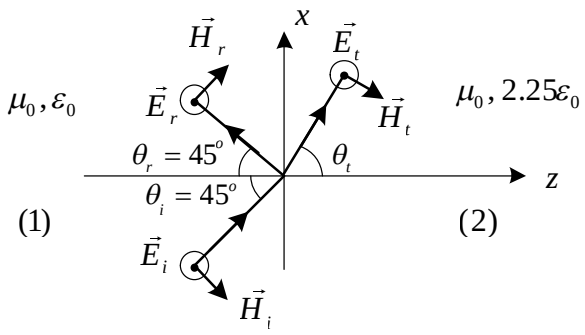
Problem 10.55



$\vec{E}_i = 50\cos(\omega t - \beta_1 x \sin 45^o - \beta_1 z \cos 45^o)\vec{a}_y$ V/m

Clearly $\theta_i = 45^o$

The incident magnetic field:

$$\vec{H}_i = \frac{50}{377}\cos(\omega t - \beta_1 x \sin 45^o - \beta_1 z \cos 45^o) \times (-\cos 45^o \vec{a}_x + \sin 45^o \vec{a}_z)$$

$$\vec{H}_i = 0.1326\cos(\omega t - \beta_1 x \sin 45^o - \beta_1 z \cos 45^o) \times (-0.707\vec{a}_x + 0.707\vec{a}_z)$$

$$\vec{H}_i = 93.762 \times 10^{-3}\cos(\omega t - \beta_1 x \sin 45^o - \beta_1 z \cos 45^o) \times (-\vec{a}_x + \vec{a}_z) \text{ A/m}$$

Using Snell' law of refraction $n_1 \sin\theta_i = n_2 \sin\theta_t \quad \Rightarrow \quad \sqrt{\mu_{1r}\varepsilon_{1r}}\sin\theta_i = \sqrt{\mu_{2r}\varepsilon_{2r}}\sin\theta_t$

$$\sqrt{1\times 1}\sin 45^o = \sqrt{1\times 2.25}\sin\theta_t \quad \Rightarrow \quad \theta_t = 28.126^o$$

$$\tau_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \quad \Rightarrow \quad \tau_\perp = \frac{377\sqrt{\frac{1}{2.25}}\cos 45^o - 377\cos 28.126^o}{377\sqrt{\frac{1}{2.25}}\cos 45^o + 377\cos 28.126^o}$$

$$\tau_\perp = \frac{\sqrt{\frac{1}{2.25}}\cos 45^o - \cos 28.126^o}{\sqrt{\frac{1}{2.25}}\cos 45^o + \cos 28.126^o} \quad \Rightarrow \quad \tau_\perp = \frac{0.4714 - 0.8819}{0.4714 + 0.8819} = -0.3033$$

Problem 10.56

Using Snell' law of refraction $n_1 \sin\theta_i = n_2 \sin\theta_t \quad \Rightarrow \quad \sqrt{\mu_{1r}\varepsilon_{1r}}\sin\theta_i = \sqrt{\mu_{2r}\varepsilon_{2r}}\sin\theta_t$

$$\sqrt{1\times 1}\sin 45^o = \sqrt{1\times 4.5}\sin\theta_{t1} \qquad \Rightarrow \qquad \theta_{t1} = 19.4712^o$$

$$\sqrt{1\times 4.5}\sin 19.4712^o = \sqrt{1\times 2.25}\sin\theta_{t2} \qquad \Rightarrow \qquad \theta_{t2} = 28.126^o$$

Problem 11.9

a) $Z_o = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

$$Z_o = \sqrt{\frac{40 + j \times 2\pi \times 10^7 \times 0.2 \times 10^{-6}}{400 \times 10^{-6} + j \times 2\pi \times 10^7 \times 0.5 \times 10^{-9}}}$$

$$Z_o = \sqrt{\frac{40 + j\,4\pi}{400 \times 10^{-6} + j10^{-2}\pi}} = \sqrt{\frac{\sqrt{1600 + 16\pi^2}\,\underline{|17.441^o}}{\sqrt{16 \times 10^{-8} + 10^{-4}\pi^2}\,\underline{|89.271^o}}}$$

$$Z_o = \sqrt{\frac{41.928\,\underline{|17.441^o}}{3.1419 \times 10^{-2}\,\underline{|89.271^o}}} \quad \Rightarrow \quad Z_o = \sqrt{1334.48\,\underline{|-71.83^o}} \quad \Rightarrow \quad Z_o = 36.531\,\underline{|-35.915^o}\ \Omega$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad \Rightarrow \quad \gamma = \sqrt{(41.928\,\underline{|17.441^o})(3.1419 \times 10^{-2}\,\underline{|89.271^o})}$$

$$\gamma = \sqrt{1.3173\,\underline{|106.712^o}} = 1.1477\,\underline{|53.356^o}$$

Expressing γ in rectangular coordinates:

$$\gamma = 1.1477\,\underline{|53.356^o} = 1.1477\cos 53.356^o + j1.1477\sin 53.356^o = 0.685 + j\,0.921$$

$$\gamma = \alpha + j\beta = 0.685 + j\,0.921$$

$$\therefore\ \beta = 0.921\ \text{rad/m}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 10^7}{0.921} = 6.822 \times 10^7\ \text{m/s}$$

b) $V^+(z) = V^+(0)e^{-\gamma z}$ (voltage variation with distance)

$$V^+(z) = V^+(0)e^{-(\alpha + j\beta)z} = V^+(0)e^{-\alpha z}e^{-j\beta z}$$

$$\left|V^+(z)\right| = \left|V^+(0)\right|e^{-\alpha z}$$ (voltage amplitude variation with distance)

$$\left|V^+(L)\right| = \left|V^+(0)\right|e^{-0.685 \times L}$$

$$20\log\frac{\left|V^+(0)\right|}{\left|V^+(L)\right|} = 30 \quad \Rightarrow \quad 20\log\frac{\left|V^+(0)\right|}{\left|V^+(0)\right|e^{-0.685 \times L}} = 30 \quad \Rightarrow \quad 20\log e^{0.685 \times L} = 30$$

$$20 \times 0.685 \times L \times \log e = 30 \quad \Rightarrow \quad 13.7 \times L \times 0.4343 = 30 \quad \Rightarrow \quad L = 5.04\,\text{m}$$

