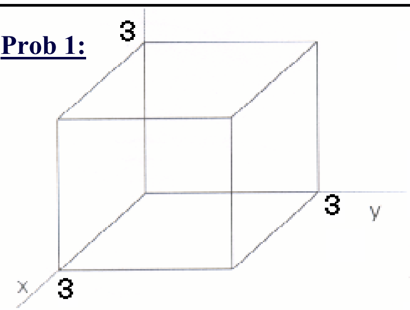
EE 340 Electromagnetics Lab

Problem Session #2

Prob 1:



a) $\int_S F = \int_{left} + \int_{right} + \int_{top} + \int_{bottom} + \int_{front} + \int_{back}$

since $\mathbf{F}=\mathbf{a}_y$, I have only y component in $\mathbf{F}$, then there is a value for the surfaces in the direction of +ve y and –ve y

$$\int_S F = \int_{left} + \int_{right} = 5\int_{z=0}^{3}\int_{x=0}^{3} dxdz - 5\int_{z=0}^{3}\int_{x=0}^{3} dxdz =$$

$$= 5\int_{z=0}^{3}[xdz]_0^3 - 5\int_{z=0}^{3}[xdz]_0^3 = 45 - 45 = 0 = 0$$

b) $\mathbf{F} = x^2 y^2 \mathbf{a}_x$

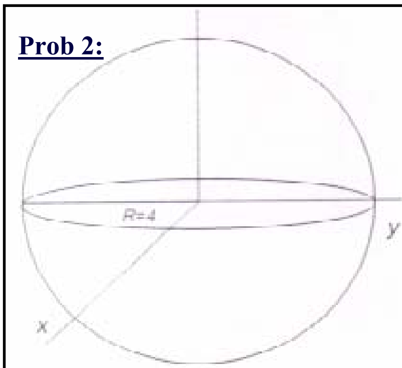
since I have only x component in $\mathbf{F}$, then there is a value for the surfaces in the direction of +ve x and –ve x

$$\int_{front} F.ds = \int_{z=0}^{3}\int_{y=0}^{3} x^2 y^2 dydz \Big|_{x=3} = 9\int_{z=0}^{3}\int_{y=0}^{3} y^2 dydz = 9\int_{z=0}^{3}\left[\frac{y^3}{3}\right]_0^3 dz = 81\int_{z=0}^{3} dz = 81[z]_0^3 = 243$$

$$\int_{back} F.ds = \int_{z=0}^{3}\int_{y=0}^{3} x^2 y^2 dydz \Big|_{x=0} = 0$$

$$\int_S F.ds = \int_{front} + \int_{back} = 243 + 0 = 243$$

Prob 2:



$\overline{ds} = r^2 \sin\theta d\theta d\phi \, \overline{\mathbf{ar}}$

a) $F = \dfrac{\mathbf{a}r}{\mathbf{r}^2}$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi = -\int_{\phi=0}^{2\pi}[\cos\theta]_{\theta=0}^{\Pi} d\phi = -\int_{\phi=0}^{2\Pi}[-1-1]d\phi = 2[\phi]_{\phi=0}^{2\Pi} = 4\Pi$$

b) $F = \dfrac{\sin^2\phi}{r^2}\overline{ar} + \cos\phi\overline{a_\theta} \Rightarrow F.ds = \dfrac{\sin^2\phi}{r^2}$

$r = 4$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} r\sin\theta\sin^2\phi d\theta d\phi = 8\pi$$

$\int_0^{\pi} \sin\theta \, d\theta = 2$

$\int_0^{2\pi} \sin^2\Phi \, d\Phi = \pi$

because of integration of sin function over one period

Prob 2: c) $F = \mathbf{a}x$

$\overline{F} = \sin\theta\cos\phi\overline{ar} + \cos\theta\cos\phi\overline{a_\theta} - \sin\phi\overline{a\phi}$

$F.ds = r^2 \sin^2 \cos\phi d\theta d\phi$

$r^2 = 16$

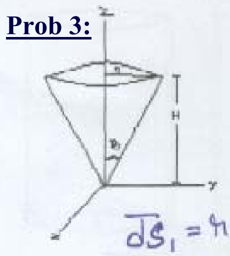
$\int_0^{\pi} \sin^2\theta \, d\theta = \dfrac{\pi}{2}$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} r^2 \sin^2\theta\cos\phi d\theta d\phi = 0$$

$\int_0^{2\pi} \cos\phi \, d\phi = 0$

because of integration of sin function over one period

Prob 3:



$\overline{dS}_1 = r\sin\theta\, dr\, d\phi\ \overline{a_\theta}$ (sp.)

