Problem 10.4

a) $$\frac{\sigma}{\omega\varepsilon} = \frac{0.08}{(2\pi \times 50 \times 10^6) \times (3.6\varepsilon_0)} = \frac{0.08}{(10^8\pi) \times (3.6 \times 8.85 \times 10^{-12})} = 7.993$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\alpha = \frac{10^8\pi}{3\times 10^8}\sqrt{\frac{2.1\times 3.6}{2}\left[\sqrt{1+(7.993)^2} - 1\right]} \quad \Rightarrow \quad \alpha = \frac{\pi}{3}\sqrt{3.78\times(8.0553-1)} = 5.408 \text{ Np/m}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} \quad \Rightarrow \quad \beta = \frac{\pi}{3}\sqrt{3.78\times(8.0553+1)} = 6.127 \text{ rad/m}$$

$$\gamma = \alpha + j\beta \quad \Rightarrow \quad \gamma = 5.408 + j6.127$$

b) $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.127} = 1.0255\ m$$

c) $$u = \frac{\omega}{\beta} = \frac{10^8\pi}{6.127} = 5.128 \times 10^7 \text{ m/s}$$

d) $$\eta = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}} \angle \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right) \quad \Rightarrow \quad \eta = \frac{377\sqrt{\frac{\mu_r}{\varepsilon_r}}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}} \angle \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$$

$$\eta = \frac{377\sqrt{\frac{2.1}{3.6}}}{[1+(7.993)^2]^{1/4}} \angle \frac{1}{2}\tan^{-1}(7.993) \quad \Rightarrow \quad \eta = \frac{377\times 0.76376}{2.8382} \angle \frac{82.869^o}{2} = 101.45\angle 41.434^o\ \Omega$$

e) $$\vec{H}_s = \frac{6e^{-\gamma x}}{\eta}(-\vec{a}_y) = \frac{6e^{-(5.408+j6.127)x}}{101.45\angle 41.434^o\ \Omega}(-\vec{a}_y)$$

$$\vec{H}_s = \frac{6e^{-\gamma x}}{\eta}(-\vec{a}_y) = -5.914\times 10^{-2} e^{-5.408x} e^{-j6.127x - j41.434^o}\vec{a}_y \quad \text{A/m}$$

Problem 10.5

a) $\beta = \frac{\omega}{c}\sqrt{\frac{\mu_r \varepsilon_r}{2}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2}+1]} \quad \Rightarrow \quad 10 = \frac{2\pi\times 5\times 10^6}{3\times 10^8}\sqrt{\frac{5\times 2}{2}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2}+1]}$

$10 = \frac{10^7\pi}{3\times 10^8}\sqrt{5\times[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2}+1]} \quad \Rightarrow \quad (95.493)^2 = 5\times[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2}+1] \quad \Rightarrow \quad \frac{\sigma}{\omega\varepsilon} = 1822.8$

b) $\frac{\sigma}{\omega\varepsilon} = 1822.8 \quad \Rightarrow \quad \sigma = 1822.8\omega\varepsilon = 1822.8\times 10^7\pi\times 2\times 8.85\times 10^{-12} = 1.013 \text{ S/m}$

c) $\varepsilon_c = \varepsilon(1 - j\frac{\sigma}{\omega\varepsilon}) \quad \Rightarrow \quad \varepsilon_c = 2\varepsilon_0(1 - j1822.8) = (2 - j3645.6)\varepsilon_0$

d) Since $\frac{\sigma}{\omega\varepsilon} = 1822.8 >> 1 \quad \Rightarrow$ the medium is a good conductor

$\therefore \alpha \simeq \beta = 10 \text{ Np/m}$

e) For a good conductor $\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ$

$\eta = \sqrt{\frac{10^7\pi\times 5\mu_0}{\sigma}}\angle 45^\circ = \sqrt{\frac{10^7\pi\times 5\times 4\pi\times 10^{-7}}{1.013}}\angle 45^\circ = 13.958\angle 45^\circ \ \Omega$

Problem 10.6

a) The medium is non-magnetic and thus $\mu_r = 1$.

The amplitude reduces by 18% every 1m.

