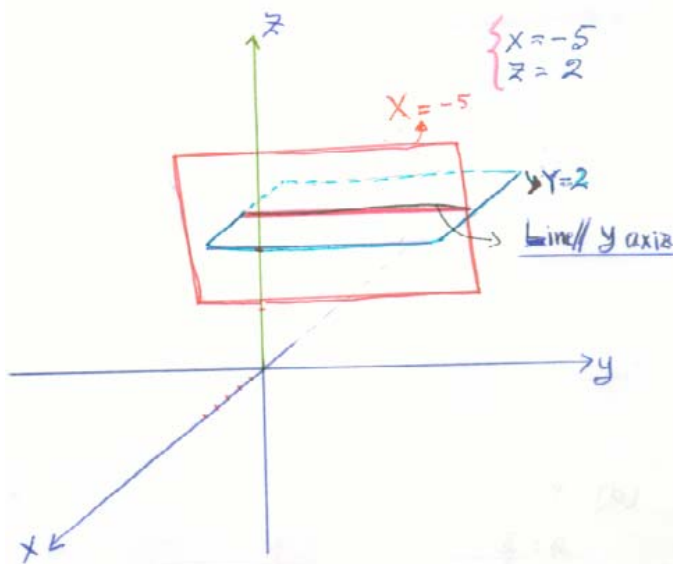# EE 340
Electromagnetics
Lab

Problem Session #1

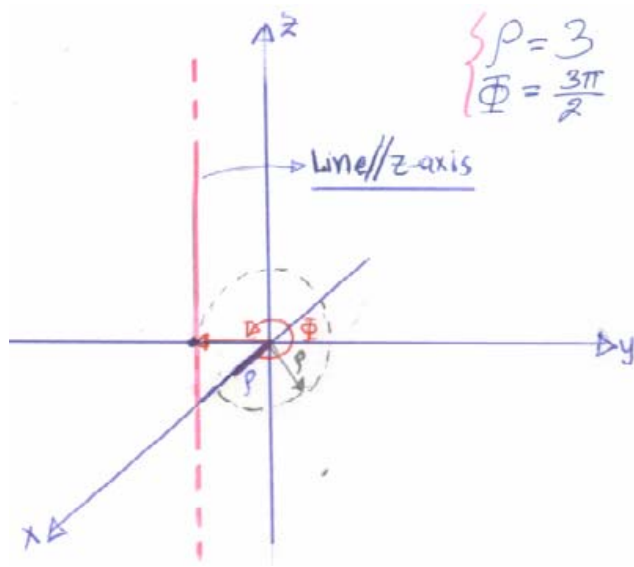## Part 1: Visualization of surfaces in 3D coordinate systems:
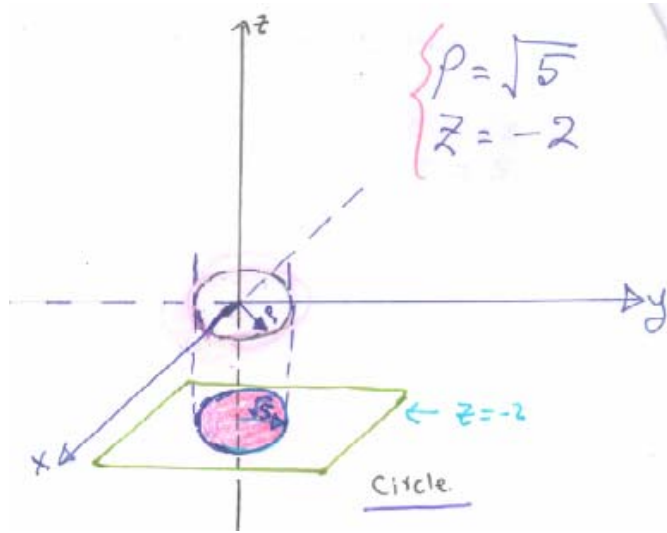
(a)

Part 1: Visualization of surfaces in 3D coordinate systems:

(b)

$$\begin{cases} \rho = 3 \\ \Phi = \frac{3\pi}{2} \end{cases}$$

Line // z-axis

Part 1: Visualization of surfaces in 3D coordinate systems:

(c)

$$\begin{cases} \rho = \sqrt{5} \\ Z = -2 \end{cases}$$

← z = -2

Circle

Part 1: Visualization of surfaces in 3D coordinate systems:

