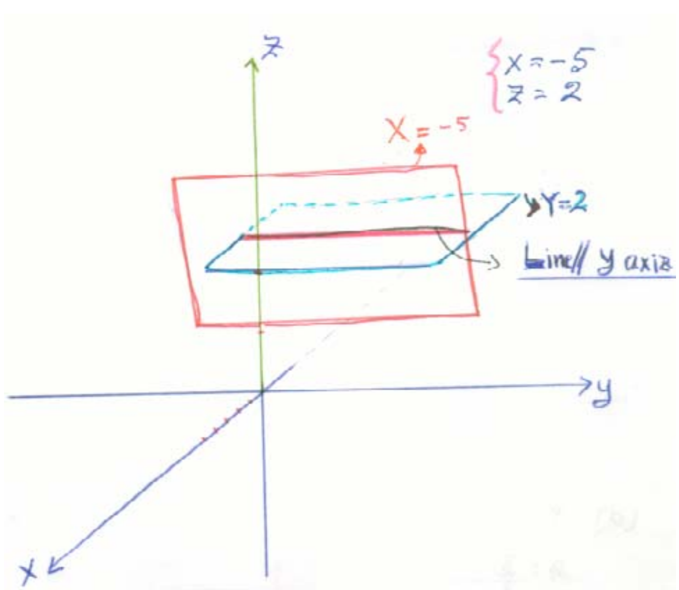
EE 340 Electromagnetics Lab

Problem Session #1

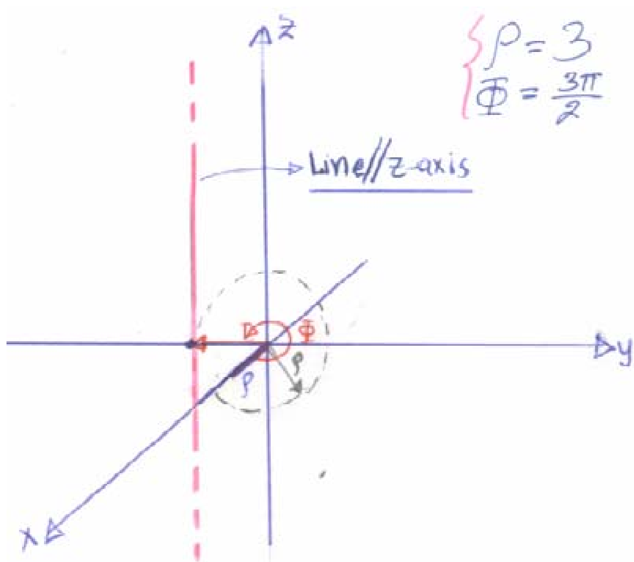
Part 1: Visualization of surfaces in 3D coordinate systems:

(a)



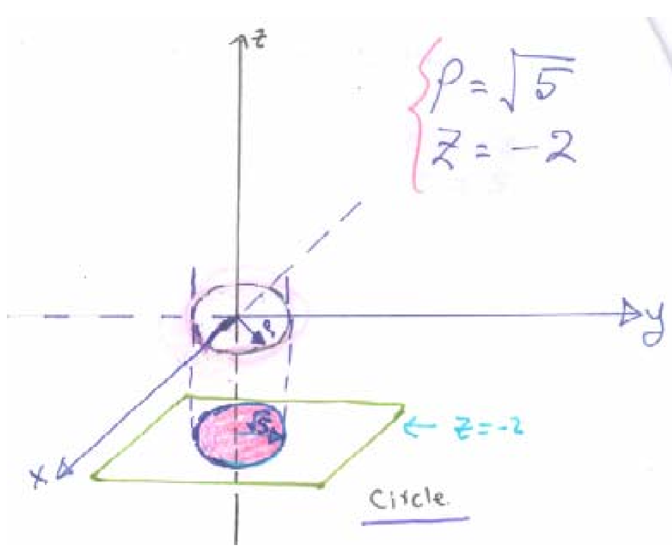
Part 1: Visualization of surfaces in 3D coordinate systems:

(b)



