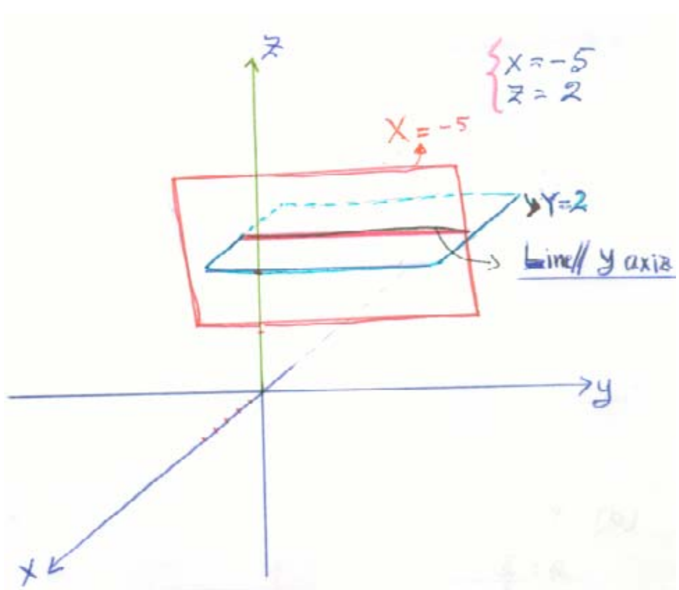
# EE 340 Electromagnetics Lab

## Problem Session #1

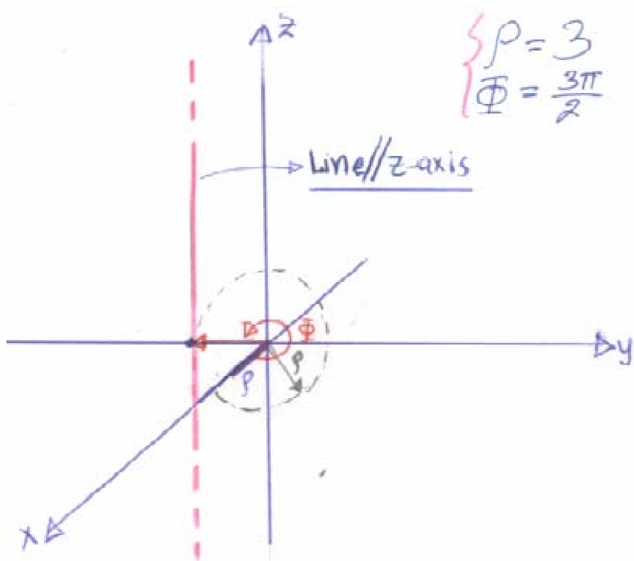
**Part 1: Visualization of surfaces in 3D coordinate systems:**

(a)



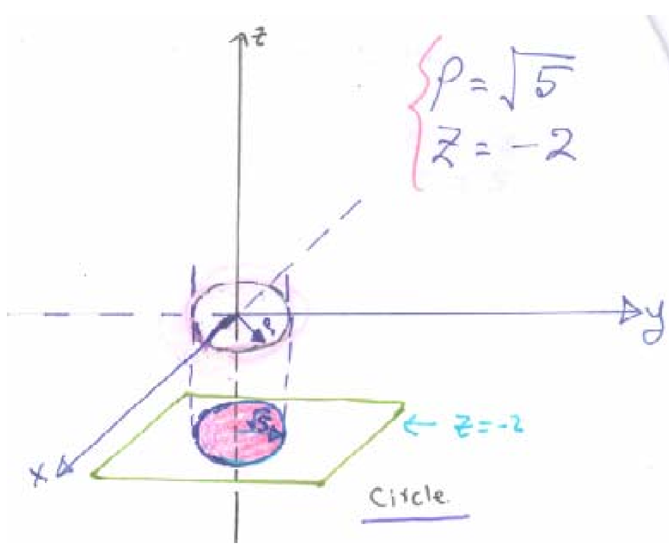
## Part 1: Visualization of surfaces in 3D coordinate systems:

(b)



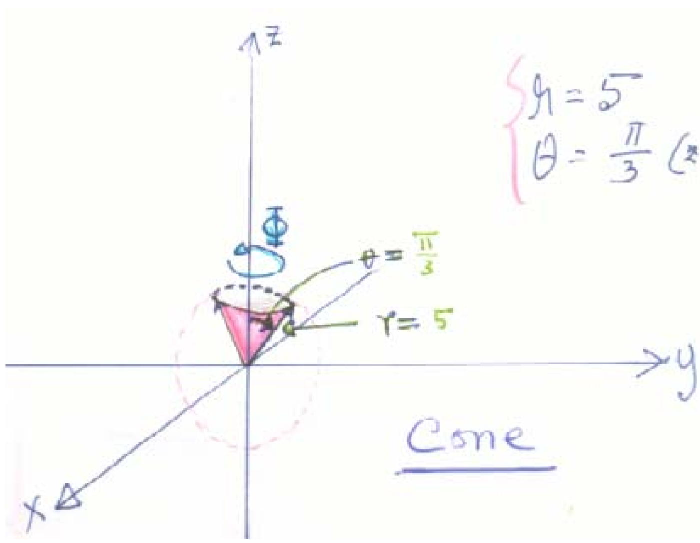

## Part 1: Visualization of surfaces in 3D coordinate systems:

(c)



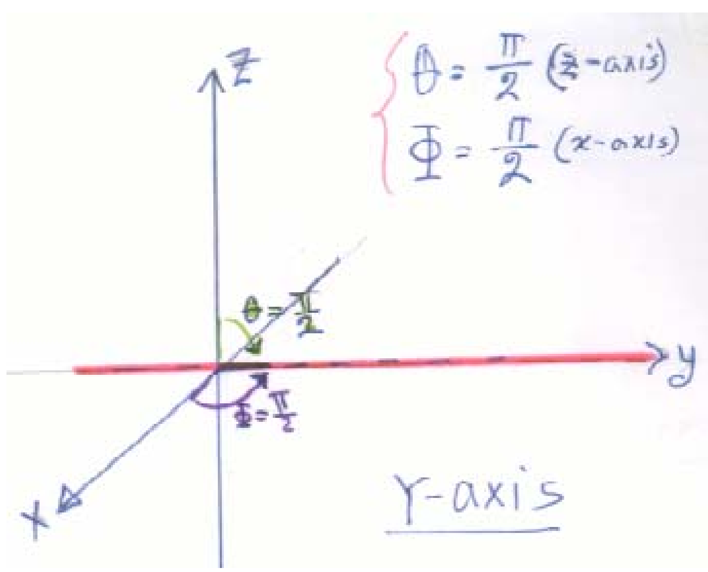
## Part 1: Visualization of surfaces in 3D coordinate systems:

(d)



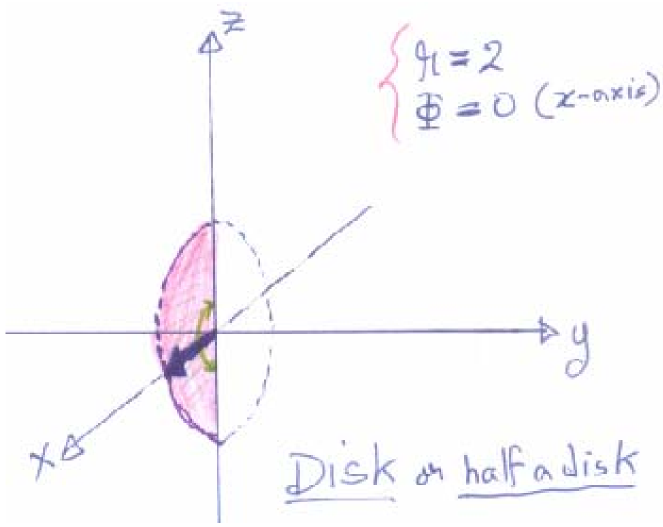

## Part 1: Visualization of surfaces in 3D coordinate systems:

(e)