(d)

(e)

Part 1: Visualization of surfaces in 3D coordinate systems:

(f)

$$\begin{cases} r = 2 \\ \Phi = 0 \ (x\text{-axis}) \end{cases}$$

Disk or half a disk

Part 1: Visualization of surfaces in 3D coordinate systems:

(g)

$$\{ y = 5$$

Plane // (Xz-plane)

Part 2: Visualization of surfaces in 3D coordinate systems:

(a)

(b)

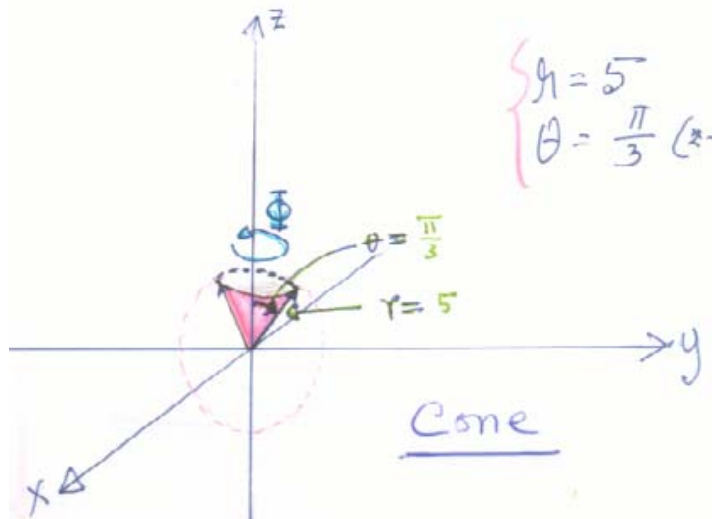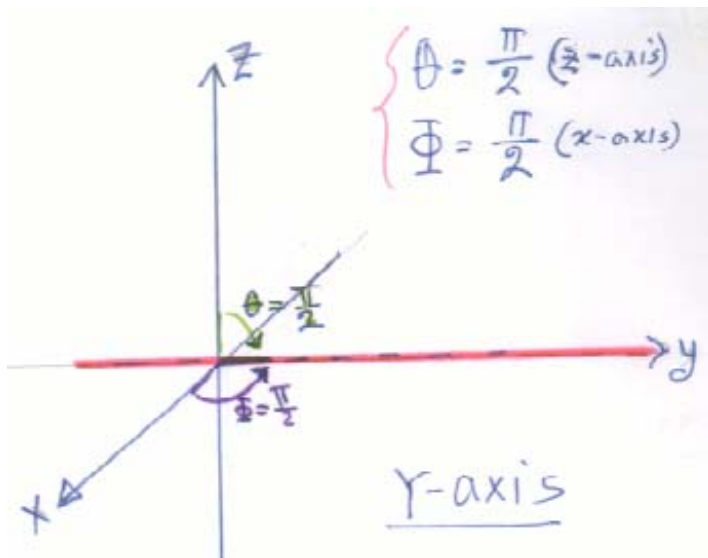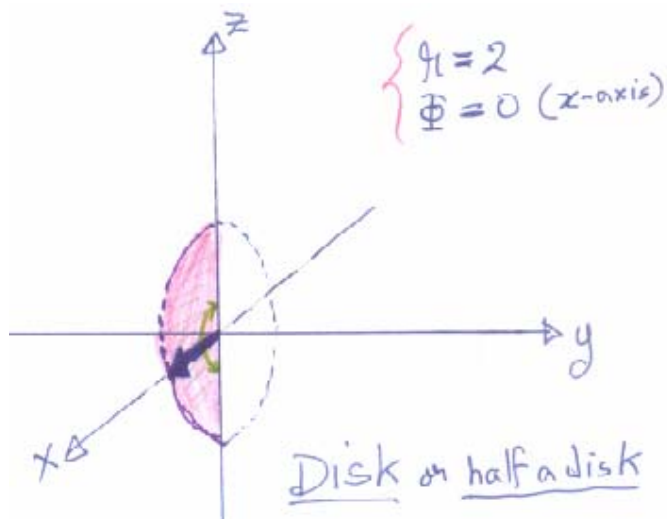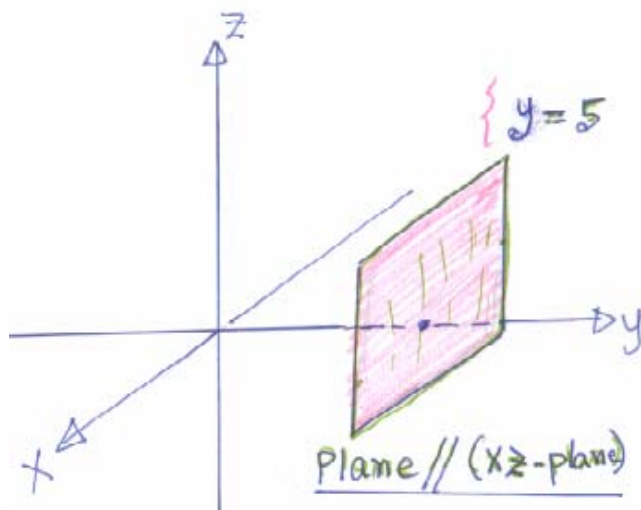## **Part 2: Visualization of surfaces in 3D coordinate systems:**

