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Appendix B: PROBLEM SESSIONS

PROBLEM SESSION I

Part (1): Visualization of surfaces in 3D coordinate systems

Describe the following surfaces separately:

a) x=-5, z=2. b) $\rho=3, \Phi=3\pi/2.$ c) $\rho=\sqrt{5}, z=-2.$ d) r=5, $\Phi=\pi/3.$ e) $\theta=\pi/2, \Phi=\pi/2.$ f) r=2, $\Phi=0.$ g) y=5.

Part (2): Visualization of surfaces in 3D coordinate systems

Describe the intersection of surfaces (1) and (2):

Part (3): Vector Algebra

Problems 1.5 and 1.10 from the text book.

- **1.5** For $\mathbf{U} = U_x \mathbf{a}_x + 5 \mathbf{a}_y \mathbf{a}_z$, $\mathbf{V} = 2 \mathbf{a}_x V_y \mathbf{a}_y + 3 \mathbf{a}_z$, and $\mathbf{W} = 6 \mathbf{a}_x + \mathbf{a}_y + W_z \mathbf{a}_z$, obtain U_x , V_y , and W_z such that \mathbf{U} , \mathbf{V} , and \mathbf{W} are mutually orthogonal.
- **1.10** Verify that
 - (a) $\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = 0 = \mathbf{B} \cdot (\mathbf{A} \times \mathbf{B})$
 - **(b)** $(\mathbf{A} \cdot \mathbf{B})^2 + |\mathbf{A} \cdot \mathbf{B}|^2 = (AB)^2$
 - (c) If $\mathbf{A} = (A_x, A_y, A_z)$, then $\mathbf{A} = (\mathbf{A} \cdot \mathbf{a}_x) \mathbf{a}_x + (\mathbf{A} \cdot \mathbf{a}_y) \mathbf{a}_y + (\mathbf{A} \cdot \mathbf{a}_z) \mathbf{a}_z$.

Part (4): Coordinate transformations

Problems 2.1, 2.2, 2.3 and 2.15 from the text.

- 2.1 Convert the following points to Cartesian coordinates:
 - (a) $P_1(5, 120^\circ, 0)$
 - **(b)** $P_2(1, 30^\circ, -10)$
 - (c) $P_3(10, 3\pi/4, \pi/2)$
 - (d) $P_4(3, 30^\circ, 240^\circ)$
- **2.2** Express the following points in cylindrical and spherical coordinates:
 - (a) P(1, -4, -3)
 - **(b)** Q(3, 0, 5)
 - (c) R(-2, 6, 0)

2.3 Express the following points in cylindrical and spherical coordinates:

- (a) $\mathbf{P} = (y + z) \mathbf{a}_{\mathbf{x}}$
- **(b)** $\mathbf{Q} = y \mathbf{a}_{x} + x z \mathbf{a}_{y} + (x + y) \mathbf{a}_{z}$
- (c) $\mathbf{T} = \left[\frac{x^2}{x^2 + y^2} y^2\right]a_x + \left[\frac{xy}{x^2 + y^2} + xy\right]a_y + a_z$

(d)
$$\mathbf{S} = \frac{\mathbf{y}}{\mathbf{x}^2 + \mathbf{y}^2} \mathbf{a}_x - \frac{\mathbf{x}}{\mathbf{x}^2 + \mathbf{y}^2} \mathbf{a}_y + 10\mathbf{a}_z$$

2.15 If $\mathbf{J} = r \sin \theta \cos \phi a_r - \cos 2\theta \sin \phi a_{\theta} + \tan \frac{\theta}{2} \ln r a_{\phi}$, determine the vector component of \mathbf{J} at $T(2, \pi/2, 3\pi/2)$ that is

- (a) Parallel to $\mathbf{a}_{\mathbf{z}}$.
- (b) Normal to the surface $\Phi = 3\pi/2$.
- (c) Tangential to the spherical surface r = 2.
- (d) Parallel to the line y = -2, z = 0.