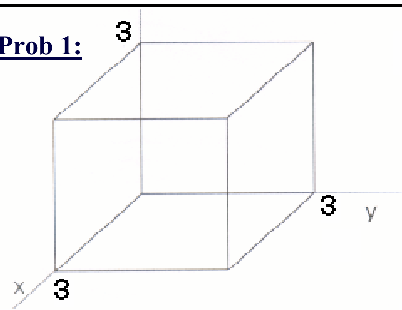
EE 340 Electromagnetics Lab

Problem Session #2

Prob 1:



a) $\int_S F = \int_{left} + \int_{right} + \int_{top} + \int_{bottom} + \int_{front} + \int_{back}$

since $\mathbf{F}=\mathbf{a}_y$, I have only y component in $\mathbf{F}$, then there is a value for the surfaces in the direction of +ve y and –ve y

$$\int_S F = \int_{left} + \int_{right} = 5\int_{z=0}^{3}\int_{x=0}^{3} dxdz - 5\int_{z=0}^{3}\int_{x=0}^{3} dxdz =$$

$$= 5\int_{z=0}^{3}\left[xdz\right]_0^3 - 5\int_{z=0}^{3}\left[xdz\right]_0^3 = 45 - 45 = 0 = 0$$

b) $\mathbf{F}=x^2y^2\,\mathbf{a}_x$

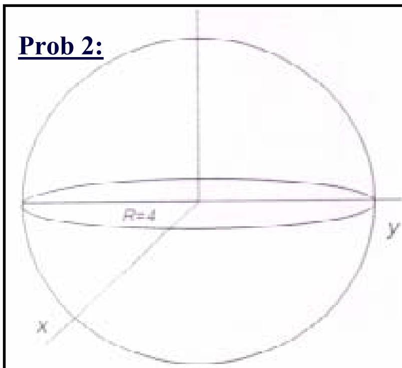
since I have only x component in $\mathbf{F}$, then there is a value for the surfaces in the direction of +ve x and –ve x

$$\int_{front} F.ds = \int_{z=0}^{3}\int_{y=0}^{3} x^2y^2 dydz \Big|_{x=3} = 9\int_{z=0}^{3}\int_{y=0}^{3} y^2 dydz = 9\int_{z=0}^{3}\left[\frac{y^3}{3}\right]_0^3 dz = 81\int_{z=0}^{3} dz = 81\left[z\right]_0^3 = 243$$

$$\int_{back} F.ds = \int_{z=0}^{3}\int_{y=0}^{3} x^2y^2 dydz \Big|_{x=0} = 0$$

$$\int_S F.ds = \int_{front} + \int_{back} = 243 + 0 = 243$$

Prob 2:



$\overline{ds} = r^2 \sin\theta d\theta d\phi \, \overline{ar}$

a) $F = \dfrac{ar}{r^2}$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi = -\int_{\phi=0}^{2\pi}[\cos\theta]_{\theta=0}^{\Pi} d\phi = -\int_{\phi=0}^{2\Pi}[-1-1]d\phi = 2[\phi]_{\phi=0}^{2\Pi} = 4\Pi$$

b) $F = \dfrac{\sin^2\phi}{r^2}\overline{ar} + \cos\phi \overline{a_\theta} \Rightarrow F.ds = \dfrac{\sin^2\phi}{r^2}$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} r \sin\theta \sin^2\phi d\theta d\phi = 8\pi$$

$r = 4$

$\int_0^{\pi} \sin\theta \, d\theta = 2$

$\int_0^{2\pi} \sin^2\Phi \, d\Phi = \pi$

because of integration of sin function over one period

Prob 2: c) F = ax

$F = \sin\theta\cos\phi \overline{ar} + \cos\theta\cos\phi \overline{a_\theta} - \sin\phi \overline{a\phi}$

$F.ds = r^2 \sin^2 \cos\phi\theta d\theta d\phi$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} r^2 \sin^2\theta \cos\phi d\theta d\phi = 0$$

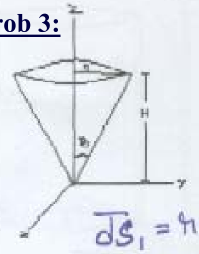
$r^2 = 16$

$\int_0^{\pi} \sin^2\theta \, d\theta = \dfrac{\pi}{2}$

$\int_0^{2\pi} \cos\Phi \, d\phi = 0$

because of integration of sin function over one period

Prob 3:



$\overline{dS}_1 = r\sin\theta\, dr\, d\phi\ \overline{a_\theta}$ (sp.)