(c)

$$\begin{cases} \rho = 5 \\ \Phi = 45^\circ \end{cases}$$

infinte Line // z -axis

Part 2: Visualization of surfaces in 3D coordinate systems:

(d)

$$\begin{cases} r = 1 \\ \theta = 60^\circ \\ \text{(z-axis)} \end{cases}$$

Circle with $r_1 = \frac{\sqrt{3}}{2}$

3D View

$$\therefore r_1 = r \cos 30^\circ = \frac{\sqrt{3}}{2}$$
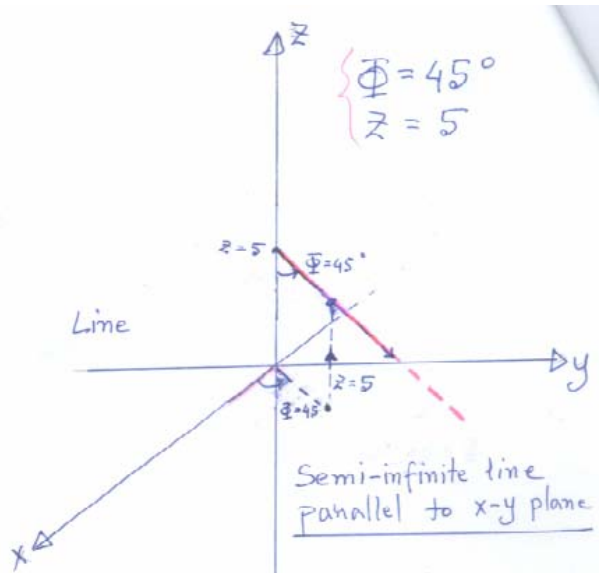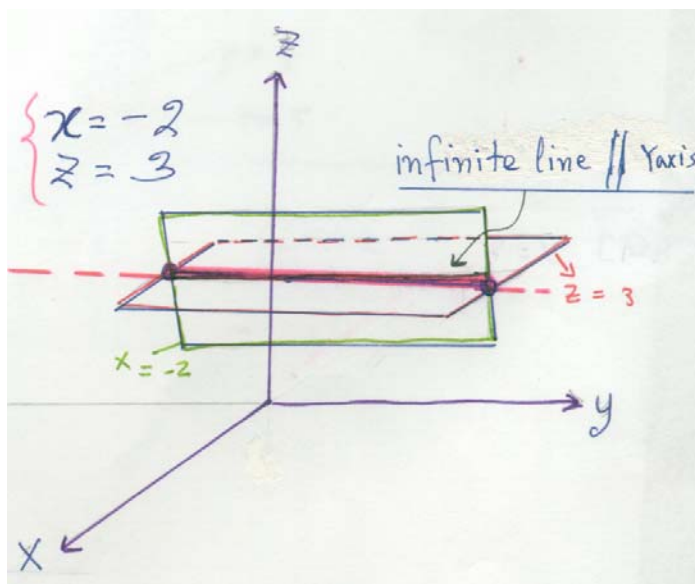
Part(3): <u>**Vector Algebra**</u>

Problem 1.5

For $\overline{U} -Ux ax + 5 ay - az$, $\overline{V}=2 ax - Vy ay + 3az$ and $\overline{W}= 6ax + ay + Wz az$, obtain Ux, Vy and Wz such that **U, V** and **W** are mutually orthogonal ?