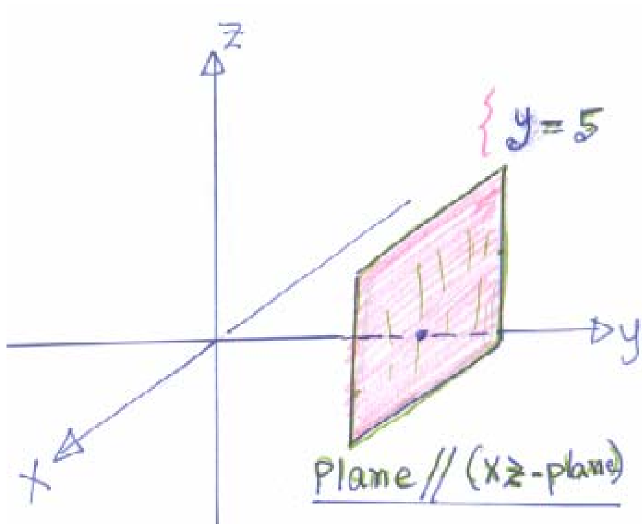
## Part 1: Visualization of surfaces in 3D coordinate systems:

(f)



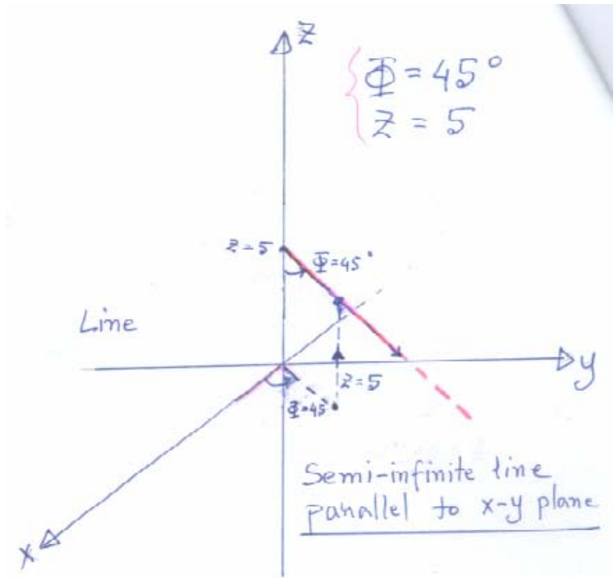

## Part 1: Visualization of surfaces in 3D coordinate systems:

(g)



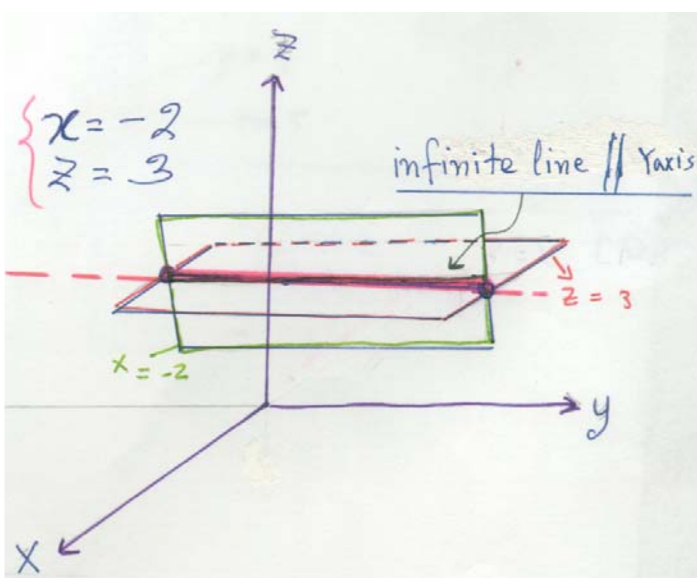
## Part 2: Visualization of surfaces in 3D coordinate systems:

(a)



