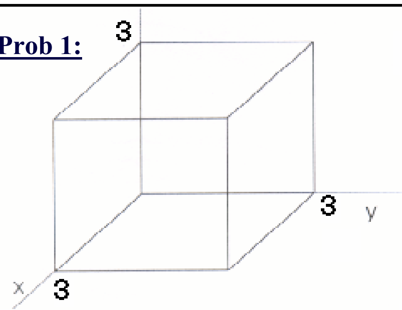
EE 340 Electromagnetics Lab

Problem Session #2

Prob 1:



a) $\int_S F = \int_{left} + \int_{right} + \int_{top} + \int_{bottom} + \int_{front} + \int_{back}$

since $\mathbf{F}=\mathbf{a}_y$, I have only y component in **F**, then there is a value for the surfaces in the direction of +ve y and –ve y

$$\int_S F = \int_{left} + \int_{right} = 5\int_{z=0}^{3}\int_{x=0}^{3} dxdz - 5\int_{z=0}^{3}\int_{x=0}^{3} dxdz =$$

$$= 5\int_{z=0}^{3} [xdz\,]_0^3 - 5\int_{z=0}^{3} [xdz\,]_0^3 = 45 - 45 = 0 = 0$$

b) $\mathbf{F}=x^2y^2\,\mathbf{a}_x$

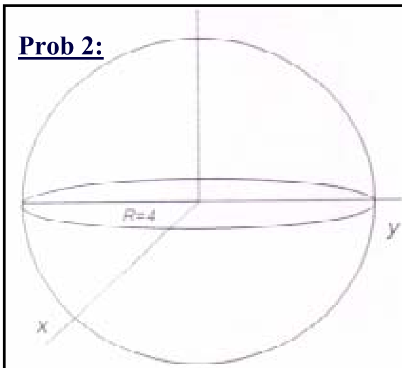
since I have only x component in **F**, then there is a value for the surfaces in the direction of +ve x and –ve x

$$\int_{front} F.ds = \int_{z=0}^{3}\int_{y=0}^{3} x^2y^2 dydz\,\Big|_{x=3} = 9\int_{z=0}^{3}\int_{y=0}^{3} y^2 dydz = 9\int_{z=0}^{3}\left[\frac{y^3}{3}\right]_0^3 dz = 81\int_{z=0}^{3} dz = 81[z]_0^3 = 243$$

$$\int_{back} F.ds = \int_{z=0}^{3}\int_{y=0}^{3} x^2y^2 dydz\,\Big|_{x=0} = 0$$

$$\int_S F.ds = \int_{front} + \int_{back} = 243 + 0 = 243$$

Prob 2:



$\bar{ds} = r^2 \sin\theta d\theta d\phi \,\bar{ar}$

a) $F = \frac{ar}{r^2}$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi = -\int_{\phi=0}^{2\pi}[\cos\theta]_{\theta=0}^{\Pi} d\phi = -\int_{\phi=0}^{2\Pi}[-1-1]d\phi = 2[\phi]_{\phi=0}^{2\Pi} = 4\Pi$$

b) $F = \frac{\sin^2\phi}{r^2}\bar{ar} + \cos\phi\,\bar{a_\theta} \Rightarrow F.ds = \frac{\sin^2\phi}{r^2}$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} r\sin\theta\sin^2\phi\, d\theta d\phi = 8\pi$$

$r = 4$

$\int_0^{\pi} \sin\theta\, d\theta = 2$

$\int_0^{2\pi} \sin^2\Phi\, d\Phi = \pi$

because of integration of sin function over one period

Prob 2: c) F=ax

$F = \sin\theta\cos\phi\,\bar{ar} + \cos\theta\cos\phi\,\bar{a_\theta} - \sin\phi\,\bar{a\phi}$

$F.ds = r^2 \sin^2 \cos\phi\theta d\theta d\phi$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} r^2 \sin^2\theta\cos\phi\, d\theta d\phi = 0$$

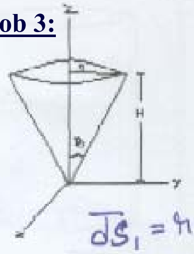
$r^2 = 16$

$\int_0^{\pi} \sin^2\theta\, d\theta = \frac{\pi}{2}$

$\int_0^{2\pi} \cos\phi\, d\phi = 0$

because of integration of sin function over one period

Prob 3:



$\overline{dS}_1 = r \sin\theta \, dr \, d\phi \; \bar{a}_\theta$ (sp.)

Surface 1:

a) $\bar{F} = r a_r = r \bar{a}_r$

$f.ds = 0$

b) $\S F.ds = \iint r^2 \sin\theta \, dr \, d\phi = \frac{2\pi}{3} a(h^2 + a^2)$

c) $F = \cos\phi \, a_\phi + r \, a_\theta$

Surface 2:

a) $F = r\bar{a}_r = r \sin\theta \, \bar{a}_p + r\cos\theta \, \bar{a}_z$
$= p a_p + z a_z$

$\bar{a}_r = \sin\theta \cos\phi \, \bar{a}_x + \sin\theta \sin\phi \, \bar{a}_y + \cos\theta \, \bar{a}_z$

$\bar{a}_r = \sin\theta \cos\phi (\cos\phi \, \bar{a}_\rho) - \sin\theta \cos\phi \cdot (\sin\phi \, a_\phi) + \sin\theta \sin\phi (\sin\phi \, \bar{a}_\rho) + \sin\theta \cdot \sin\phi (\cos\phi \, a_\phi) + \cos\theta \, a_z$

