Evasion of Multiple, Intelligent Pursuers in a Stationary, Cluttered Environment Using a Poisson Potential Field

Ahmad A. Masoud
Electrical Engineering Department, KFUPM, P.O. Box 287, Dhahran 31261, Saudia Arabia
Tel.: 03-860-3740, E-mail: masoud@kfupm.edu.sa

Abstract

In this paper a new potential field approach is suggested for the evasive navigation of an agent that is engaging multiple pursuers in a stationary environment. Here, the gradient of a potential field that is generated by solving the Poisson equation subject to a set of mixed boundary conditions is used to generate a sequence of directions to guide the motion of an evader so that it will escape a group of pursuers while avoiding a set of forbidden regions (clutter). The focus here is on continuous evasion where the agent does not have the benefit of a target zone (e.g., a shelter) which up on reaching it can discontinue engaging the pursuers. The capabilities of the approach are demonstrated using simulation experiments.

I. Introduction

Evasive navigation is an important tactical aid that is needed to enhance survivability of an agent operating in an adversarial environment [1]. It is also a non-determinate game in which an agent (evader) has to move from an initial location to a final one while avoiding a number of pursuers in an environment that may be populated by forbidden regions. The presence of clutter complicates the evasion strategy which is usually studied for one pursuer in an open environment [2]. Clutter (figure-1) excludes simple solutions to the evasion problem such as running along a straight line toward infinity using the highest possible speed. It also excludes commonly used maneuvers such as protean behavior [3] in which an evader turns in an unpredictable manner to confuse a faster, but less maneuverable, pursuer. In this case, it is highly likely that simple reflexive control will not work. A high level controller is needed to fuse the context in which the actors are operating, the strategy and intentions of the pursuers with the decision making process used to guide the evader's actions.

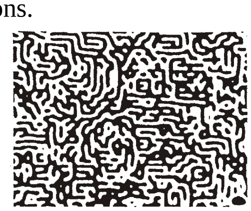


Figure-1: A cluttered environment.

Many aspects of pursuit-evasion have been investigated. In [4] the problem of a group of pursuers searching a cluttered environment for an intruder using flash lights is investigated. On the other hand, in [5] the behavior of an agent trying to hide from a pursuer is evaluated in terms of the amount of protection provided by the environment. In [6] an algorithm is suggested for a group of robots to capture a fugitive agent moving on a grid. In [7] an intelligent controller is suggested for intercepting a known, well-informed target that is intelligently maneuvering in a cluttered environment to evade capture. An intensive literature survey on pursuit evasion may be found in [8].

The focus here is on evasive navigation of an agent that is being tracked by multiple pursuers in a cluttered environment for the case where the evader does not have a target point present. A target point is equivalent to a shelter which when reached the evader no longer have to engage the pursuers. The situation necessitates that the evader continuously engage the pursuers. This presents the evader with a considerable intellectual burden, especially when facing intelligent pursuers who may be cooperating and have the ability to learn regularities or patterns in the evader's behavior and evolve a capture strategy.

The approach suggested in this paper for tackling the above problem is an alternative to a recent approach suggested by the author [13] that utilizes a modified version of the harmonic potential field (HPF) approach to behavior synthesis [9,10,11] for constructing an intelligent controller to guide an evader in a situation such as the one described above. While the approach in [13] utilizes a vector boundary value problem (VBVP) to solve for the phase field of the harmonic potential (the solution of the magnitude field of the HPF is bypassed), the approach in this paper generates the field by solving a standard Poisson equation subject to an appropriate set of mixed boundary conditions. The use of Poisson equation is motivated by the fact that solution of this type of BVPs is much simpler than that of a VBVP. Highly stable, off-the-shelf numerical packages exist for solving the Poisson BVP (e.g., PDE tool-box of MATLAB), which is not the case for VBVPs. Also the properties of the solution of the Poisson equation are well-understood, and are thoroughly documented in the literature.

The paper is organized as follows: in section II the evasion problem is formulated. Section III gives a brief background of the harmonic potential field approach, the difficulties in its application to the evasion problem, and a recent attempt by the author to modify this approach so it can be used for continuously evading multiple pursuers. In section IV the Poisson approach is introduced and the BVP that generates the evasion field is provided. Simulation results are provided in section V, and conclusions are stated in section VI.