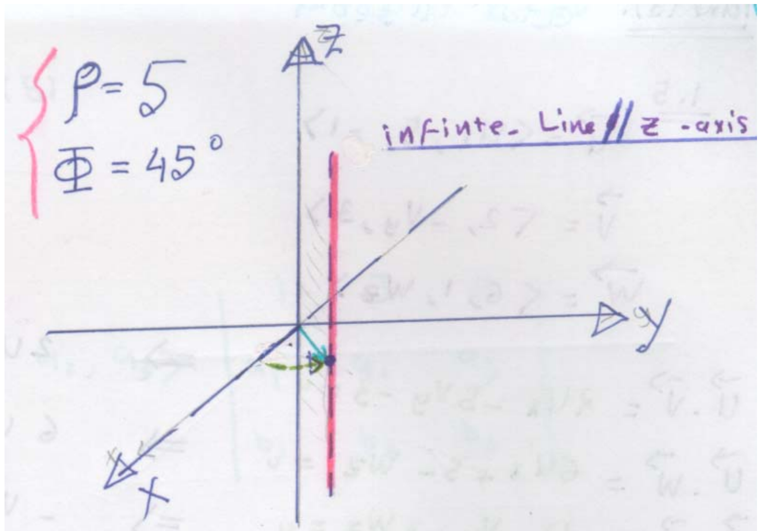

## Part 2: Visualization of surfaces in 3D coordinate systems:

(b)



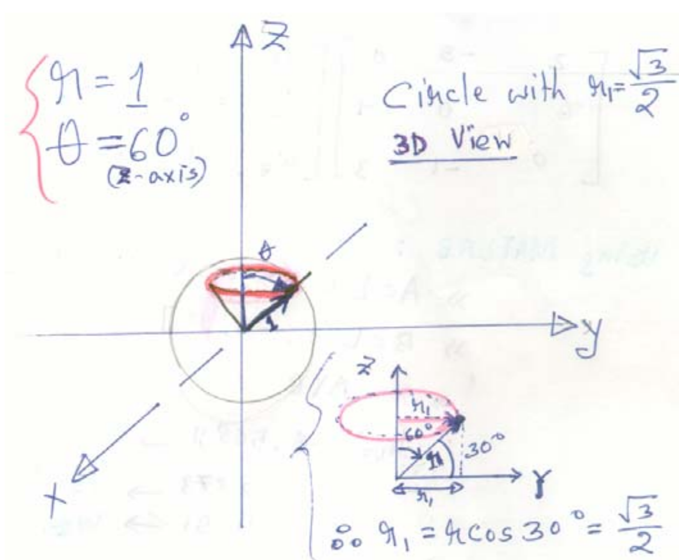
## Part 2: Visualization of surfaces in 3D coordinate systems:

(c)




## Part 2: Visualization of surfaces in 3D coordinate systems:

(d)



Part(3): **Vector Algebra**

**Problem 1.5**

For $\bar{U} = U_x a_x + 5 a_y - a_z$, $\bar{V} = 2 a_x - V_y a_y + 3 a_z$ and $\bar{W} = 6 a_x + a_y + W_z a_z$, obtain Ux, Vy and Wz such that **U**, **V** and **W** are mutually orthogonal ?

$\bar{U}.\bar{V} = (u_X a_x + 5 a_y - a_z).(2a_x - v_y a_y + 3 a_z)$

$= 2U_x - 5V_y - 3 = 0$ ........................ (1)

$\bar{U}.\bar{W} = (U_x a_x + 5a_y - a_z).(6a_x + a_y + W_z a_z)$

$= 6U_x + 5 - W_z = 0$ ........................ (2)

$\bar{V}.\bar{W} = (2a_x - V_y a_y + 3 a_z).(6a_x + a_y + W_z a_z)$

$= 12 - V_y + 3W_z = 0$ ........................ (3)

eq.1 $*(-3)$ + eq.2 $\Rightarrow 15 v_y - w_z + 14 =$ —④

eq.3 $*(15)$ + eq.4 $\Rightarrow w_z = -97/22$

**Problem 1.10**

(a) $\bar{A}.(\bar{A}X\bar{B}) = 0 = \bar{B}(\bar{A}X\bar{B})$

$$(\bar{A}X\bar{B}) = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Remember (dot prod.)