Assuming the wave travels in the $+z$ direction. Then the field amplitude varies as $E = E_0 e^{-\alpha z}$

Then $E = E_o$ at $z = 0$ and $E = 0.82E_0$ at $z = 1$.

$$0.82E_0 = E_0 e^{-\alpha \times 1} \quad \Rightarrow \quad 0.82 = e^{-\alpha} \quad \Rightarrow \quad \alpha = 0.1985 \text{ Np/m}$$

Since the electric field leads the magnetic field by 24^o $\Rightarrow$ $\theta_\eta = 24^o$

$$\theta_\eta = \frac{1}{2}\tan^{-1}(\frac{\sigma}{\omega\varepsilon}) = 24^o \quad \Rightarrow \quad \tan^{-1}(\frac{\sigma}{\omega\varepsilon}) = 48^o \quad \Rightarrow \quad \frac{\sigma}{\omega\varepsilon} = \tan(48^o) = 1.1106$$

$$\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2} - 1]} \quad \Rightarrow \quad 0.1985 = \frac{2\pi\times 10\times 10^6}{3\times 10^8}\sqrt{\frac{1\times\varepsilon_r}{2}[\sqrt{1+(1.1106)^2} - 1]}$$

$$0.1985 = \frac{2\pi}{30}\sqrt{\frac{\varepsilon_r}{2}[1.4945 - 1]} \quad \Rightarrow \quad 0.1985\times\frac{30}{2\pi} = \sqrt{\frac{\varepsilon_r}{2}\times 0.4945} \quad \Rightarrow \quad \varepsilon_r = 3.633$$

$$\beta = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2} + 1]} \quad \Rightarrow \quad \beta = \frac{2\pi}{30}\sqrt{\frac{3.633}{2}[1.4945 + 1]} = 0.4458 \text{ rad/m}$$

$$\gamma = \alpha + j\beta \quad \Rightarrow \quad \gamma = 0.1985 + j0.4458$$

b) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.4458} = 14.094\ m$

c) $\delta = \frac{1}{\alpha} = \frac{1}{0.1985} = 5.038$ m

d) $\frac{\sigma}{\omega\varepsilon} = 1.1106$

$$\sigma = 1.1106\omega\varepsilon = 1.1106\times 2\pi\times 10^7\times(3.633\times 8.85\times 10^{-12}) = 2.244\times 10^{-3} \text{ S/m}$$

Problem 10.31

$$\vec{E} = 50\cos(10^9 t - 8x)\vec{a}_y + 40\sin(10^9 t - 8x)\vec{a}_z \quad \text{V/m}$$

$\mu_r = 1$ (nonmagnetic medium).

$$u = \frac{\omega}{\beta} = \frac{c}{\sqrt{\varepsilon_r \mu_r}} \quad \Rightarrow \quad \frac{10^9}{8} = \frac{3\times 10^8}{\sqrt{\varepsilon_r}} \quad \Rightarrow \quad \sqrt{\varepsilon_r} = 2.4 \quad \Rightarrow \quad \varepsilon_r = 5.76$$

$$\eta = 377\sqrt{\frac{\mu_r}{\varepsilon_r}} \quad \Rightarrow \quad \eta = \frac{377}{2.4} = 157.08\,\Omega$$

$$\vec{H} = \frac{50}{157.08}\cos(10^9 t - 8x)\vec{a}_z + \frac{40}{157.08}\sin(10^9 t - 8x)(-\vec{a}_y)$$

$$\vec{H} = 0.318\cos(10^9 t - 8x)\vec{a}_z - 0.255\sin(10^9 t - 8x)\vec{a}_y \quad \text{A/m}$$

Problem 10.35

a) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{3.5\times 10^{7}}{(2\pi \times 150\times 10^{6})\times \varepsilon_0} = 4.196\times 10^{9}$ $\Rightarrow$ aluminum is a good conductor at 150 MHz

$$\alpha = \sqrt{\pi f \ \mu\sigma} \quad \Rightarrow \quad \alpha = \sqrt{\pi\times 150\times 10^{6}\times(4\pi\times 10^{-7})\times(3.5\times 10^{7})}$$

$$\alpha = 1.44\times 10^{5} \simeq \beta$$

$$\gamma = \alpha + j\beta \quad \Rightarrow \quad \gamma = 1.44\times 10^{5} + j1.44\times 10^{5}$$

b) $\delta = \dfrac{1}{\alpha} = \dfrac{1}{1.44\times 10^{5}} = 6.946\times 10^{-6}\ \text{m} = 6.946\,\mu\text{m}$

c) $u = \dfrac{\omega}{\beta} = \dfrac{2\pi\times 150\times 10^{6}}{1.44\times 10^{5}} = 6.545\times 10^{3}\ \text{m/s}$