# King Fahd University of Petroleum and Minerals
## Electrical Engineering Department

# PROBLEM SESSION # 3

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**3.15** Determine the gradient of the following scalar fields:
- **(a)** $U = 4xz^2 + 3yz$.
- **(b)** $V = e^{(2x+3y)} \cos 5z$.
- **(c)** $W = 2\rho(z^2 + 1) \cos \varphi$.
- **(d)** $T = 5\rho\, e^{-2z} \sin \varphi$.
- **(e)** $H = r^2 \cos \theta \cos \varphi$.
- **(f)** $Q = (\sin \theta \sin \varphi) / r^3$.

**3.18** Find the divergence and curl of the following vector fields:
- **(a)** $\mathbf{A} = e^{xy} \mathbf{a_x} + \sin xy\, \mathbf{a_y} + \cos^2 xz\, \mathbf{a_z}$
- **(b)** $\mathbf{B} = \rho\, z^2 \cos \varphi\, \mathbf{a_\rho} + z \sin^2 \varphi\, \mathbf{a_z}$
- **(c)** $\mathbf{C} = r \cos \theta\, \boldsymbol{a_r} - \dfrac{1}{r} \sin \theta\, \boldsymbol{a_\theta} + 2\, r^2 \sin \theta\, \boldsymbol{a_\phi}$

**3.30** Given that $\boldsymbol{E} = \dfrac{1}{r^4} \sin^2 \phi\, \boldsymbol{a_r}$, evaluate

- **(a)** $\displaystyle \oint_S E \cdot dS$

- **(b)** $\displaystyle \int_V (\nabla \cdot E)\, dv$

over the region between the spherical surfaces $r = 2$ and $r = 4$.

**3.33** Calculate the total outward flux of vector
$$\boldsymbol{F} = \rho^2 \sin \phi\, \boldsymbol{a_\rho} + z \cos \phi\, \boldsymbol{a_\phi} + \rho\, z\, \boldsymbol{a_z}$$
through the hollow cylinder defined by $2 \le \rho \le 3$, $0 \le z \le 5$.

**3.39** Given the vector field
$$R = (2\, x^2 y + yz)a_x + (xy^2 - xz^3)a_y + (c\, xyz - 2x^2 y^2)a_z$$
determine the value of $c$ for **R** to be solenoidal.

**3.40** If the vector field
$$T = (\alpha xy + \beta z^3)a_x + (3x^2 - \gamma z)a_y + (3\, xz^2 - y)a_z$$
is irrotational, determine $\alpha$, $\beta$, and $\gamma$. Find $\nabla \cdot \mathbf{T}$ at $(2, -1, 0)$.