$a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$

$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$

$= a_x (A_yB_z - A_zB_y) - a_y (A_xB_z - A_zB_x) + a_z (A_xB_y - A_yB_x)$

$A.(AXB) = A_xA_yB_z - A_xA_zB_y - A_yA_xB_z + A_yA_zB_x A_zA_xB_y - A_yA_zB_x = 0$

$B.(AXB) = A_yB_xB_z - A_zB_xB_y - A_xB_yB_z + A_zB_xB_y + A_xB_yB_z - A_yB_xB_z = 0$

So this statement is true.

# Part 3: Vector Algebra

**Problem 1.5**

(b) $(\bar{A}.\bar{B})^2 + |\bar{A}X\bar{B}|^2 = (AB)^2$

$|AXB|^2 = (AB)^2 \sin^2\Theta$ $\longrightarrow$ direction ignored.

$(A.B)^2 + |AXB|^2 = (AB\cos\Theta)^2 + (AB\sin\Theta)^2 = (AB)^2\cos^2\Theta + (AB)^2\sin^2\Theta$

$= (AB)^2(\cos^2\Theta + \sin^2\Theta) = (AB)^2 * 1 = RHS$

(c) $\bar{A} = A_x \bar{a_x} + A_y \bar{a_y} + A_z \bar{a_z}$

Because $A.a_x \Leftarrow A_x$

$A.a_y \Leftarrow A_y$

$A.a_z \Leftarrow A_z$

subs. Ax, Ay & Az leads;

$\bar{A} = (\bar{A}\cdot\bar{a}_x)\bar{a}_x + (\bar{A}\cdot\bar{a}_y)a_y + (\bar{A}\cdot\bar{a}_z)\bar{a}_z$

## Part 4: Coordinate Transformations

Problem 2.1 convert the following points to Cartesian coordinates:

$X=\rho\cos\Theta$ & $Y=\rho\sin\Theta$ & $Z=Z$

(a) $P(5,\ 120^o,\ 0)$

$X=5\cos 120=-2.5 \qquad Y=5\sin 120=4.33 \qquad Z=0$

$P(-2.5, 4.33,\ 0)$

(b) $P(1, 30^o, -10)$

$X=1\cos 30=0.866 \qquad Y=1\sin 30=0.5 \qquad Z=-10$

$P(0.866, 0.5, -10)$

(c) $P(10, 3\pi/4,\ \pi/2)$

$X=r\sin\Theta\cos\Phi \qquad Y=r\sin\Theta\sin\Phi \qquad Z=r\cos\Theta$

$X=10\sin 3\pi/4\cos\pi/2=0$

$Y=10\sin 3\pi/4\sin\pi/2=7.071$

$Z=10\cos 3\pi/4=-7.071$

$P(0,\ 7.071,\ -7.071)$

## Part 4: Coordinate Transformations

Problem 2.2 Express in Cylindrical and spherical coord

(a) $P(1,\ -4,\ -3)$

$\rho=\sqrt{1+16}=\sqrt{17}$, $\Phi=\tan^{-1}(-4/1)=-75.94^o=284.06^o$, & $z=-3$

$P(\sqrt{17}, -75.94, -3) \longrightarrow$ Cylindrical

$r=\sqrt{(1+16+9)}=\sqrt{26}$ as $r=\sqrt{x^2+y^2+z^2}$

$\Theta=\tan^{-1}(\sqrt{17}/-3)=-53.96^o$

$P(\sqrt{26}, -53.96^o, -75.94^o) \longrightarrow$ Spherical

(b) $Q(3,\ 0,\ 5)$

$\rho=\sqrt{9+0}=3$, $\Phi=\tan^{-1}(0/3)=0^o$ & $z=5$

$Q(3, 0,\ 5) \longrightarrow$ Cylindrical

$Q(\sqrt{34}, 30.96^o, 0) \longrightarrow$ Spherical

(c) $R(-2,6,0)$

$\rho=\sqrt{4+36}=\sqrt{40}$, $\Phi=\tan^{-1}(6/-2)=-71.565^o$ & $z=0$

$P(\sqrt{40}, -71.565^o,\ 0)$

## Part 4: Coordinate Transformations

**2.3(a)**

$$\begin{pmatrix} P_\rho \\ P_\phi \\ P_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y+z \\ 0 \\ 0 \end{pmatrix} \quad (eq. 2.13)$$