II. Problem Formulation

The pursuers and evader are assumed to be operating in a multidimensional environment (R^N) that is populated by stationary forbidden regions (O, $\Gamma=\partial O$). All actors are required to restrict their activities to the subset, Ω, of the multidimensional space known as the workspace ($\Omega=R^N$-O). The location of the i'th pursuer is xp_i. A group of L pursuers is assumed to be operating in Ω. The location of the group is described using the vector $XP=[xp_1\ xp_2\ ...\ xp_L]^t$.

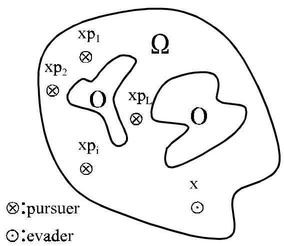


Figure-2: the pursuit-evasion environment

The evader has full knowledge of the environment and the location of the pursuers. Likewise, the pursuers, who may be communicating, are assumed to have full knowledge of the environment and the location of the evader, figure-3.

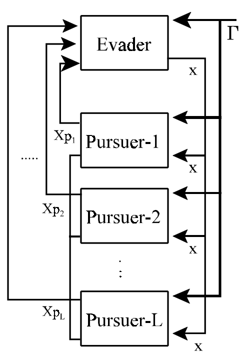


Figure-3: interaction between the evader and the pursuers

The high-level guidance mechanism which the evader is using aggregates the data about the environment (Γ), and the locations of the pursuers (XP) in order to advise the evader regarding the direction it needs to head along if it is to be safe and escape capture. Although the controller yields only a reference trajectory for the evader to follow, many techniques exist for translating a trajectory marked by the gradient field of an HPF into a control signal. A summery of some of these techniques may be found in [12]. Mathematically speaking, constructing the evasion control requires the construction of the gradient dynamical system:

$$\dot{x} = -\nabla V(x, XP(t), \Gamma) \qquad (1)$$

such that: $x(t) \in \Omega \qquad \forall t$

and $\sum_{i=1}^{L} |xp_i - x| > 0 \qquad \forall t,$

where ∇ is the gradient operator, and V is the HPF.

III. Background

Difficulties with the HPF approach:

In the HPF approach, the navigation field is synthesized using the BVP: $\nabla \cdot \nabla V(x) = \nabla^2 V(x) \equiv 0 \quad \forall\, x \in \Omega \qquad (2)$

$$V(x) = 1\big|_{x=\Gamma}, \quad V(x) = 1\big|_{x=xp_i(t), i=1,..,L}, \quad V(x) = 0\big|_{x=x_t}$$

where x_t is the target point, the potential, V(x), is valid for only the time instant t. The control at time t is derived from the negative gradient of V(x):

$$u = -\nabla V(x) . \qquad (3)$$

Harmonic functions assume their minima and maxima on their boundaries (here Γ, XP(t), and x_t). There are no stagnating points in Ω where ∇V vanishes. Therefore, the highest potential, V=1, will be at x = Γ, and $x=xp_i(t)$, i=1,..,L; while the lowest potential, V=0, will be at $x=x_t$. Figure-4a shows a rectangular forbidden region confining the motion of three pursuers and an evader whose goal is to reach the target without running into the pursuers or the walls of the workspace. Figuers-4b, and c show the HPF, and its negative gradient field respectively.

Removing the goal point, x_t, where the potential is fixed to zero, from the BVP in (2) makes the maxima and minima of

