17}$$

In the following, it is shown that the above control can ensure convergence to ξ_r. The system equation in Q (on Γ) has the form

$$\dot{\xi} = f_t(q(\xi)) + D^{-1}(q(\xi))u\Gamma_{1t}(\xi) \tag{18}$$

Where $f_t(q(\xi)) = e_t^t(q(\xi))f_q(q(\xi))$. To prove convergence, the following Liapunov function is constructed

$$\Xi(\xi) = (1/2)\cdot(\xi - \xi_r)^t(\xi - \xi_r) \tag{19}$$

We need to show that the control can make the time derivative of this function negative definite

$$\begin{aligned}\dot{\Xi} &= (\xi-\xi_r)^t\,\dot{\xi} \\ &= (\xi-\xi_r)^t(f_{qt}(q(\xi)) + D^{-1}(q(\xi))u\Gamma_{1t}(\xi)) \\ &= -\alpha_2(q(\xi))\frac{(\xi-\xi_r)^tD^{-1}(q(\xi))(\xi-\xi_r)}{\|\xi-\xi_r\|} \\ &\quad + (\xi-\xi_r)^tf_{qt}(q(\xi))\end{aligned} \tag{20}$$

Let us consider the radial coordinates $e_\rho(\xi)$ in Q, and the tangent to it $e_\tau(\xi)$

$$e_\rho(\xi) = \frac{\xi - \xi_r}{\|\xi - \xi_r\|} \tag{21}$$

Let f_{qt} be represented in terms of these components

$$f_{qt} = \eta_1(q(\xi))e_\rho(\xi) + \eta_2(q(\xi))e_\tau(\xi)$$

Substituting the above term in $\dot{\Xi}$ we get

$$\begin{aligned}\dot{\Xi} &= -\alpha_2(q(\xi))\frac{(\xi-\xi_r)^tD^{-1}(q(\xi))(\xi-\xi_r)}{\|\xi-\xi_r\|} \\ &\quad + \|\xi-\xi_r\|e_\rho^t(\xi)(\eta_1\ e_\rho(\xi) + \eta_2\ e_\tau(\xi)) \\ &= -\alpha_2(q(\xi))\frac{(\xi-\xi_r)^tD^{-1}(q(\xi))(\xi-\xi_r)}{\|\xi-\xi_r\|} \\ &\quad + \eta_1(q(\xi))\cdot\|\xi-\xi_r\|\end{aligned} \tag{22}$$

It can be seen that an α_2 can be chosen to guarantee that $\dot{\Xi}$ is negative definite This ensures that the motion is driven to Γ_T where it is swept to the target.

V. The Steering Control

To gradually decelerate the robot till it is stopped before colliding with the obstacle, and to smoothly deflect the motion toward Γ_T, the steering control is required to occupy a finite region around A ($A\delta$) where its strength is set to zero at $\Gamma\delta$, then gradually builds up to full strength at Γ. The control inside $A\delta$ is generated from its values at the boundaries. Other constraints may be added to control its strength. Such a task is carried out by solving a Vector Boundary Value Problem

(VBVP). Solving a VBVP is, in general, a difficult task. In [19] Morse and Feshbach solved a number of VBVP in the 2-D and 3-D case for a limited class of natural coordinates. Since the interest is in the N-D case, a different approach is sought. The PPC and LAC components are constructed separately in a manner that guarantees orthogonality. Both components have the form

$$u_{\ }(q,\dot{q}) = M_n(q,\dot{q})\cdot\mathbf{Q}_n(q)$$
$$u_{1t}(q,\dot{q}) = M_t(q,\dot{q})\cdot\mathbf{Q}_t(q) \qquad (23)$$

Where $\mathbf{Q}$ is the basis vector phase field, and M is a scalar magnitude field that modulates the strength of $\mathbf{Q}$. Each field is generated by solving a Scalar BVP (SBVP). Collectively, these SBVP's are equivalent to solving a VBVP.