**Cylindrical**

$$P_\rho = \cos\phi\,(y+z) = \cos\phi\,(\rho\sin\phi + z)$$

$$P_\phi = -\sin\phi\,(y+z) = -\sin\phi\,(\rho\sin\phi + z)$$

$$P_z = 0$$

$$\longrightarrow \vec{P} = (\rho\sin\phi + z)(\cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi)$$

**Spherical**

$$P_r = (y+z)(\sin\theta\cos\phi) = (r\sin\theta\sin\phi + r\cos\theta)(\sin\phi\cos\phi)$$

$$P_\theta = (y+z)(\cos\theta\cos\phi) \Rightarrow \quad (eq. 2.27)$$

$$P_\phi = (y+z)(-\sin\phi) \Rightarrow$$

$$\longrightarrow \vec{P} = r(\sin\theta\sin\phi + \cos\theta)(\sin\theta\cos\phi\,\vec{a}_r + \cos\theta\cos\phi\,\vec{a}_\theta - \sin\phi\,\vec{a}_\phi)$$

## Part 4: Coordinate Transformations

**2.3(b)** **Cylindrical**

$$\vec{Q} = (y\cos\phi + xz\sin\phi)\,\vec{a}_\rho + (-y\sin\phi + xz\cos\phi)\,\vec{a}_\phi + (x+y)\,\vec{a}_z$$

sub. $x = \rho\cos\phi \quad y = \rho\sin\phi$

$$\Rightarrow \vec{Q} = \frac{1}{2}\rho\sin 2\phi\,(1+z)\,\vec{a}_\rho + \rho(z\cos^2\phi - \sin^2\phi)\,\vec{a}_\phi + \rho(\cos\phi + \sin\phi$$

**Spherical**

$$\vec{Q} = \left[\frac{1}{2} r\sin 2\phi\sin^2\theta + \frac{1}{2} r^2\sin^2\theta\cos\theta\sin^2\phi + \frac{1}{2} r(\cos\phi + \sin\phi)\cos 2\theta\right]$$

$$+\left[\frac{1}{4} r\sin 2\theta\sin 2\phi + \frac{1}{2} r^2\cos^2\theta\sin\theta\sin 2\phi - \frac{r}{2}(\cos\phi + \sin\phi)\sin^2\theta\right]$$

$$+\left(-r\sin\theta\sin^2\phi + \frac{1}{2} r^2\sin 2\theta\cos^2\phi\right)\vec{a}_\phi$$

## Part 4: Coordinate Transformations (Cylindrical case)

2.3 (c) $T = \left[\frac{x^2}{x^2+y^2} - y^2\right] a_x + \left[\frac{xy}{x^2+y^2} + xy\right] a_y + a_z$

$$\begin{bmatrix} A\rho \\ A\theta \\ Az \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ax \\ ay \\ az \end{bmatrix}$$

$$\begin{vmatrix} Ar \\ A\Theta \\ A\Phi \end{vmatrix} = \begin{vmatrix} \sin\Theta\cos\Phi & \sin\Theta\sin\Phi & \cos\Theta \\ \cos\Theta\cos\Phi & \cos\Theta\sin\Phi & -\sin\Theta \\ -\sin\Phi & \cos\Phi & 0 \end{vmatrix} \begin{vmatrix} Ax \\ Ay \\ Az \end{vmatrix}$$

$$\boxed{A\rho} = \cos\Phi\left[\frac{x^2}{x^2+y^2} - y^2\right] + \sin\Phi\left[\frac{xy}{x^2+y^2} + xy\right] + 0$$

$$= \cos^3\Phi - \rho^2\sin^2\Phi\cos\Phi + \sin^2\Phi\cos\Phi + \rho^2\sin^2\Phi\cos\Phi = \cos\Phi$$