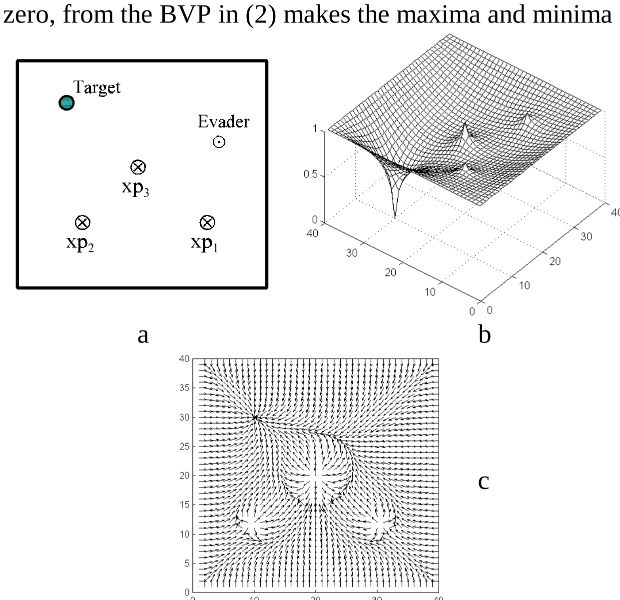


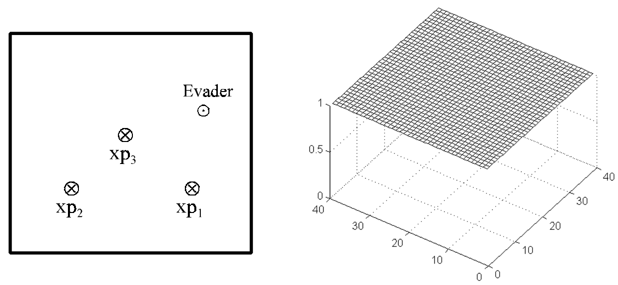
Figure-4: a: environment, b: potential, c: negative gradient

V(x) equal to 1. In other words the value of V(x) is a constant equal to 1 for all points in Ω. This causes the gradient field to degenerate every where in Ω ($\nabla V(x) \equiv 0$, $\forall\ x \in \Omega$) making it impossible to construct the evasion field, figures-5a,b.

The Modified HPF Approach:

The removal of the goal point, x_t, from (2) causes the potential to become flat and the gradient field to degenerate. While the magnitude field of ∇V, a(x), degenerates, the phase field, **G**(x), remains stable and computable. This makes it possible to adapt the generating BVP to work for the case where no target point is explicitly specified.

$$\nabla V(x) = a(x)\mathbf{G}(x)\ . \qquad (4)$$



a b

Figure-5: a: the environment, b: the potential field

TheBVP is: solve $\nabla \cdot \mathbf{G}(x) \equiv 0 \quad X \in \Omega$ (5)

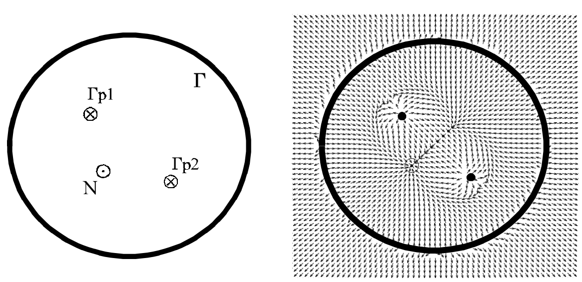
subject to: $\mathbf{G}(x) = n\Gamma \,\big|_{x=\Gamma}$, $\mathbf{G}(x) = n\Gamma i \,\big|_{x=\Gamma i}$,

and $|\mathbf{G}(x)| = 1$, i=1,...L

Where $n\Gamma_i$ is a unit vector field orthogonal to Γi,

$$\Gamma_i = \{x : |x - xp_i| = \delta, \delta > 0\}, \qquad (6)$$

and nΓ is a unit vector field orthogonal to Γ. The above vector BVP (VBVP) is solved for the environment shown in figure-6a. The corresponding evasion field is shown in figure-6b.



a b

Figure-6: a. environment, b. evasion field

Although a target point was not specified in the modified BVP, a stable equilibrium point, N (i.e. a target point), spontaneously emerged in the synthesized field (a north pole, figure-6b). Unlike (2) where target location is a priori specified, in the modified VBVP, the target location is free to move in a manner dependant on the environment and the locations of the pursuers. The target keeps adapting its location positioning itself as far as possible from the pursuers and the forbidden regions. The intelligent, high-level controller suggested here for continuously steering the evader away from harm, accepts Γ and XP(t) as inputs, and generates N(t) as an output. The generated time sequence of locations , N(t), is the one the evader has to follow in order to avoid harm and capture, figure-7.

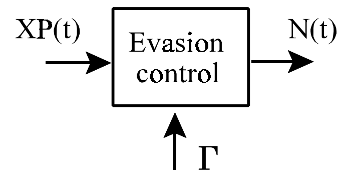


Figure-7: suggested evasion controller

IV. The Suggested Approach

When the potential field was first suggested, the most serious problem facing it was deadlock, or local minima at which ∇V vanishes trapping motion short of reaching the target. The HPF approach to motion planning solved the deadlock problem by forcing ∇V to satisfy Laplace equation for each point inside Ω,

$$\nabla^2 V(x) = \nabla \cdot \nabla V(x) \equiv 0 \qquad x \in \Omega \qquad (7)$$

By satisfying the Laplace equation, the divergence of the gradient of the potential is forced to zero inside Ω. Physically speaking, the divergence of a vector field is defined as the outflow of the flux generated by the field when the volume of the closed area the field is passing through shrinks to zero. In other words, the flux that goes inside that close area must leave (figure-8a). This prevents stagnation of the flux and in turn prevents deadlock.