The PPC

To generate $\mathbf{Q}_n$, the following SBVP is solved (Figure-3)

$$\nabla^2 V_{1n}(q) = 0 \quad \text{subject to}$$
$$V_{1n}\big|_{\Gamma} = C, \text{ and } V_{1n}\big|_{\Gamma\delta} = 0 \qquad C > 0$$
$$\mathbf{Q}_n(q) = \frac{\nabla V_{1n}(q)}{\|\nabla V_{1n}(q)\|} \qquad (24)$$

M_n is obtained by solving :

$$\nabla^2 V_{2n}(q) = 0 \quad \text{subject to}$$
$$V_{2n}\big|_{\Gamma,\Gamma\delta d} = 1, \quad \text{and} \quad V_{2n}\big|_{\Gamma\delta} = 0$$
$$M_n(q,\dot{q}) = \begin{cases} \alpha_1(q,\dot{q})\cdot V_{2n}(q) & q\in A\delta d \\ \alpha_1(\Gamma\delta d,\dot{q})\cdot V_{2n}(q) & q\in A\delta \end{cases} \qquad (25)$$

If the PPC is to, as well, clamp the robot to Γ_o, an additional boundary condition is added

$$V_{2n}\big|_{\Gamma_o'} = -1$$

where Γ_o' is the portion of Γ' that corresponds to Γ_o, and Γ' is an equipotential surface inside $A\delta$ chosen equal to C/2.

The LAC

The following steps are use for constructing the LAC component (Figure-4)

1- Choose ξ_r inside Γ_T, and ξ_n inside Γ_o

2- Construct the following lines on the surface of the obstacle

$$\rho_r = \{q : \dot{q}(t) = -\mathbf{Q}_n(q),\ 0\le t\le\tau,\ q(0)=\xi_r,\ q(\tau)\in\Gamma\delta\}$$
$$\rho_n = \{q : \dot{q}(t) = -\mathbf{Q}_n(q),\ 0\le t\le\tau,\ q(0)=\xi_n,\ q(\tau)\in\Gamma\delta\} \qquad (26)$$

3- Solve the following BVP

$$\nabla^2 V_{1t}(q) = 0 \quad \text{subject to}$$
$$V_{1t}\big|_{\rho r} = 0, \text{ and } V_{1t}\big|_{\rho n} = C \qquad C > 0$$
$$\partial V_{1t}/\partial n = 0 \quad \text{at } \Gamma, \Gamma', \text{ and } \Gamma\delta \qquad (27)$$
$$\mathbf{Q}_t(q) = \frac{\nabla V_{1t}(q)}{\|\nabla V_{1t}(q)\|}$$

4- Compute the magnitude field by solving the following BVP

$$\nabla^2 V_{2t}(q) = 0 \quad \text{subject to}$$
$$V_{2t}\big|_{\Gamma,\Gamma'} = 1, \quad \text{and} \quad V_{2t}\big|_{\Gamma\delta} = 0$$
$$M_t(q,\dot{q}) = \begin{cases} \alpha_2(q,\dot{q})\cdot V_{2t}(q) & \text{inside } \Gamma' \\ \alpha_2(\Gamma\delta d,\dot{q})\cdot V_{2t}(q) & \text{outside } \Gamma' \end{cases} \qquad (28)$$

Existence and uniqueness of the solution of the BVP were proven [19].It can also be shown that $\mathbf{Q}_t$ is orthogonal to $\mathbf{Q}_n$.

The following formulae can be used to generate the solution of the Laplace SBVP from its boundary conditions [20]

$$V(r) = \frac{1}{2\pi}\oint_S \left(\frac{\partial V(q)}{\partial n}\cdot G(r,q) - V(q)\cdot\frac{\partial G(r,q)}{\partial n}\right)dS$$
$$\frac{\partial V(r)}{\partial X_i(r)} = \frac{1}{2\pi}\oint_S \left(\frac{\partial V(q)}{\partial n}\cdot\frac{\partial G(r,q)}{\partial X_i(r)} - V(q)\cdot\frac{\partial}{\partial X_i(r)}\frac{\partial G(r,q)}{\partial n}\right)dS \qquad (29)$$

where S is the closed surface surrounding $A\delta$, r is a point inside $A\delta$, q is confined to the boundary, and G(r,q) is the fundamental solution of the Laplace BVP (Green's function) in the specified dimension.

VI. RESULTS

Simulation is done for a polar manipulator with the following system equation

$$\begin{bmatrix} Mr^2 & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{r} \end{bmatrix} + \begin{bmatrix} 2Mr\dot{r}\dot{\theta} \\ -Mr\dot{\theta}^2 \end{bmatrix} = \begin{bmatrix} \tau \\ F \end{bmatrix} \qquad (30)$$