*Remember :* $x = \rho\cos\phi : y = \rho\sin\phi : z = z$

$$\boxed{A\Phi} = \left[\frac{x^2}{x^2+y^2} - y^2\right](-\sin\Phi) + \left[\frac{xy}{x^2+y^2} + xy\right](\cos\Phi) + 0$$

$$= -\sin\Phi\cos^2\Phi + \rho^2\sin^3\Phi + \sin\Phi\cos^2\Phi + \rho^2\sin\Phi\cos^2\Phi = \rho^2\sin\Phi$$

$\boxed{Az} = 1$ So, $\boxed{T = \cos\Phi\,\overline{a\rho} + \rho^2\sin\Phi\,\overline{a\Phi} + \overline{az}}$

## Part 4: Coordinate Transformations (Spherical case)

$$Ar = \left[\frac{x^2}{x^2+y^2} - y^2\right]\sin\Theta\cos\phi + \left[\frac{xy}{x^2+y^2} + xy\right]\sin\Theta\sin\phi + \cos\Theta$$

$$= \sin\Theta\cos^3\phi - r^2\sin^3\Theta\sin^2\Phi\cos\phi + \cos\phi\sin^2\phi\sin\Theta + r^2\sin^3\Theta\cos\phi\sin^2\phi + \cos\Theta$$

$$= \sin\Theta\cos\Phi(\cos^2\Phi + \sin^2\Phi + r^2\sin^2\Theta\sin^2\Phi - r^2\sin^2\Phi\sin^2\Theta) + \cos\Theta$$

$$= \sin\Theta\cos\Phi + \cos\Theta$$

*Remember :* $x = r\sin\theta\cos\phi$

$$A\Theta = \left[\frac{x^2}{x^2+y^2} - y^2\right]\cos\Theta\cos\phi + \left[\frac{xy}{x^2+y^2} + xy\right]\cos\Theta\sin\phi - \sin\Theta$$

$$= (\cos^2\phi - r^2\sin^2\Theta\cos^2\phi)(\cos\Theta\cos\phi) + (\cos\phi\sin\phi + r^2\sin^2\Theta$$
$$\cos\phi\sin\phi)(\cos\Theta\sin) + \cos\Theta$$

$$= r^2\sin^2\Theta\cos\phi\cos\Theta(\cos^2\Theta + \sin^2\phi) + \cos\Theta\cos\phi(\sin^2\phi - \cos^2\phi) + \cos\Theta$$

$$= \cos\Theta\cos^3\Phi - r^2\sin^2\Theta\sin^2\Phi\cos\Theta\cos\Phi + \cos\Theta\sin^2\Phi\cos\Phi + r^2\sin^2\Theta\cos\Phi\sin^2\Phi\cos\Theta - \sin\Theta$$

$$= \cos\Theta\cos\Phi - \sin\Theta$$

## Part 4: Coordinate Transformations (Spherical case)

$$A\phi = -\left[\frac{x^2}{x^2+y^2} - y^2\right](-\sin\phi) + \left[\frac{xy}{x^2+y^2} + xy\right]\cos\phi + 0$$

$$= (\cos^2\phi - r^2\sin^2\Theta\cos^2\phi)(-\sin\phi) + (\cos\phi\ \sin\phi + r^2\sin^2\Theta\cos\phi\sin\phi)\cos\phi + 0$$
$$= -\sin\Phi\cos^2\Phi + r^2\sin^2\Theta\sin^3\Phi + \cos^2\Phi\sin\Phi + r^2\sin^2\Theta\cos^2\Phi\sin\Phi$$
$$= r^2\sin^2\Theta\sin\phi$$

$$\boxed{T = (\sin\Theta\cos\Phi + \cos\Theta)\,\overline{a_r} + (\cos\Theta\cos\Phi - \sin\Theta)\,\overline{a_\Theta} + r^2\sin^2\Theta\sin\Phi\,\overline{a_\Phi}}$$