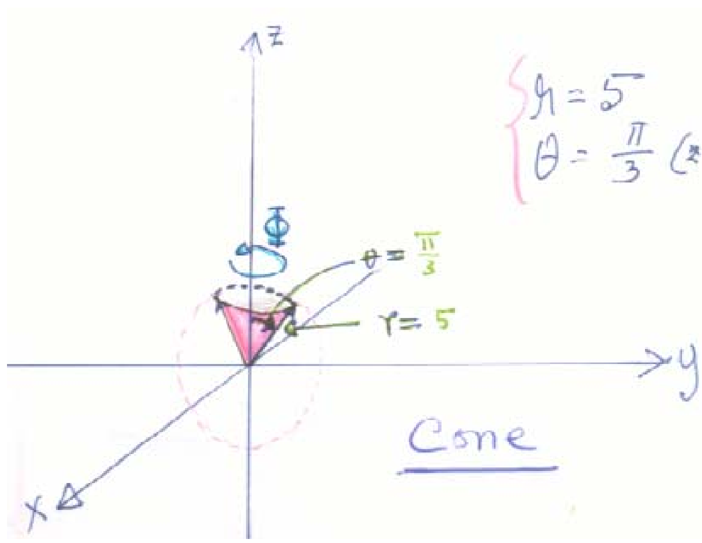
Part 1: Visualization of surfaces in 3D coordinate systems:

(c)



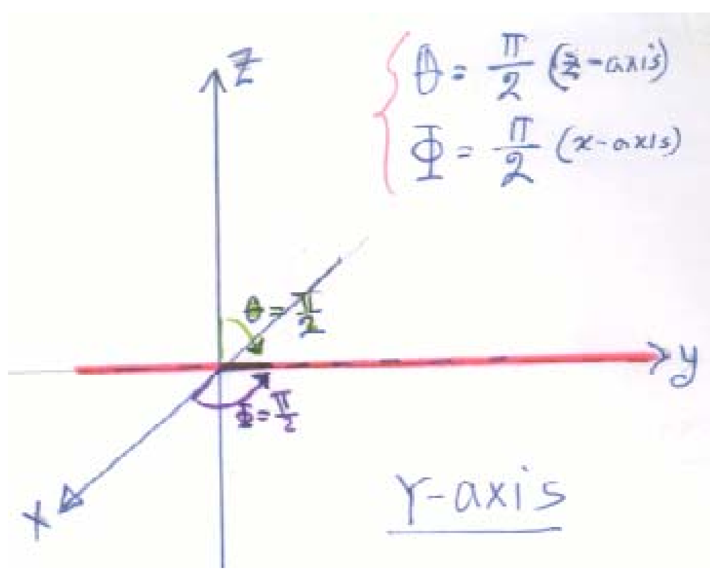
Part 1: Visualization of surfaces in 3D coordinate systems:

(d)



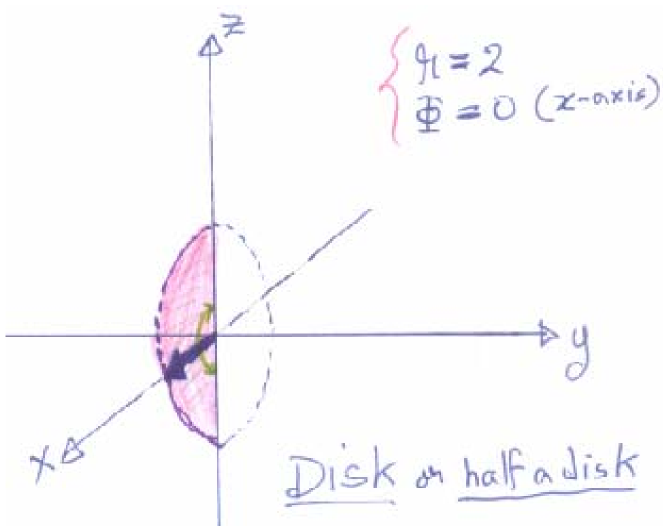
Part 1: Visualization of surfaces in 3D coordinate systems:

(e)