where M is the mass (M=1kg), r is the radial distance, θ is the angle from the X-axis.u_g is a PD controller ($\tau=k_p(\theta-\theta_d)+k_d\cdot\dot{\theta}$, $F=k_p(r-r_d)+k_d\cdot\dot{r}$). $k_p=.5$, $k_d=5$, $\theta(0)=45^\circ$, $r(0)=\sqrt{8}$, $\theta_d=0$, $r_d=2$.Figure-5 shows the path of the robot gripper in the free space ($u_l=0$). A rectangular slap ($.6\leq x\leq 3$, $.8\leq y\leq 1.2$) is placed between the gripper and the target. In Figure-6 an SPF (LAC= 0) is used to prevent collision. As can be seen the robot got trapped in a local equilibrium position before it could reach its target. In Figure-7 a VPF is used for navigation. As can be seen u_l was able to prevent collision,yanked the arm from the local equilibrium zone, and drove it around the obstacle so that u_g was able to sweep it to the target. Figure-8 demonstrates the decoupled nature of the steering control.For this case a blind navigation strategy is used where the LAC is a simple unidirectional field circulating the obstacle.It can be shown that this strategy can be successfully used with convex obstacle and circularly symmetric global attractors. Although the path is of a lesser quality than the one that can be obtain using the above procedure of generating LAC, its simplicity, invariance to rotation and transslation of the obstacles, and the little amount of information need to construct it makes it an attractive choice.

VII. Conclusions

The use of artificial VPF's is explored for navigation in a cluttered envirironment. We demonstrate the advantage VPF's has over SPF's in terms of their ability to project the navigation forces along more than one degree of freedom in the space to better navigate the robot. Such an advantage enables the use of a flexible navigation strategy, that isn't possible to implement with SPF's. We strongly believe that the VPF approach to navigation has a promising future.

REFERENCES

[1] O. Khatib,The Int. J. of Rob. Res. Vol.5, No.1, Spring 1986, pp 90-98.

[2] F. Miyazaki, S. Arimoto, Proc. of the 9th IFAC, Budapest, Hungary, 1984.

[3] V. Pavlov, A. Voronin, Soviet Automatic Cont.,Vol. 17,No.6,1984, pp 45-51.

[4] P. Khosla, R. Volpe,IEEE Conf. on Rob. and Autom., 1988,pp 1778-84.

[5] C. Warren,IEEE Int. Conf. on Rob. & Autom.,May 14-9, Arizona,89,pp 316-21.

[6] B. Krogh, Rob. Res. :The Next Five Years and Beyond, August 14-16, 1984, Bethlehem, Pennsylvania, USA, pp 1-15.

[7] B. Krogh,The American Cont. Conf. June 19-21,Boston MA,1985, pp 950-955.

[8] R. Tilove, Proc. IEEE Int. Conf. on Rob. and Autom. May 13-18, Cincinnati Ohio, 1990, pp 566-71.

[9] E. Rimon,D.Koditchek,IEEE Int. Conf. on Rob. Autom. May 14-9, 1989, pp 21-26.

[10] D. E. Koditschek, E. Rimon,Advances in Appl. Math., 11, 1990, pp 412-442.

[11] E. Rimon,D. Koditschek,IEEE Int. Conf. on Rob. and Autom. Cincinnati Ohio, 1990, pp 1937-1942.

[12] D. Koditschek, Dynamics and Control of Multibody Systems, Contemporary Math. Vol. 97,Proc. of a Summer Res. Conf., July 30-Aug. 5 1988, 1989, pp 131-157.

[13] C. Connolly, J. Burns, IEEE Int. Conf. on Rob. and Autom., May 13-18, Cincinnati Ohio, 1990, pp 2102-2106.

[14] J. Kim, P. Khosla,IEEE Int. Conf. on Rob. Autom. Sacramento California, April 1991, pp 790-796.

[15] L. Tarassenko, A. Blake,IEEE Int. Conf. on Rob. and Autom., Sacramento California, April 1991, pp 540-545.

[16] J. Chuang, N. Ahuja,IEEE Int. Conf. on Rob. and Autom.,Sacramento,California April 1991,pp 558-563.

[17] V. Utkin,S. Drakunov,H. Hashimoto, F. Harashima,IEEE/RSJ Int. Workshop on Intel. Rob. and Sys.,IROS'91, Nov. 3-5, Osaka Japan,1991,pp 1287-1290.

[18] S. Arimoto, F. Miyazaki,Rob. Res. 1'st Int. Symp.,MIT Press, 84,pp 385-407

[19] P. Morse, H. Feshbach,Vol.1 & 2, McGraw-Hill Book Company Inc. 1953.

[20] C. Brebbia, J. Telles, L. Worbel Appl. in Eng.,Springer- Verlag, Berlin, New York, Tokyo, 1984.