$\overline{U}.\overline{V} = (uX ax + 5 ay - az).(2ax - vy ay + 3 az)$

$= 2Ux - 5 Vy - 3 = 0$ .... (1)

$\overline{U}.\overline{W} = (Ux ax + 5ay - az).(6ax + ay + Wz az)$

$= 6Ux + 5 - Wz = 0$ .... (2)

$\overline{V}.\overline{W} = (2ax - Vy ay + 3 az).(6ax + ay + Wz az)$

$= 12 - Vy + 3Wz = 0$ .... (3)

eq.1 $* (-3)$ + eq.2 $\Rightarrow 15 vy - w_2 +14 =$
$-\textcircled{4}$
eq.3 $* (15)$ + eq.4 $\Rightarrow w_2 = \frac{-97}{22}$

Problem 1.10

(a) $\breve{A} .(\overline{A}X\overline{B})=0=\overline{B} (\overline{A}X\overline{B})$

$(\overline{A}X\overline{B}) = \begin{vmatrix} ax & ay & az \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$

Remember (dot phod.)

$a_x \cdot a_x = a_y \cdot a_y = a_{\hat{z}} \cdot a_{\hat{z}} = 1$

$a_x \cdot a_y = a_y \cdot a_{\hat{z}} = a_{\hat{z}} \cdot a_x = 0$

$- ax (AyBz - AzBy) - ay (AxBz - AzBx) + az (AxBy - AyBx)$

$A . (AXB) = AxAyBz - AxAzBy - AyAxBz + AyAzBx AzAxBy - AyAzBx = 0$

$B . (AXB) = AyBxBz - AzBxBy - AxByBz + AzBxBy + AxByBz - AyBxBz = 0$

So this statement is true.

Part 3: Vector Algebra

Problem 1.5

(b) $(\overline{A}.\overline{B})^2 + |\overline{A}X\overline{B}|^2 = (AB)^2$ → direction ignored.

$|\overline{A}X\overline{B}|^2 - (AB)^2 sin^2\Theta$

$(A.B)^2 + |AXB|^2 = (AB cos\Theta)^2 + (ABsin\Theta) = (AB)^2 cos^2\Theta + (AB)^2 sin^2\Theta$

$= (AB)^2 (cos^2\Theta + sin^2\Theta) = (AB)^2 * 1 = RHS$

(c) $\overline{A} = Ax \overline{ax} + Ay \overline{ay} + Az \overline{az}$

Because A . $ax \Leftarrow Ax$

$A . ay \Leftarrow Ay$

$A . az \Leftarrow Az$

subs. Ax, Ay & Az leads;

$\overline{A} = (\overline{A}\cdot\overline{a}_x)\overline{a}_x + (\hat{A}\cdot\overline{a}_y)a_y + (\overline{A}\cdot\overline{a}_{\hat{z}})\overline{a}_{\hat{z}}$

Part 4: Coordinate Transformations

Problem 2.1 convert the following points to Cartesian coordinates:

$X=\rho\,cos\Theta$ & $Y=\rho\,sin\Theta$ & $Z=Z$

(a) $P(5, 120^{\circ}, 0)$

$X=5cos\,120 =-2.5$ $Y=5sin\,120 =4.33$ $Z=0$

$P(-2.5, 4.33, 0)$

(b) $P(1, 30^{\circ}, -10)$

$X=1cos\,30 =0.866$ $Y=1sin\,30= 0.5$ $Z=-10$

$P(0.866, 0.5, -10)$

(c) $P(10, 3\pi/4, \pi/2)$

$X=r\,sin\Theta\,cos\Phi$ $Y= r\,sin\Theta\,sin\Phi$ $Z=r\,cos\Theta$

$X=10\,sin\,3\pi/4\,cos\,\pi/2 = 0$

$Y= 10\,sin3\pi/4\,sin\,\pi/2 = 7.071$ $P(0, 7.071, -7.071)$

$Z=10\,cos3\pi/4 = -7.071$

Part 4: Coordinate Transformations

Problem 2.2 Express in Cylindrical and spherical coord

(a) $P(1, -4, -3)$

$\rho = \sqrt{1+16} = \sqrt{17}$, $\Phi=tan^{-1}(-4/1)=-75.94^{\circ}= 284.06^{\circ}$, & $z=-3$