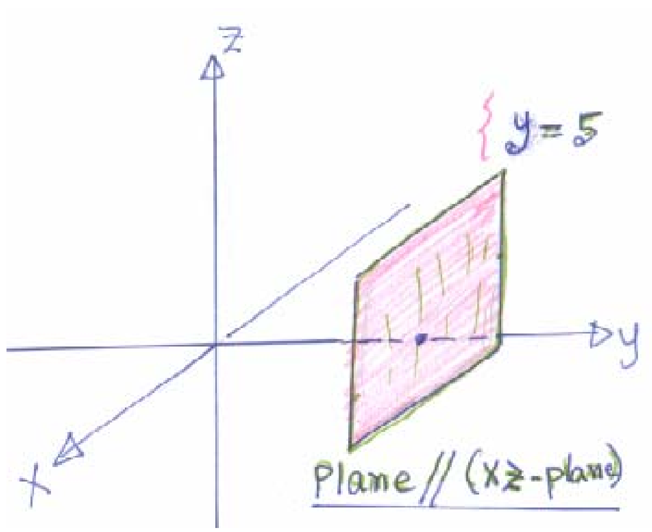
Part 1: Visualization of surfaces in 3D coordinate systems:

(f)



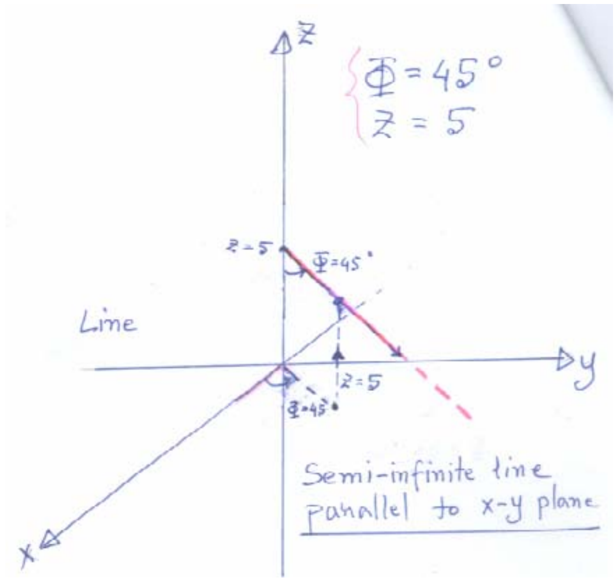
Part 1: Visualization of surfaces in 3D coordinate systems:

(g)



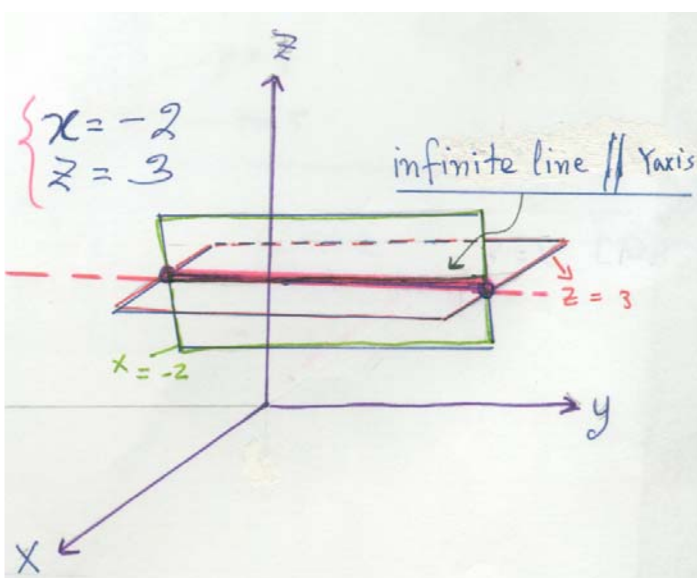
Part 2: Visualization of surfaces in 3D coordinate systems:

(a)