Problem 11.17

For a distortionless line, we have:

$$\frac{L}{R}=\frac{C}{G} \qquad , \qquad Z_o=\sqrt{\frac{L}{C}}=\sqrt{\frac{R}{G}} \qquad ,$$

$$\gamma=\sqrt{RG}\,(1+j\,\omega\frac{C}{G})=\sqrt{RG}+j\,\omega\sqrt{LC}$$

$$u=\frac{1}{\sqrt{LC}}=2.8\times10^{8}\ \text{m/s} \quad \Rightarrow \quad LC=1.27551\times10^{-17}$$

$$Z_o=\sqrt{\frac{L}{C}}=75\,\Omega \quad \Rightarrow \quad \frac{L}{C}=5625 \quad \Rightarrow \quad L=5625C$$

$$(5625C)C=1.27551\times10^{-17} \quad \Rightarrow \quad C^2=2.2676\times10^{-21} \quad \Rightarrow \quad C=4.762\times10^{-11}\ \text{F/m}$$

$$L=5625C \quad \Rightarrow \quad L=5625\times4.762\times10^{-11}=2.679\times10^{-7} \quad \text{H/m}$$

$$\alpha=\sqrt{RG}=0.06\ \text{Np/}m \quad \Rightarrow \quad RG=36\times10^{-4}$$

$$\frac{L}{R}=\frac{C}{G} \quad \Rightarrow \quad \frac{2.679\times10^{-7}}{R}=\frac{4.762\times10^{-11}}{G} \quad \Rightarrow \quad R=5625.79G$$

$$RG=36\times10^{-4} \quad \Rightarrow \quad (5625.79G)G=36\times10^{-4} \quad \Rightarrow \quad G^2=6.399\times10^{-7} \quad \Rightarrow \quad G=7.999\times10^{-4}\ \text{S/m}$$

$$R=5625.79G \quad \Rightarrow \quad R=5625.79\times7.999\times10^{-4}=4.5\,\Omega/\text{m}$$

Problem 11.18

$$Z_o = \sqrt{\frac{L}{C}} \qquad \Rightarrow \qquad \sqrt{\frac{2.4\times10^{-6}}{C}} = 85$$

$$C = \frac{2.4\times10^{-6}}{85^2} = 3.322\times10^{-10} = 332.2 \text{ pF/m}$$

$$u = \frac{1}{\sqrt{LC}} \qquad \Rightarrow \qquad u = \frac{1}{\sqrt{2.4\times10^{-6}\times332.2\times10^{-12}}} = 3.542\times10^{7} \text{ m/s}$$

$$u = \frac{\omega}{\beta} \qquad \Rightarrow \qquad 3.542\times10^{7} = \frac{2\pi\times4.5\times10^{9}}{\beta}$$

$$\beta = \frac{2\pi\times4.5\times10^{9}}{3.542\times10^{7}} = 798.26 \text{ rad/m}$$

Problem 11.19

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_o = \sqrt{\frac{65 + j \times 2\pi \times 2 \times 10^6 \times 3.4 \times 10^{-6}}{8.4 \times 10^{-3} + j \times 2\pi \times 2 \times 10^6 \times 21.5 \times 10^{-12}}}$$

$$Z_o = \sqrt{\frac{65 + j13.6\pi}{8.4 \times 10^{-3} + j8.6 \times 10^{-5}\pi}} = \sqrt{\frac{\sqrt{65^2 + (13.6\pi)^2}\,\underline{|33.318^o}}{\sqrt{(8.4 \times 10^{-3}) + (8.6 \times 10^{-5}\pi)^2}\,\underline{|1.842^o}}}$$

$$Z_o = \sqrt{\frac{77.785\underline{|33.318^o}}{8.404 \times 10^{-3}\underline{|1.842^o}}} \quad \Rightarrow \quad Z_o = \sqrt{9255.71\underline{|31.476^o}} \quad \Rightarrow \quad Z_o = 96.207\underline{|15.738^o}\ \Omega$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad \Rightarrow \quad \gamma = \sqrt{(77.785\underline{|33.318^o})(8.404 \times 10^{-3}\underline{|1.842^o})}$$

$$\gamma = \sqrt{0.6537\underline{|35.16^o}} = 0.8085\underline{|17.58^o}$$

$$\gamma = 0.8085\underline{|17.58^o} = 0.8085\cos 17.58^o + j\,0.8085\sin 17.58^o = 0.7707 + j0.2442$$

$$\gamma = \alpha + j\beta = 0.7707 + j\,0.2442$$

$$\therefore\ \beta = 0.2442 \text{ rad/m}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 2 \times 10^6}{0.2442} = 5.146 \times 10^7 \text{ m/s}$$

End to end propagation delay = $t = \frac{l}{u} = \frac{5.6}{5.146 \times 10^7} = 1.0882 \times 10^{-7} = 108.82$ ns