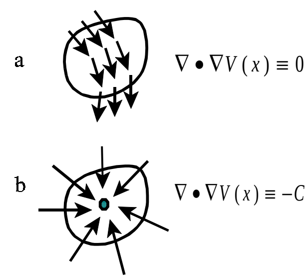


Figure-8: Physical interpretation of a divergence,
a: Laplace equation, b: Poisson equation

Unfortunately when motion is to be planned in order to continuously avoid multiple pursuers, the target point of the evader cannot be *a priori* specified. A target point which is accounted for using a point source is responsible for inducing a field that fills Ω. For motion to be actuated there has to be a vector field everywhere in Ω. Also, for a field to exist a source must be present. The Poisson equation offers an alternative to the Laplace equation in this regard. The Poisson equation:

$$\nabla^2 V(x) = \nabla \cdot \nabla V(x) \equiv -C\ , \qquad (8)$$

constrains the divergence of ∇V to a negative constant value. This amounts to densely covering the workspace with point sources (figure-9) with a field sinking in each (figure-8b). Under the influence of the proper boundary conditions the fields from the micro-sources aggregate to yield a global field pattern. Although not *a priori* specified, a context-dependant, equilibrium point emerges marking the goal point which the evader should move toward.

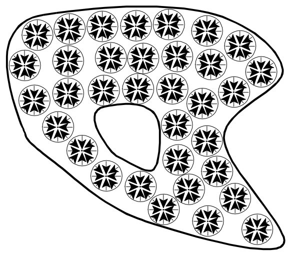


Figure-9: A workspace filled with point field sources

The Generating BVP:

The BVP used for generating the potential whose gradient constitutes the evasion field is: solve

$$\nabla^2 V(x) \equiv -C \qquad x \in \Omega$$

subject to: $$\frac{\partial V(x)}{\partial n} + q \cdot V(x) = g \Big|_{x \in \Gamma \cup \Gamma_i} \qquad \text{i=1,..,L} \qquad (9)$$

Where C, q, and g are positive constants.

V. Results

The capabilities of the Poisson potential field approach are demonstrated by simulation. In figure-10 an agent, whose location is marked by a + sign, is attempting to evade capture by two pursuers (marked by circles). Although the stationary environment, and the locations of the pursuers are known, their tactics, future moves, and any coalitions are not *a priori* known to the evader. As can be seen, the evasion, gradient field from the Poisson potential keeps adapting to the movements of the pursuers in a manner that accounts for the contents of the environment. The adaptation takes place so that the stable equilibrium point of the field is situated as far as possible away from the pursuers and the forbidden regions.

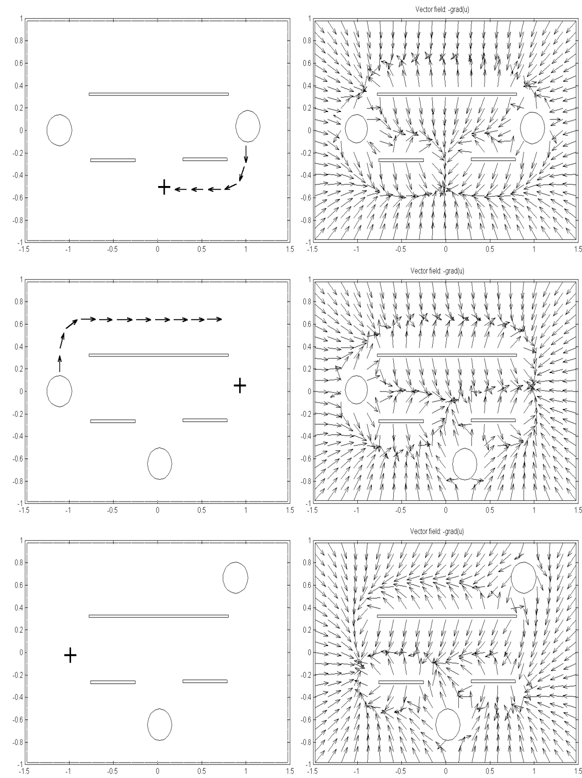


Figure-10: Movements of the evader and pursuers and the corresponding gradient evasion field.