$P(\sqrt{17}, -75.94, -3)$ ⟶ Cylindrical

$r = \sqrt{(1+16+9)}= \sqrt{26}$ as $r = \sqrt{x^2+y^2+z^2}$

$\Theta= tan^{-1}(\sqrt{17}/-3) = -53.96^{\circ}$

$P(\sqrt{26}, -53.96^{\circ}, -75.94)$ ⟶ Spherical

(b) $Q(3, 0, 5)$

$\rho = \sqrt{9+0} = 3$, $\Phi=tan^{-1}(0/3)=0^{\circ}$ & $z=5$

$Q(3, 0, 5)$ ⟶ Cylindrical

$Q(\sqrt{34}, 30.96^{\circ}, 0)$ ⟶ Spherical

(c) $R(-2, 6, 0)$

$\rho = \sqrt{4+36} = \sqrt{40}$, $\Phi=tan^{-1}(6/-2)=-71.565^{\circ}$ & $z=0$

$P(\sqrt{40}, -71.565^{\circ}, 0)$

Part 4: Coordinate Transformations

2.3(a)

$$\begin{pmatrix} P_\rho \\ P_\phi \\ P_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y+z \\ 0 \\ 0 \end{pmatrix} \qquad \text{(eq. 2.13)}$$

Cylindrical

$$P_\rho = \cos\phi\,(y+z) = \cos\phi\,(\rho\sin\phi + z)$$

$$P_\phi = -\sin\phi\,(y+z) = -\sin\phi\,(\rho\sin\phi + z)$$

$$P_z = 0$$

$$\longrightarrow \vec{P} = (\rho\sin\phi + z)(\cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi)$$

Spherical

$$P_r = (y+z)(\sin\theta\cos\phi) = (r\sin\theta\sin\phi + r\cos\theta)(\sin\phi\cos\phi)$$

$$P_\theta = (y+z)(\cos\theta\cos\phi) \Rightarrow \qquad \text{(eq. 2.27)}$$

$$P_\phi = (y+z)(-\sin\phi) \Rightarrow$$

$$\longrightarrow \vec{P} = r(\sin\theta\sin\phi + \cos\theta)(\sin\theta\cos\phi\,\vec{a}_r + \cos\theta\cos\phi\,\vec{a}_\theta - \sin\phi\,\vec{a}_\phi)$$

Part 4: Coordinate Transformations

2.3(b) ## Cylindrical

$$\vec{Q} = (y\cos\phi + xz\sin\phi)\vec{a}_\rho + (-y\sin\phi + xz\cos\phi)\vec{a}_\phi + (x+y)\vec{a}_z$$

sub. $x = \rho\cos\phi \qquad y = \rho\sin\phi$

$$\Rightarrow \vec{Q} = \tfrac{1}{2}\rho\sin 2\phi\,(1+z)\vec{a}_\rho + \rho(z\cos^2\phi - \sin^2\phi)\vec{a}_\phi + \rho(\cos\phi + \sin\phi)$$

Spherical

$$\vec{Q} = \left[\tfrac{1}{2}r\sin 2\phi\sin^2\theta + \tfrac{1}{2}r^2\sin^2\theta\cos\theta\sin^2\phi + \tfrac{1}{2}r(\cos\phi + \sin\phi)\cos 2\theta\right]$$

$$+ \left[\tfrac{1}{4}r\sin 2\theta\sin 2\phi + \tfrac{1}{2}r^2\cos^2\theta\sin\theta\sin 2\phi - \tfrac{r}{2}(\cos\phi + \sin\phi)\sin^2\theta\right]$$

$$+ \left(-r\sin\theta\sin^2\phi + \tfrac{1}{2}r^2\sin 2\theta\cos^2\phi\right)\vec{a}_\phi$$

Part 4: Coordinate Transformations   (Cylindrical case)

2.3
(c) $T = \left[\dfrac{x^2}{x^2 + y^2} - y^2 \right] ax + \left[\dfrac{xy}{x^2 + y^2} + xy \right] ay + az$

$$\begin{bmatrix} A\rho \\ A\theta \\ Az \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ax \\ ay \\ az \end{bmatrix}$$

$$\begin{vmatrix} Ar \\ A\theta \\ A\Phi \end{vmatrix} = \begin{vmatrix} \sin\theta\cos\Phi & \sin\theta\sin\Phi & \cos\theta \\ \cos\theta\cos\Phi & \cos\theta\sin\Phi & -\sin\theta \\ -\sin\Phi & \cos\Phi & 0 \end{vmatrix} \begin{vmatrix} Ax \\ Ay \\ Az \end{vmatrix}$$