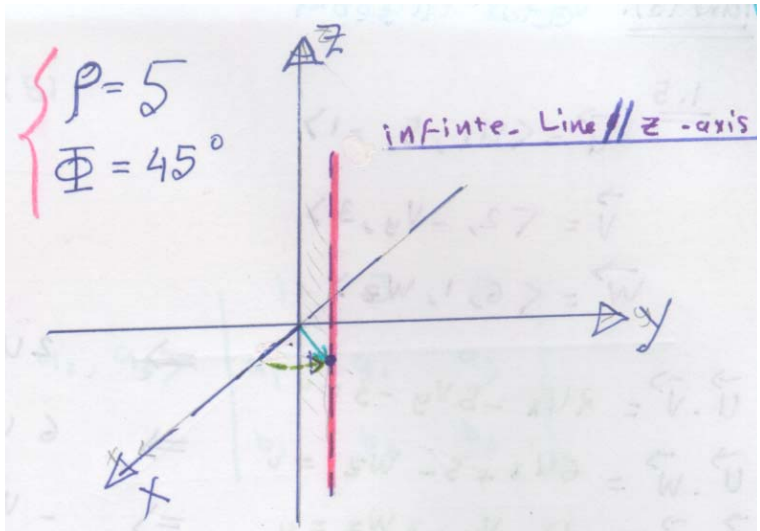
Part 2: Visualization of surfaces in 3D coordinate systems:

(b)



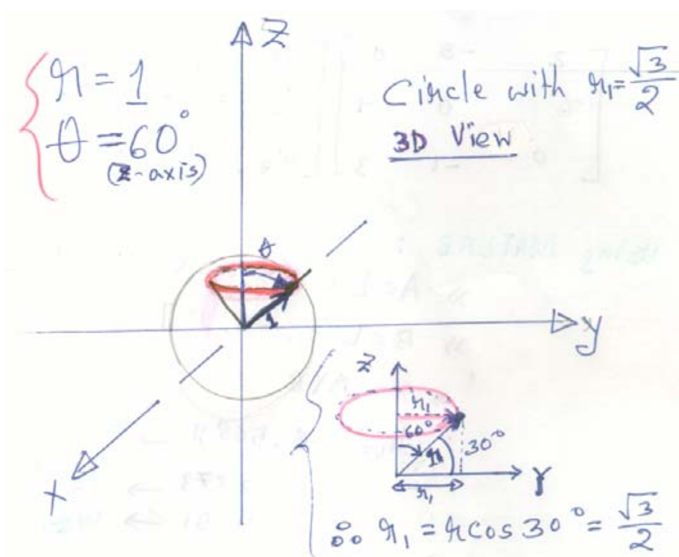
Part 2: Visualization of surfaces in 3D coordinate systems:

(c)



Part 2: Visualization of surfaces in 3D coordinate systems:

(d)



Part(3): Vector Algebra

Problem 1.5

*For $\overline{U} = U_x a_x + 5\,a_y - a_z$, $\overline{V} = 2\,a_x - V_y a_y + 3a_z$ and $\overline{W} = 6a_x + a_y + W_z a_z$, obtain U_x, V_y and W_z such that **U**, **V** and **W** are mutually orthogonal ?*

$\overline{U}.\overline{V} = (uXa_x + 5\,a_y - a_z).(2a_x - vy\,a_y + 3\,a_z)$

$= 2Ux - 5\,Vy - 3 = 0$ ……………………… (1)

$\overline{U}.\overline{W} = (Ux\,a_x + 5a_y - a_z).(6a_x + a_y + Wz\,a_z)$

$= 6Ux + 5 - Wz = 0$ ……… ………… (2)

$\overline{V}.\overline{W} = (2a_x - Vy\,a_y + 3\,a_z).(6a_x + a_y + Wz\,a_z)$

$= 12 - Vy + 3Wz = 0$ ……………………… (3)

eq.1 $*(-3)$ + eq.2 $\Rightarrow 15v_y - w_z + 14 =$ — (4)

eq.3 $*(15)$ + eq.4 $\Rightarrow w_z = -97/22$

Problem 1.10

(a) $\overline{A}.(\overline{A}X\overline{B}) = 0 = \overline{B}\,(\overline{A}X\overline{B})$

$$(\overline{A}X\overline{B}) = \begin{vmatrix} a_x & a_y & a_z \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$$

$= a_x\,(AyBz - AzBy) - a_y\,(AxBz - AzBx) + a_z\,(AxBy - AyBx)$

$A.\ (AXB) = AxAyBz - AxAzBy - \underline{AyAxBz} + AyAzBxAzAxBy - AyAzBx = 0$

$B.\ (AXB) = AyBxBz - AzBxBy - AxByBz + AzBxBy + AxByBz - AyBxBz = 0$

So this statement is true.