Cooperative surround and block pursue:

Here (Figure-11), the pursuing agents attempt to form a ring around the evader and gradually reduce its radius. Moreover, they monitor the direction along which the evader is heading and attempt to group along that direction to block its escape route

$$\dot{\rho}_i = -C \cdot en_i \qquad (10)$$
$$\dot{\theta}_i = K \cdot SGN(\dot{X}^t \, et_i)$$

where ρ_i is the distance between the evader and the i'th pursuer, θ_i the angle of the i'th pursuer, en_i and et_i are the unit vectors normal and tangent to the i'th pursuer, and SGN(x) = [+1 for x>0, 0 for x=0, -1 for x<0].

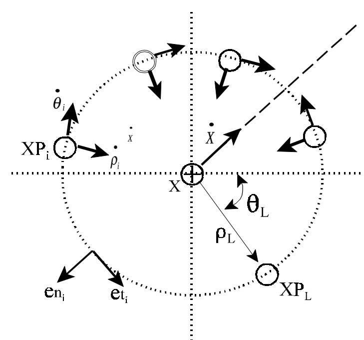


Figure-11: cooperative surround and block pursue

The distance between the evader and the i'th pursuer is:

$$D_i(t) = |X - XP_i| \qquad i = 1,\ldots, L . \ (11)$$

The distance of the evader to the nearest pursuer (DM(t)) is taken to be the measure of safety:

$$DM(t) = \min|_i \ D_i(t) \qquad i = 1,\ldots, L. \ (12)$$

Success of the pursuers is indicated by their ability to drive DM(t) close to zero.

It is a widely accepted belief that if an evader moves in a purely random manner, its chance of escaping capture is improved. Figure-12 shows snapshots of an agent utilizing random walk to evade capture by eight pursuers utilizing the block and surround strategy. A uniformly distributed random number generator is used to generate, at each time step, independent increments along the x and y directions. As can be seen, the pursuers manage to close in on the evader and surround it frequently reducing the radius of the ring to a very small value. The trajectory of the evader is shown in figure-13, and the corresponding DM(t) is shown in figure-14.

In figure-15 the evader replaces the random walk approach with the gradient, evasion field from the Poisson potential. Despite being initially surrounded by a large number of pursuers, the evader manages to outmaneuver them, break the ring, and escape keeping a steady distance away from the pursuers. Moreover, the evader manages to strip the pursuers of their advantage as a group that is capable of utilizing many patterns of behavior for capture. The manner in which the evader maneuvers to escape capture causes the formation of the pursuers to clump in effect reducing the group action into the action of a single agent that is lagging behind the evader. The trajectory of the evader is shown in figure-16, and the corresponding DM(t) is shown in figure-17.

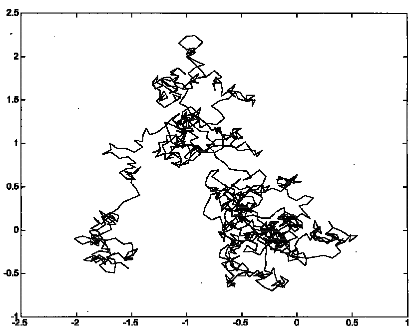


Figure-13: Trajectory of the evader

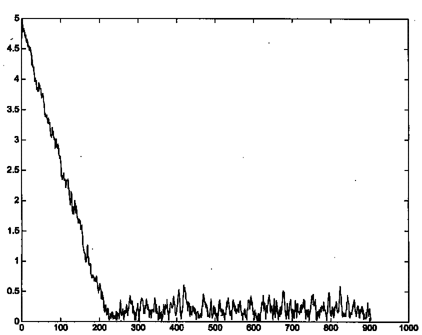


Figure-14: DM(t)

VI. Conclusions

In this paper the problem of continuously evading multi-pursuers in a stationary, cluttered environment is addressed. The high-level controller sensitizing the evader to the contents of its environment is constructed from the gradient of a potential field that satisfy the Poisson equation (also known as the Laplace-poisson equation). The gradient field is supposed to guide the actions of the evader in an attempt to escape capture by the pursuers. In addition to providing tactical