Surface 1:

a) $\overline{F} = ra_r = r\overline{a_r}$

$f.ds = 0$

b) $\S F.ds = \iint r^2 \sin\theta\, dr\, d\phi = \frac{2\pi}{3} a(h^2+a^2)$

$\overline{a_r} = \sin\theta\cos\phi\,\overline{a_x} + \sin\theta\sin\phi\,\overline{a_y} + \cos\theta\,\overline{a_z}$

$\overline{a_r} = \sin\theta\cos\phi(\cos\phi\,\overline{a_\rho}) - \sin\theta\cos\phi\cdot(\sin\phi\, a_\phi) + \sin\theta\sin\phi(\sin\phi\,\overline{a_\rho}) + \sin\theta\cdot\sin\phi(\cos\phi\, a_\phi) + \cos\theta\, a_z$

$\overline{a_r} = \sin\theta(\cos^2\phi + \sin^2\phi)\overline{a_\rho} + \cos\theta\, a_z$

$\overline{a_r} = \boxed{\sin\theta\ \overline{a_\rho} + \cos\theta\,\overline{a_z}}$

$\therefore\ \overline{F} = r\overline{a_r} = r\sin\theta\,\overline{a_\rho} + r\cos\theta\,\overline{a_z} = \rho\,\overline{a_\rho} + z\,\overline{a_z}$

c) $F = \cos\phi a_\phi + r a_\theta$

Surface 2:

a) $F = r\overline{ar} = r\sin\theta\overline{ap} + r\cos\theta\overline{az}$

$= pap + zaz$

Prob 3:

$\S F.ds = \iint zp\,dp\,d\phi$

$= z\int \frac{p^2}{2}\Big|\, d\phi = \pi h a^2$

b) $\overline{F} = r\overline{a\theta} = r\cos\theta\overline{ap} - r\sin\theta\overline{az} = z\,\overline{ap} - p\overline{az}$

$\S F.ds = -\iint p^2 dp\, d\phi = -\frac{2\pi a^3}{3}$

c) $\overline{F} = \cos\phi\overline{a\phi} + r\overline{a\theta} = r\cos\theta\overline{ap} + \cos\phi\overline{a\phi} - r\sin\theta\overline{az}$

$= z\overline{ap} + \cos\phi\overline{a\phi} - p\overline{az}$

$\S F.ds = -\iint p^2 dp\, d\phi = -\frac{\pi a^3 2}{3}$

$\boxed{\overline{dS} = \rho\, d\phi\, d\rho\ \overline{a_z}}$ (cyl.)

$\oint \overline{F}\cdot\overline{ds} = \iint(\rho\overline{a_\rho} + z\overline{a_z})\cdot(\rho\, d\phi\, d\rho\,\overline{a_z})$

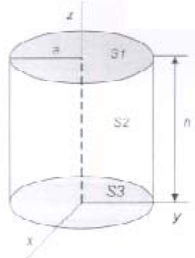
$= \iint z\rho\, d\phi\, d\rho$

$= z\int_0^{2\pi}\int_0^a \rho\, d\rho\, d\phi$

$= (h)\int_0^{2\pi} \frac{\rho^2}{2}\Big]_0^a d\phi$

$= \frac{ha^2}{2}\cdot 2\pi = \boxed{\pi h a^2}$

Prob 4a:



$ds1 = \rho\, d\phi\, d\rho\, \overline{az}$

$ds2 = \rho\, d\phi\, dz\, \overline{a\rho}$

$ds3 = -\rho\, d\phi\, d\rho\, \overline{az}$

$ds3 =$ "$-$" due to direction

a) $F = \rho^2 a_\rho + \rho \sin\phi\, a_\phi + \rho^2 \sin\phi\, \overline{az}$

$$\int_{S1} = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \rho^3 \sin\phi\, d\phi\, d\rho = 0 \longrightarrow (\overline{a_z})$$

$$\int_0^a \rho^3 d\rho = \frac{\rho^4}{4}\Big]_0^a = \frac{a^4}{4}$$

$$\int_0^{2\pi} \sin\phi\, d\phi = -\cos\phi\Big]_0^{2\pi} = -(1-1) = 0$$

integration of sin over one period

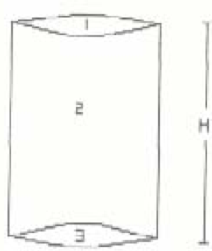
$$\int_{S2} = \int_{z=0}^{h} \int_{\phi=0}^{2\pi} \rho^3 d\phi dz = [a^3][2\pi][h] = 2\pi h a^3$$

$$\int_{S3} = -\int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \rho^3 \sin\phi\, d\phi\, d\rho = 0$$

integration of sin over one period

$$\int F = \int_{S1} + \int_{S2} + \int_{S3} = 2\pi h a^3$$

Prob 4b:



a) $F = p^2 ap + p\sin\phi\, a\phi + p^2 \sin\phi\, az$

$\S F.ds1 = \iint p^3 d\phi dz$
$= 2\pi a^3 h$

$\S F.ds2 = \iint p^3 \sin\phi\, d\phi\, dp$
$= \int -\cos\phi|\, p^3\, dp = 0$

$F.ds3 = 0$

$\Rightarrow F.ds = 2\pi a^3 h$

b) $F = x\overline{ax} + z\overline{az}$
$= x\cos\phi\, \overline{ap} - x\sin\phi\, \overline{a\phi} + z\overline{az}$
$= \rho\cos^2\phi\, \overline{ap} - \rho\cos\phi\sin\phi\, \overline{a\phi} + z\overline{az}$

$\S F.ds1 = \iint \rho^2 \cos^2\phi\, d\phi\, dz$ $\quad \left\{ \begin{matrix} z = 0 \to h \\ \phi = 0 \to 2\pi \end{matrix} \right\}$
$= \pi a^2 h$

$\S F.ds2 = \iint \rho z\, d\phi\, d\rho\, az$ $\quad \left\{ \begin{matrix} \rho = 0 \to a \\ \phi = 0 \to 2\pi \end{matrix} \right\}$
$= h \iint \rho\, d\rho\, d\phi$
$= \pi a^2 h$

$\S F.ds3 = 0$ (as $z = 0$)