$\bar{a}_r = \sin\theta (\cos^2\phi + \sin^2\phi) \bar{a}_\rho + \cos\theta \, a_z$

$\bar{a}_r = \boxed{\sin\theta \, \bar{a}_\rho + \cos\theta \, \bar{a}_z}$

$\therefore \; \bar{F} = r \bar{a}_r$
$= r \sin\theta \, \bar{a}_\rho + r \cos\theta \, \bar{a}_z$
$= \rho \, \bar{a}_\rho + z \, \bar{a}_z$

Prob 3:

$\S F.ds = \iint z p \, dp \, d\phi$
$= z \int \frac{p^2}{2} \Big| \, d\phi = \pi h a^2$

b) $\bar{F} = r\bar{a}_\theta = r\cos\theta \, \bar{a}_p - r\sin\theta \, \bar{a}_z = z \, \bar{a}_p - p \, \bar{a}_z$

$\S F.ds = -\iint p^2 \, dp \, d\phi = -\frac{2\pi a^3}{3}$

c) $\bar{F} = \cos\phi \, \bar{a}_\phi + r\bar{a}_\theta = r\cos\theta \, \bar{a}_p + \cos\phi \, \bar{a}_\phi - r\sin\theta \, \bar{a}_z$
$= z \bar{a}_p + \cos\phi \, \bar{a}_\phi - p \bar{a}_z$

$\S F.ds = -\iint p^2 \, dp \, d\phi = -\frac{\pi a^3 2}{3}$

$\boxed{\overline{dS} = \rho \, d\phi \, d\rho \; \bar{a}_z}$ (cyl.)

$\oint \bar{F} \cdot \overline{ds} = \iint (\rho \bar{a}_\rho + z \bar{a}_z) \cdot (\rho \, d\phi \, d\rho \, \bar{a}_z)$

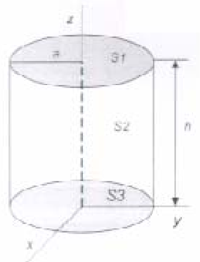
$s_1 \quad = \iint z \rho \, d\phi \, d\rho$

$s_1 \quad = z \int_0^{2\pi} \int_0^a \rho \, d\rho \, d\phi$

$s_1 \quad = (h) \int_0^{2\pi} \left. \frac{\rho^2}{2} \right]_0^a d\phi$

$s_1 \quad = \frac{h a^2}{2} \cdot 2\pi = \boxed{\pi h a^2}$

Prob 4a:



$ds1 = \rho d\phi d\rho \overline{a_z}$

$ds2 = \rho d\phi dz \overline{a_\rho}$

$ds3 = -\rho d\phi d\rho \overline{a_z}$

ds3 = "—" due to direction

a) $F = \rho^2 a_\rho + \rho \sin\phi a_\phi + \rho^2 \sin\phi \overline{a_z}$

$$\int_{S1} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{a} \rho^3 \sin\phi d\phi d\rho = 0 \rightarrow (\overline{a_z})$$

$$\int_0^a \rho^3 d\rho = \frac{\rho^4}{4}\Big]_0^a = \frac{a^4}{4}$$

$$\int_0^{2\pi} \sin\phi d\phi = -\cos\phi\Big]_0^{2\pi} = -(1-1) = 0$$

integration of sin over one period

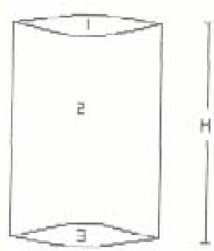
$$\int_{S2} = \int_{z=0}^{h}\int_{\phi=0}^{2\pi} \rho^3 d\phi dz = [a^3][2\pi][h] = 2\pi h a^3$$

$$\int_{S3} = -\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a} \rho^3 \sin\phi d\phi d\rho = 0$$

integration of sin over one period

$$\int F = \int_{S1} + \int_{S2} + \int_{S3} = \underline{2\pi h a^3}$$

Prob 4b:



a) $F = \rho^2 a_\rho + \rho \sin\phi a_\phi + \rho^2 \sin\phi a_z$

$§F.ds1 = \iint \rho^3 d\phi dz$
$= 2\pi a^3 h$

$§F.ds2 = \iint \rho^3 \sin\phi d\phi d\rho$
$= \int -\cos\phi| \rho^3 d\rho = 0$

$F.ds3 = 0$

⇨ $F.ds = 2\pi a^3 h$

b) $F = x\overline{a_x} + z\overline{a_z}$
$= x\cos\phi \overline{a_\rho} - x\sin\phi \overline{a_\phi} + z\overline{a_z}$
$= \rho\cos^2\phi \overline{a_\rho} - \rho\cos\phi\sin\phi \overline{a_\phi} + z\overline{a_z}$

$§F.ds1 = \iint \rho^2\cos^2\phi d\phi dz$ $\{z = 0 \rightarrow h, \ \phi = 0 \rightarrow 2\pi\}$
$= \pi a^2 h$

$§F.ds2 = \iint \rho z d\phi d\rho a_z$ $\{\rho = 0 \rightarrow a, \ \phi = 0 \rightarrow 2\pi\}$
$= h\iint \rho d\rho d\phi$
$= \pi a^2 h$

$§F.ds3 = 0$ (as $z = 0$)