Problem 8.29

$$B = \mu H = \mu n I = \mu \frac{N}{d} I$$

$$\psi = BS = \mu \frac{N}{d} IS \quad \Rightarrow \quad L = \frac{\Lambda}{I} = \frac{N\psi}{I} = \mu \frac{N^2}{d} S$$

Where N , d and S are the number of turns, length, and cross-sectional area of the solenoid, respectively.

$$L = \mu \frac{N^2}{d} S = \mu_0 \frac{(10^3)^2}{0.4} [(0.05)^2 \times \pi] = 1.9635 \times 10^4 \mu_0 = 24.67\ mH$$

The core material is assumed to be air, because the problem does not mention the material filling the core.

Problem 8.34

For a toroid whose mean radius ρ_o is much larger than the dimensions of the core, we have:

$$H = \frac{NI}{l} = \frac{NI}{2\pi\rho_o} \quad \Rightarrow \quad B = \mu H = \mu \frac{NI}{2\pi\rho_o}$$ (*uniform* magnetic field in the core of the toroid)

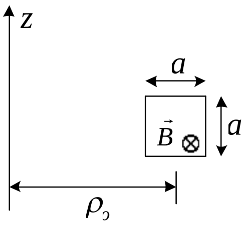
$$\psi = BS = \mu \frac{NI}{2\pi\rho_o} S \quad \Rightarrow \quad L = \frac{\Lambda}{I} = \frac{N\mu \frac{NI}{2\pi\rho_o} S}{I} = \frac{\mu N^2 S}{2\pi\rho_o}$$

$$L = \frac{\mu N^2 S}{2\pi\rho_o} \quad \Rightarrow \quad 2.5 = \frac{200\mu_o N^2 (12\times10^{-4})}{2\pi\times0.5} \quad \Rightarrow \quad 2.5 = 9.6\times10^{-8} N^2$$

$$N^2 = \frac{2.5}{9.6\times10^{-8}} = 2.604\times10^7$$

$$\therefore \quad N = \sqrt{2.604\times10^7} \approx 5103 \text{ turns}$$

Problem 8.38



$$\vec{H} = \frac{NI}{2\pi\rho}\vec{a}_\phi \qquad \Rightarrow \qquad \vec{B} = \frac{\mu NI}{2\pi\rho}\vec{a}_\phi$$

$$\psi = \int_S \vec{B}.\vec{ds} \qquad \Rightarrow \qquad \psi = \int_S \vec{B}.(d\rho dz\vec{a}_\phi) \qquad \Rightarrow \qquad \psi = \int_{z=0}^{a} \int_{\rho=\rho_o-\frac{a}{2}}^{\rho_o+\frac{a}{2}} \frac{\mu NI}{2\pi\rho} d\rho dz$$

$$\psi = \frac{\mu NIa}{2\pi}\ln\frac{\rho_o+\frac{a}{2}}{\rho_o-\frac{a}{2}} = \frac{\mu NIa}{2\pi}\ln\frac{2\rho_o+a}{2\rho_o-a}$$

$$L = \frac{\Lambda}{I} = \frac{N\psi}{I} = \frac{\mu N^2 a}{2\pi}\ln\frac{2\rho_o+a}{2\rho_o-a}$$

Assuming an air core, $\mu = \mu_0$

$$L = \frac{\mu_0 N^2 a}{2\pi}\ln\frac{2\rho_o+a}{2\rho_o-a}$$

Problem 8.41

The magnetic field in the insulated medium (between the inner and outer conductor) is given by:

$\vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi$ (can easily be derived from Ampere's law $\oint_l \vec{H}.\vec{dl} = I$).

$w_m = \frac{1}{2}\mu H^2 = \frac{1}{2}\mu(\frac{I}{2\pi\rho})^2 = \frac{\mu I^2}{8\pi^2\rho^2}$ (which is clearly nonuniform).

$$W_m = \int_V w_m dv = \int_{z=0}^{l}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b} (\frac{\mu I^2}{8\pi^2\rho^2})\rho d\rho d\phi dz$$

$$W_m = \frac{\mu I^2 l}{4\pi}\ln\frac{b}{a}$$

$$W_m = \frac{4\mu_0(25\times10^{-3})^2\times3}{4\pi}\ln\frac{1.8}{1.2} = \frac{\mu_0\times625\times10^{-6}\times3}{\pi}\times0.40547 = 3.041\times10^{-10}\text{ J}$$