

**ANALYSIS OF  
METAL-CLAD TM-PASS POLARIZERS  
USING THE METHOD OF LINES**

by

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*Dedicated*

*to*

*my beloved mother*

*and*

*to the memory of my late father.*

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# Nomenclature

## English Symbols

$\mathbf{E}$	electric field vector, volts/meter
$\mathbf{H}$	magnetic field vector, amperes/meter
$N$	diagonal matrix of refractive-index squared at mesh grids
$I$	identity matrix
$B$	Phase Parameter
$P$	time-averaged power per unit length in the y-direction, watts/m
$Q$	matrix of the eigen-value equation
$n$	refractive index
$k_o$	free space wavenumber, radians/meter
$h$	mesh size, meter
$d$	thickness of a layer, meter
$b$	buffer layer thickness, meter
$L$	length of metal-clad section, meter

$L_{gr}$	length of grating section in the polarizer, meter
$L_S$	spacing between metal-clad and grating section, meter
$j$	$\sqrt{-1}$
$t$	time, sec
$\mathbf{A} \cdot \mathbf{B}$	scalar (dot) product of vectors $\mathbf{A}$ and $\mathbf{B}$
$\mathbf{A} \times \mathbf{B}$	vector (cross) product of vectors $\mathbf{A}$ and $\mathbf{B}$

### Greek Symbols

$\psi$	general field component of the $\mathbf{E}$ or $\mathbf{H}$ field
$\Psi$	general field component of the $\mathbf{E}$ or $\mathbf{H}$ sampled field, column vector
$\epsilon_o$	free space permittivity, $4\pi \cdot 10^{-7}$ Vs/Am
$\epsilon_r$	relative permittivity
$\mu_o$	free space permeability, $8.8541 \cdot 10^{-12}$ As/Vm
$\mu_r$	relative permeability
$\omega = 2\pi f$	angular frequency, rad/sec
$\alpha_m$	mth modal field coefficient
$\beta$	propagation constant. radians/meter
$\lambda$	wavelength, meter
$\nabla$	nabla operator, $\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$
$\nabla^2$	Laplace operator, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

### Abbreviations

TE	Transverse Electric
----	---------------------

TM	Transverse Magnetic
MOL	Method of Lines
PML	Perfectly Matched Layer
BPM	Beam Propagation Method
ABC	Absorbing Boundary Condition
PER	Polarizer Extinction Ratio
PIL	Polarizer Insertion Loss
FOM	Polarizer Figure of Merit
$n_{eff}$	Modal Effective Index
$\lambda_B$	Bragg Wavelength
$Re$	Real part of a complex number
$Im$	Imaginary part of a complex number

### Subscripts

$A_x, A_y, A_z$	x, y, z components of a vector $\mathbf{A}$
$\psi_0, \psi_{\pm 1}, \psi_{\pm 2}$	sample number of field $\psi$
$\psi_{0\pm}, \psi_{1\pm}, \psi_{2\pm}$	field immediately to the left or to the right of a sample point

### Superscript

$\psi'$	first derivative of $\psi$
$\psi''$	second derivative of $\psi$
$\psi'''$	third derivative of $\psi$

$\psi''''$	fourth derivative of $\psi$
$\psi'''''$	fifth derivative of $\psi$
$\psi''''''$	sixth derivative of $\psi$
$\beta''$	mode attenuation, $N_p/\mu\text{m}$
$n_{eff}'$	real part of the effective index
$n_{eff}''$	imaginary part of the effective index
$A^*$	complex conjugate of $A$