$\bar{a}_r = \sin\theta\cos\phi\,\bar{a}_x + \sin\theta\sin\phi\,\bar{a}_y + \cos\theta\,\bar{a}_z$

$\overline{a_r} = \sin\theta\cos\phi(\cos\phi\,\overline{a_\rho}) - \sin\theta\cos\phi\cdot(\sin\phi\, a_\phi) + \sin\theta\sin\phi(\sin\phi\,\overline{a_\rho}) + \sin\theta\cdot\sin\phi(\cos\phi\, a_\phi) + \cos\theta\, a_z$

$\bar{a}_r = \sin\theta(\cos^2\phi+\sin^2\phi)\overline{a_\rho} + \cos\theta\, a_z$

$\bar{a}_r = \boxed{\sin\theta\,\overline{a_\rho} + \cos\theta\,\overline{a_z}}$

$\therefore \bar{F} = r\overline{a_r} = r\sin\theta\,\bar{a}_\rho + r\cos\theta\,\bar{a}_z = \rho\,\bar{a}_\rho + z\,\bar{a}_z$

Surface 1:

a) $\bar{F}$ = rar $= r\overline{a_r}$

f.ds =0

b) §F.ds = ∫∫ r²sinθdrdø = $\frac{2\pi}{3}$ a(h²+a²)

c) F = cosøaø +r aθ

Surface 2:

a)F = r$\overline{ar}$ = r sinθ$\overline{ap}$ + rcosθ$\overline{az}$

= pap+zaz

Prob 3:

§F.ds = ∫∫zpdpdø

= z∫p²| dø = πha²
  2

b)F = raθ = r cosθ$\overline{ap}$-rsinθ$\overline{az}$ = z $\overline{ap}$ − p$\overline{az}$

§F.ds = -∫∫p²dpdø = $-\frac{2\pi a^3}{3}$

c) $\bar{F}$ = cosøaø+raθ = rcosθ$\overline{ap}$ +cosøaø-rsinθ$\overline{az}$

= z$\overline{ap}$+cosøaø −p$\overline{az}$

§F.ds = -∫∫p²dpdø = $-\frac{\pi a^3 2}{3}$

$\boxed{\overline{dS} = \rho\, d\phi\, d\rho\ \overline{a_z}}$ (cyl.)

$\oint \bar{F}\cdot\bar{ds} = \iint(\rho\bar{a}_\rho + z\bar{a}_z)\cdot(\rho\, d\phi\, d\rho\,\bar{a}_z)$

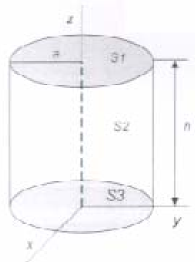
$\therefore = \iint z\rho\, d\phi\, d\rho$

$\therefore = z\int_0^{2\pi}\int_0^a \rho\, d\rho\, d\phi$

$\therefore = (h)\int_0^{2\pi} \frac{\rho^2}{2}\Big]_0^a d\phi$

$\therefore = \frac{ha^2}{2}\cdot 2\pi = \boxed{\pi h a^2}$

Prob 4a:



$ds1 - \rho d\phi d\rho \bar{a_z}$
$ds2 - \rho d\phi dz \bar{a_\rho}$
$ds3 = - \rho d\phi d\rho \bar{a_z}$

$ds3 =$ "—" due to direction

a) $F = \rho^2 a_\rho + \rho \sin\phi a_\phi + \rho^2 \sin\phi \bar{a_z}$

$$\int_{S1} = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \rho^3 \sin\phi d\phi d\rho = 0 \quad (\bar{a_z})$$

$$\int_0^a \rho^3 d\rho = \left.\frac{\rho^4}{4}\right]_0^a = \frac{a^4}{4}$$

$$\int_0^{2\pi} \sin\phi d\phi = -\cos\phi\Big]_0^{2\pi} = -(1-1) = 0$$

integration of sin over one period

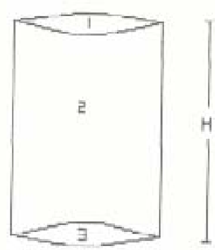
$$\int_{S2} = \int_{z=0}^{h} \int_{\phi=0}^{2\pi} \rho^3 d\phi dz = [a^3][2\pi][h] = 2\pi h a^3$$

$$\int_{S3} = -\int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \rho^3 \sin\phi d\phi d\rho = 0$$

integration of sin over one period

$$\int F = \int_{S1} + \int_{S2} + \int_{S3} = 2\pi h a^3$$

Prob 4b:



a) $F = \rho^2 a_\rho + \rho \sin\phi a_\phi + \rho^2 \sin\phi a_z$

$\oint F.ds1 = \iint \rho^3 d\phi dz$
$= 2\pi a^3 h$

$\oint F.ds2 = \iint \rho^3 \sin\phi d\phi d\rho$
$= \int -\cos\phi| \rho^3 d\rho = 0$

$F.ds3 = 0$

⇨ $F.ds = 2\pi a^3 h$

b) $F = x\bar{a_x} + z\bar{a_z}$
$= x\cos\phi \bar{a_\rho} - x\sin\phi \bar{a_\phi} + z\bar{a_z}$
$= \rho\cos^2\phi \bar{a_\rho} - \rho\cos\phi\sin\phi \bar{a_\phi} + z\bar{a_z}$

$\oint F.ds1 = \iint \rho^2\cos^2\phi d\phi dz$ $\left\{ \begin{matrix} z = 0 \to h \\ \phi = 0 \to 2\pi \end{matrix} \right\}$
$= \pi a^2 h$

$\oint F.ds2 = \iint \rho z d\phi d\rho a_z$ $\left\{ \begin{matrix} \rho = 0 \to a \\ \phi = 0 \to 2\pi \end{matrix} \right\}$
$= h\iint \rho d\rho d\phi$
$= \pi a^2 h$

$\oint F.ds3 = 0$ (as $z = 0$)