Remember (dot prod.)

$a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$

$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$

Part 3: Vector Algebra

Problem 1.5

(b) $(\overline{A}.\overline{B})^2 + |\overline{A}X\overline{B}|^2 = (AB)^2$

$|AXB|^2 = (AB)^2\sin^2\Theta$ ⟶ direction ignored.

$(A.B)^2 + |AXB|^2 = (AB\cos\Theta)^2 + (AB\sin\Theta)^2 = (AB)^2\cos^2\Theta + (AB)^2\sin^2\Theta$

$= (AB)^2(\cos^2\Theta + \sin^2\Theta) = (AB)^2 * 1 = RHS$

(c) $\overline{A} = Ax\,\overline{a_x} + Ay\,\overline{a_y} + Az\,\overline{a_z}$

Because $A.\,a_x \Leftarrow Ax$

$A.\,a_y \Leftarrow Ay$

$A.\,a_z \Leftarrow Az$

subs. Ax, Ay & Az leads;

$$\overline{A} = (\overline{A}\cdot\overline{a}_x)\overline{a}_x + (\overline{A}\cdot\overline{a}_y)a_y + (\overline{A}\cdot\overline{a}_z)\overline{a}_z$$

Part 4: Coordinate Transformations

Problem 2.1 convert the following points to Cartesian coordinates:

$X = \rho\cos\Theta$ & $Y = \rho\sin\Theta$ & $Z = Z$

(a) $P(5,\ 120^o,\ 0)$

$X = 5\cos 120 = -2.5$ $\quad Y = 5\sin 120 = 4.33$ $\quad Z = 0$

$P(-2.5, 4.33,\ 0)$

(b) $P(1, 30^o, -10)$

$X = 1\cos 30 = 0.866$ $\quad Y = 1\sin 30 = 0.5$ $\quad Z = -10$

$P(0.866, 0.5, -10)$

(c) $P(10, 3\pi/4,\ \pi/2)$

$X = r\sin\Theta\cos\Phi$ $\quad Y = r\sin\Theta\sin\Phi$ $\quad Z = r\cos\Theta$

$X = 10\sin 3\pi/4\cos\pi/2 = 0$

$Y = 10\sin 3\pi/4 \sin\pi/2 = 7.071$

$Z = 10\cos 3\pi/4 = -7.071$

$P(0,\ 7.071,\ -7.071)$

Part 4: Coordinate Transformations

Problem 2.2 Express in Cylindrical and spherical coord

(a) $P(1,\ -4,\ -3)$

$\rho = \sqrt{1+16} = \sqrt{17}$, $\quad \Phi = \tan^{-1}(-4/1) = -75.94^o = 284.06^\circ$, $\quad$ & $z = -3$

$P(\sqrt{17}, -75.94, -3) \longrightarrow$ Cylindrical

$r = \sqrt{(1+16+9)} = \sqrt{26}$ $\quad$ as $r = \sqrt{x^2+y^2+z^2}$

$\Theta = \tan^{-1}(\sqrt{17}/-3) = -53.96^o$

$P(\sqrt{26}, -53.96^o, -75.94^o) \longrightarrow$ Spherical

(b) $Q(3,\ 0,\ 5)$

$\rho = \sqrt{9+0} = 3$, $\quad \Phi = \tan^{-1}(0/3) = 0^o$ & $z = 5$

$Q(3, 0, 5) \longrightarrow$ Cylindrical

$Q(\sqrt{34}, 30.96^\circ, 0) \longrightarrow$ Spherical

(c) $R(-2, 6, 0)$

$\rho = \sqrt{4+36} = \sqrt{40}$, $\quad \Phi = \tan^{-1}(6/-2) = -71.565^o$ & $z = 0$