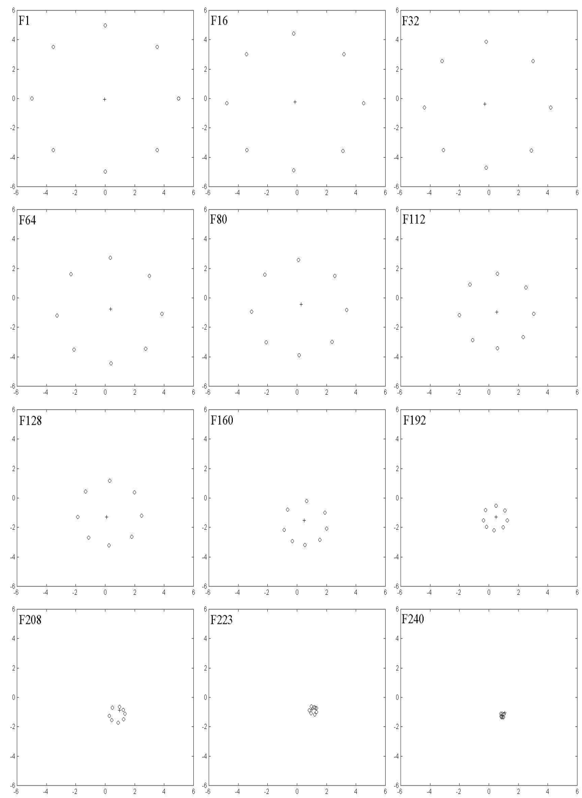


Figure-12: Evasion using random walk.

capabilities for an agent attempting to survive in an adversarial environment, the suggested high-level controller may help to shed light on the origin of purposive behavior. It is believed that any mechanism concerned with the generation of goal-oriented behavior must be supplied with an *a priori* specified goal around which the guidance field is constructed. The suggested controller is a proof that goal-oriented, purposive behavior can be synthesized without having to *a priori* specify a goal. While the preliminary results regarding the performance of the suggested evasion controller are encouraging, in-depth mathematical analysis and simulation experiments remain to be done.

Acknowledgment:

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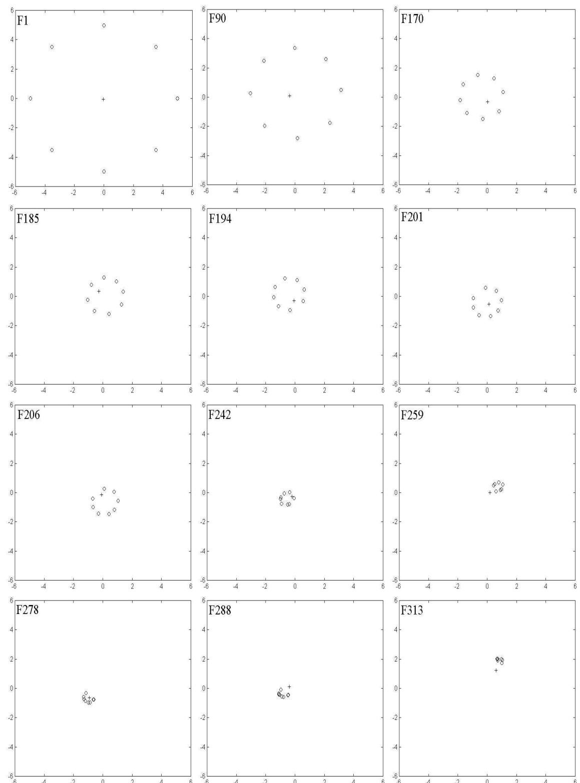

Figure-15: evader utilizing the gradient of a Poisson potential for escape

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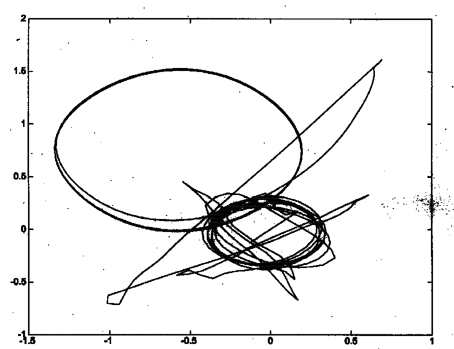

Figure-16: Trajectory of the evader

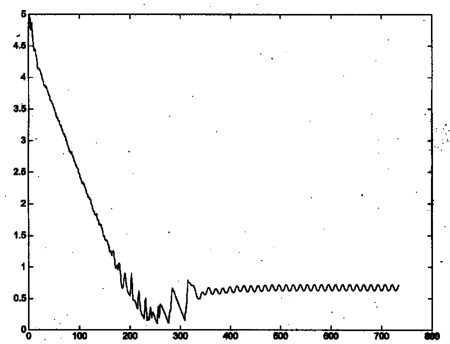

Figure-17: DM(t)

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