

King Fahd University of Petroleum and Minerals
Electrical Engineering Department

PROBLEM SESSION # 3

3.15 Determine the gradient of the following scalar fields:

- (a) $U = 4xz^2 + 3yz.$
- (b) $V = e^{(2x+3y)} \cos 5z.$
- (c) $W = 2\rho(z^2 + 1) \cos \phi.$
- (d) $T = 5\rho e^{-2z} \sin \phi.$
- (e) $H = r^2 \cos \theta \cos \phi.$
- (f) $Q = (\sin \theta \sin \phi) / r^3.$

3.18 Find the divergence and curl of the following vector fields:

- (a) $\mathbf{A} = e^{xy} \mathbf{a}_x + \sin xy \mathbf{a}_y + \cos^2 xz \mathbf{a}_z$
- (b) $\mathbf{B} = \rho z^2 \cos \phi \mathbf{a}_\rho + z \sin^2 \phi \mathbf{a}_z$
- (c) $\mathbf{C} = r \cos \theta \mathbf{a}_r - \frac{1}{r} \sin \theta \mathbf{a}_\theta + 2r^2 \sin \theta \mathbf{a}_\phi$

3.30 Given that $\mathbf{E} = \frac{1}{r^4} \sin^2 \phi \mathbf{a}_r$, evaluate

- (a) $\oint_S \mathbf{E} \cdot d\mathbf{S}$
- (b) $\int_V (\nabla \cdot \mathbf{E}) dv$

over the region between the spherical surfaces $r = 2$ and $r = 4$.

3.33 Calculate the total outward flux of vector

$$\mathbf{F} = \rho^2 \sin \phi \mathbf{a}_\rho + z \cos \phi \mathbf{a}_\phi + \rho z \mathbf{a}_z$$

through the hollow cylinder defined by $2 \leq \rho \leq 3, 0 \leq z \leq 5$.

3.39 Given the vector field

$$\mathbf{R} = (2x^2y + yz)\mathbf{a}_x + (xy^2 - xz^3)\mathbf{a}_y + (cxyz - 2x^2y^2)\mathbf{a}_z$$

determine the value of c for \mathbf{R} to be solenoidal.

3.40 If the vector field

$$\mathbf{T} = (\alpha xy + \beta z^3)\mathbf{a}_x + (3x^2 - \gamma z)\mathbf{a}_y + (3xz^2 - y)\mathbf{a}_z$$

is irrotational, determine α , β , and γ . Find $\nabla \cdot \mathbf{T}$ at $(2, -1, 0)$.