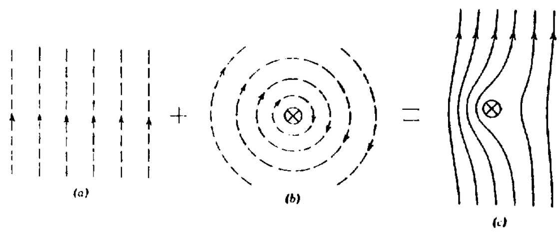

Figure-1: Superposition of fields from VPF and SPF

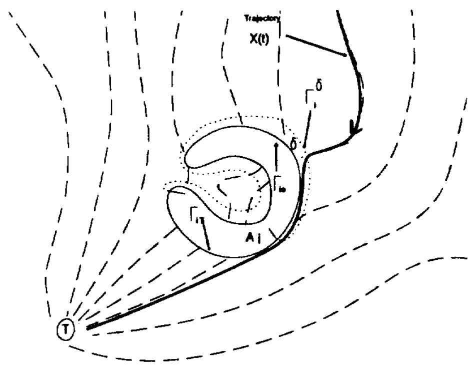

Figure-2: The navigation strategy.

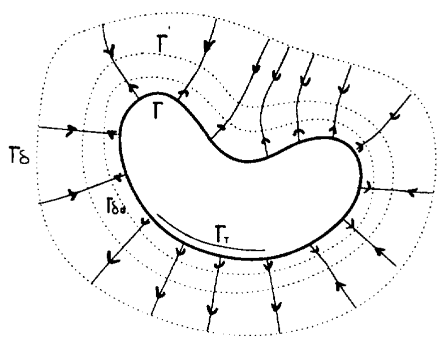

Figure-3: The PPC component

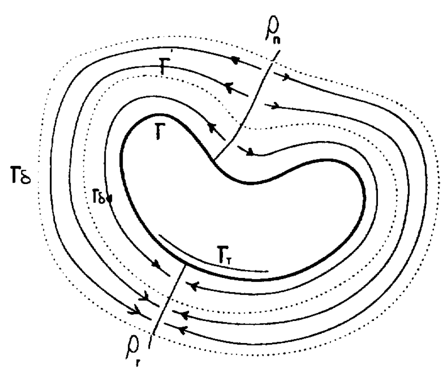

Figure-4: The LAC component

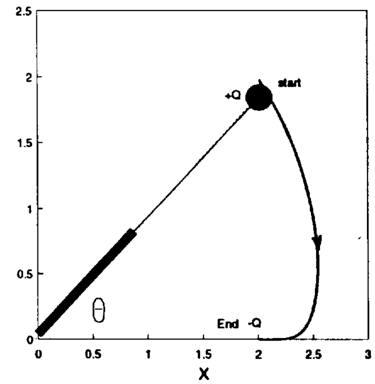

Figure-5: Robot path in free space

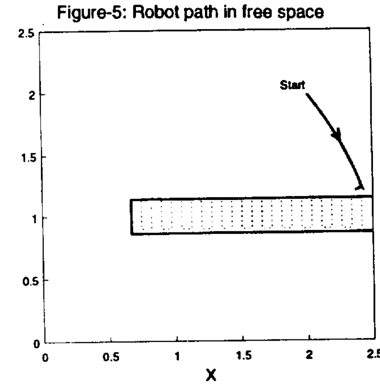

Figure-6: Scalar potential used.

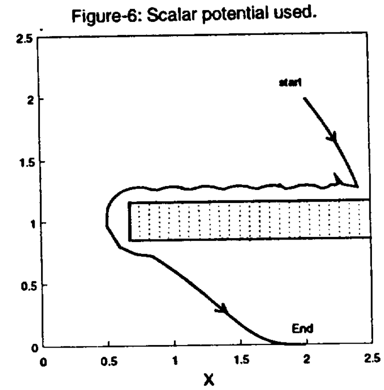

Figure-7: Vector potential used.

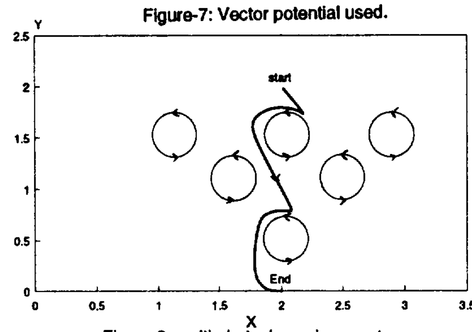

Figure 8: multi-obstacle environment.