$P(\sqrt{40}, -71.565^o,\ 0)$

Part 4: Coordinate Transformations

2.3(a)

$$\begin{pmatrix} P_\rho \\ P_\phi \\ P_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y+z \\ 0 \\ 0 \end{pmatrix} \quad (eq. 2.13)$$

Cylindrical

$$P_\rho = \cos\phi\,(y+z) = \cos\phi\,(\rho\sin\phi + z)$$

$$P_\phi = -\sin\phi\,(y+z) = -\sin\phi\,(\rho\sin\phi + z)$$

$$P_z = 0$$

$$\longrightarrow \vec{P} = (\rho\sin\phi + z)(\cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi)$$

Spherical

$$P_r = (y+z)(\sin\theta\cos\phi) = (r\sin\theta\sin\phi + r\cos\theta)(\sin\phi\cos\phi)$$

$$P_\theta = (y+z)(\cos\theta\cos\phi) \Rightarrow \quad (eq. 2.27)$$

$$P_\phi = (y+z)(-\sin\phi) \Rightarrow$$

$$\longrightarrow \vec{P} = r(\sin\theta\sin\phi + \cos\theta)(\sin\theta\cos\phi\,\vec{a}_r + \cos\theta\cos\phi\,\vec{a}_\theta - \sin\phi\,\vec{a}_\phi)$$

Part 4: Coordinate Transformations

2.3(b) **Cylindrical**

$$\vec{Q} = (y\cos\phi + xz\sin\phi)\vec{a}_\rho + (-y\sin\phi + xz\cos\phi)\vec{a}_\phi + (x+y)\vec{a}_z$$

Sub. $x = \rho\cos\phi \quad y = \rho\sin\phi$

$$\Rightarrow \vec{Q} = \frac{1}{2}\rho\sin 2\phi\,(1+z)\,\vec{a}_\rho + \rho(z\cos^2\phi - \sin^2\phi)\,\vec{a}_\phi + \rho(\cos\phi + \sin\phi$$

Spherical

$$\vec{Q} = \left[\frac{1}{2}r\sin 2\phi\sin^2\theta + \frac{1}{2}r^2\sin^2\theta\cos\theta\sin^2\phi + \frac{1}{2}r(\cos\phi + \sin\phi)\cos 2\theta\right]$$

$$+\left[\frac{1}{4}r\sin 2\theta\sin 2\phi + \frac{1}{2}r^2\cos^2\theta\sin\theta\sin 2\phi - \frac{r}{2}(\cos\phi + \sin\phi)\sin^2\theta\right]$$

$$+\left(-r\sin\theta\sin^2\phi + \frac{1}{2}r^2\sin 2\theta\cos^2\phi\right)\vec{a}_\phi$$

Part 4: Coordinate Transformations (Cylindrical case)

2.3 (c) $T=\left[\frac{x^2}{x^2+y^2}-y^2\right]ax+\left[\frac{xy}{x^2+y^2}+xy\right]ay+az$

$$\begin{bmatrix} A\rho \\ A\theta \\ Az \end{bmatrix}=\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} ax \\ ay \\ az \end{bmatrix}$$

$$\begin{vmatrix} Ar \\ A\Theta \\ A\Phi \end{vmatrix}=\begin{vmatrix} \sin\Theta\cos\Phi & \sin\Theta\sin\Phi & \cos\Theta \\ \cos\Theta\cos\Phi & \cos\Theta\sin\Phi & -\sin\Theta \\ -\sin\Phi & \cos\Phi & 0 \end{vmatrix}\begin{vmatrix} Ax \\ Ay \\ Az \end{vmatrix}$$

$$\boxed{A\rho}=\cos\Phi\left[\frac{x^2}{x^2+y^2}-y^2\right]+\sin\Phi\left[\frac{xy}{x^2+y^2}+xy\right]+0$$

$$=\cos^3\Phi-\rho^2\sin^2\Phi\cos\Phi+\sin^2\Phi\cos\Phi+\rho^2\sin^2\Phi\cos\Phi=\cos\Phi$$