$\boxed{A\rho} = \cos\Phi \left[\dfrac{x^2}{x^2 + y^2} - y^2 \right] + \sin\Phi \left[\dfrac{xy}{x^2 + y^2} + xy \right] + 0$

$= \cos^3\Phi - \rho^2 \sin^2\Phi\cos\Phi + \sin^2\Phi\cos\Phi + \rho^2\sin^2\Phi\cos\Phi = \cos\Phi$

Remember : $x = \rho\cos\phi$; $y = \rho\sin\phi$; $z = z$

$\boxed{A\Phi} = \left[\dfrac{x^2}{x^2 + y^2} - y^2 \right](-\sin\Phi) + \left[\dfrac{xy}{x^2 + y^2} + xy \right](\cos\Phi) + 0$

$= -\sin\Phi\cos^2\Phi + \rho^2\sin^3\Phi + \sin\Phi\cos^2\Phi + \rho^2\sin\Phi\cos^2\Phi = \rho^2\sin\Phi$

$\boxed{Az} = 1$ So, $\boxed{T = \cos\Phi\ \overline{a\rho} + \rho^2\sin\Phi\ \overline{a\Phi} + \overline{az}}$

Part 4: Coordinate Transformations   (Spherical case)

$Ar = \left[\dfrac{x^2}{x^2 + y^2} - y^2 \right] \sin\theta\cos\phi + \left[\dfrac{xy}{x^2 + y^2} + xy \right] \sin\theta\sin\phi + \cos\theta$

$= \sin\theta\cos^3\phi - r^2\sin^3\theta\sin^2\Phi\cos\phi + \cos\phi\sin^2\phi\sin\theta + r^2\sin^3\theta\cos\phi\sin^2\phi$
$\quad + \cos\theta$

$= \sin\theta\cos\Phi(\cos^2\Phi + \sin^2\Phi + r^2\sin^2\theta\sin^2\Phi - r^2\sin^2\Phi\sin^2\theta) + \cos\theta$

$= \sin\theta\cos\Phi + \cos\theta$

Remember : $x = r\sin\theta\cos\phi$

$A\theta = \left[\dfrac{x^2}{x^2 + y^2} - y^2 \right] \cos\theta\cos\phi + \left[\dfrac{xy}{x^2 + y^2} + xy \right] \cos\theta\sin\phi - \sin\theta$

$= (\cos^2\phi - r^2\sin^2\theta\cos^2\phi)(\cos\theta\cos\phi) + (\cos\phi\sin\phi + r^2\sin^2\theta$
$\cos\phi\sin\phi)(\cos\theta\sin) + \cos\theta$

$= r^2\sin^2\theta\cos\phi\cos\theta(\cos^2\theta + \sin^2\phi) + \cos\theta\cos\phi(\sin^2\phi - \cos^2\phi) + \cos\theta$
$= \cos\theta\cos^3\phi - r^2\sin^2\theta\sin^2\Phi\cos\theta\cos\Phi + \cos\theta\sin^2\Phi\cos\Phi + r^2\sin^2\theta\cos$
$\quad \Phi\sin^2\Phi\cos\theta - \sin\theta$
$= \cos\theta\cos\Phi - \sin\theta$

Part 4: Coordinate Transformations (Spherical case)

$$A\phi = -\left[\frac{x^2}{x^2 + y^2} - y^2\right](-sin\phi) + \left[\frac{xy}{x^2 + y^2} + xy\right]cos\phi + 0$$

$$= (cos^2\phi - r^2sin^2\Theta cos^2\phi)(-sin\phi) + (cos\phi\ sin\phi + r^2sin^2\Theta cos\phi sin\phi)cos\phi + 0$$
$$= -sin\Phi cos^2\Phi + r^2 sin^2\Theta sin^3\Phi + cos^2\Phi sin\Phi + r^2 sin^2\Theta cos^2\Phi sin\Phi$$
$$= r^2sin^2\Theta sin\phi$$

$$T = (sin\Theta cos\Phi + cos\Theta)\ \overline{a_n} + (cos\Theta cos\Phi - sin\Theta)\ \overline{a\Theta} + r^2 sin^2\Theta sin\Phi\ \overline{a\Phi}$$