Remember : $x=\rho\cos\phi$: $y=\rho\sin\phi$: $z=z$

$$\boxed{A\Phi}=\left[\frac{x^2}{x^2+y^2}-y^2\right](-\sin\Phi)+\left[\frac{xy}{x^2+y^2}+xy\right](\cos\Phi)+0$$

$$=-\sin\Phi\cos^2\Phi+\rho^2\sin^3\Phi+\sin\Phi\cos^2\Phi+\rho^2\sin\Phi\cos^2\Phi=\rho^2\sin\Phi$$

$\boxed{Az}=1$ So, $\boxed{T=\cos\Phi\,\overline{a\rho}+\rho^2\sin\Phi\,\overline{a\Phi}+\overline{az}}$

Part 4: Coordinate Transformations (Spherical case)

$$Ar=\left[\frac{x^2}{x^2+y^2}-y^2\right]\sin\Theta\cos\phi+\left[\frac{xy}{x^2+y^2}+xy\right]\sin\Theta\sin\phi+\cos\Theta$$

$$=\sin\Theta\cos^3\phi-r^2\sin^3\Theta\sin^2\Phi\cos\phi+\cos\phi\sin^2\phi\sin\Theta+r^2\sin^3\Theta\cos\phi\sin^2\phi+\cos\Theta$$

$$=\sin\Theta\cos\Phi(\cos^2\Phi+\sin^2\Phi+r^2\sin^2\Theta\sin^2\Phi-r^2\sin^2\Phi\sin^2\Theta)+\cos\Theta$$

$$=\sin\Theta\cos\Phi+\cos\Theta$$

Remember : $x=r\sin\theta\cos\phi$

$$A\Theta=\left[\frac{x^2}{x^2+y^2}-y^2\right]\cos\Theta\cos\phi+\left[\frac{xy}{x^2+y^2}+xy\right]\cos\Theta\sin\phi-\sin\Theta$$

$$=(\cos^2\phi-r^2\sin^2\Theta\cos^2\phi)(\cos\Theta\cos\phi)+(\cos\phi\sin\phi+r^2\sin^2\Theta\cos\phi\sin\phi)(\cos\Theta\sin\,)+\cos\Theta$$

$$=r^2\sin^2\Theta\cos\phi\cos\Theta(\cos^2\Theta+\sin^2\phi)+\cos\Theta\cos\phi(\sin^2\phi-\cos^2\phi)+\cos\Theta$$

$$=\cos\Theta\cos^3\Phi-r^2\sin^2\Theta\sin^2\Phi\cos\Theta\cos\Phi+\cos\Theta\sin^2\Phi\cos\Phi+r^2\sin^2\Theta\cos\Phi\sin^2\Phi\cos\Theta-\sin\Theta$$

$$=\cos\Theta\cos\Phi-\sin\Theta$$

Part 4: Coordinate Transformations (Spherical case)

$$A\phi = -\left[\frac{x^2}{x^2+y^2} - y^2\right](-\sin\phi) + \left[\frac{xy}{x^2+y^2} + xy\right]\cos\phi \; +0$$

$$=(\cos^2\phi - r^2\sin^2\Theta\cos^2\phi)(-\sin\phi) + (\cos\phi\ \sin\phi + r^2\sin^2\Theta\cos\phi\sin\phi)\cos\phi + 0$$
$$= -\sin\Phi\cos^2\Phi + r^2\sin^2\Theta\sin^3\Phi + \cos^2\Phi\sin\Phi + r^2\sin^2\Theta\cos^2\Phi\sin\Phi$$
$$= r^2\sin^2\Theta\sin\phi$$

$$\boxed{T = (\sin\Theta\cos\Phi + \cos\Theta)\,\overline{a_r} + (\cos\Theta\cos\Phi - \sin\Theta)\,\overline{a_\Theta} + r^2\sin^2\Theta\sin\Phi\,\overline{a_\Phi}}$$