Experiment # 3

SIMULATION OF ELECTRIC FIELD AND POTENTIAL INSIDE CAPACITORS

OBJECTIVE

The electric field and potential inside capacitors of different shapes are obtained numerically. The finite-difference method is used to solve Laplace's equation in two dimensions.

INTRODUCTION

The electric potential distribution inside any given structure can be analytically obtained by solving Laplace's (or Poisson's) equation subject to some boundary conditions. If we assume no volume charge inside the structure, Laplace's equation is given by:

$$\nabla^2 V = 0 \quad \text{...} (1)$$

In two dimensions (rectangular coordinates), equation (1) becomes:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \quad \text{..} (2)$$

The ability to solve equation (2) depends in a great deal on the nature of the structure under consideration. In some cases, a closed-form (analytical) solution to equation (2) is difficult to obtain. Alternatively, numerical methods can be used, especially for large structures. Numerical methods utilize the speed of computers and the flexibility of programming.

In this experiment, the Finite-Difference (*FD*) method will be used. The *FD* method is one of the most popular numerical methods in the field of electromagnetics. Its main advantages are the following:
1. Easy to formulate.
2. Suitable for many structures.
3. Flexible for modifications.

The most serious disadvantage of the *FD* method is its computational intensity relative to other methods. However, this is becoming less of a disadvantage with the advent of powerful computers.

The *FD*-based solution is performed in the following steps:
1. Discretize the given structure (gridding) using a suitable step size in both dimensions (Δx and Δy). The accuracy of the results improves with smaller step sizes.
2. Approximate the partial derivatives in equation (2) by the following:

$$\frac{\partial^2 V}{\partial x^2} = \frac{V(i+1, j) - 2V(i, j) + V(i-1, j)}{(\nabla x)^2} \quad \dots \dots \dots \dots \dots \dots (3)$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{V(i, j+1) - 2V(i, j) + V(i, j-1)}{(\nabla y)^2} \quad \dots \dots \dots \dots \dots \dots (4)$$

where *i* and *j* are the indices along the *x*-axis and the *y*-axis respectively. Substituting equations (3) and (4) in equation (2), we get

$$V(i, j) = \frac{1}{4}[V(i+1, j) + V(i-1, j) + V(i, j+1) + V(i, j-1)] \dots (5)$$

3. Solve the resulting linear system of equations.

Example
Find the potential distribution inside the structure given in figure 1. Take a step size of *5cm* in both dimensions. The boundary conditions are as shown in the figure.
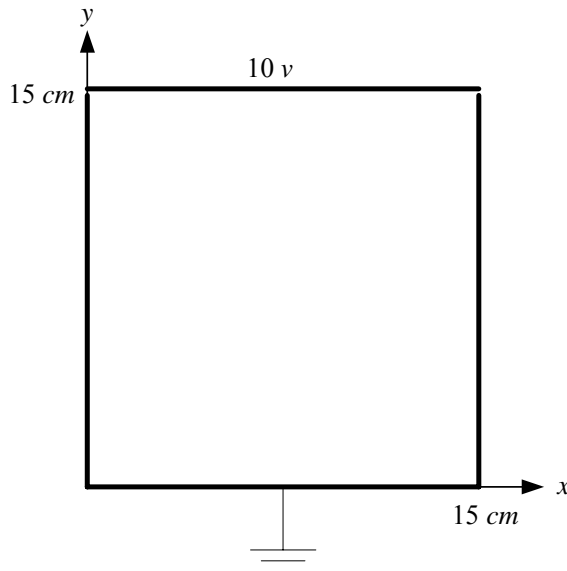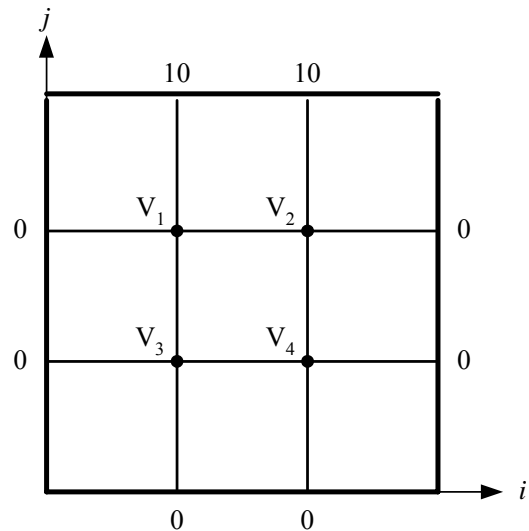

Figure 1: The structure of the example   Figure 2: The grid used for solution

The gridding is shown in figure 2. At each node in the figure, the value of the potential is labeled. Only four unknown values (V_1, V_2, V_3 and V_4) are to be determined; all other potentials are given as boundary conditions.
Applying the algorithm of equation (5), we get the following system of linear equations:

$$+ 4V_1 - V_2 - V_3 - 0V_4 = 10$$
$$- V_1 + 4V_2 - 0V_3 - V_4 = 10$$
$$- V_1 - 0V_2 + 4V_3 - V_4 = 0$$
$$- 0V_1 - V_2 - V_3 + 4V_4 = 0$$

The solution of the above system can be obtained using different methods (e.g., matrix formulation in MA TLAB). The result is:

$$V_1 = 3.75 \text{ V}, \quad V_2 = 3.75 \text{ V}, \quad V_3 = 1.25 \text{ V}, \quad V_4 = 1.25 \text{ V}$$

PROCEDURE

1. Solve Laplace's equation using the *FD* method for the structure given in figure 3. The dimensions of the structure are 20 *cm* x 30 *cm*. Use a step size of 5 *cm* in both dimensions.

2. Decrease the step size to 2.5 cm and repeat part (1).

3. Compare the results obtained in parts (1) and (2) at some points inside the structure.

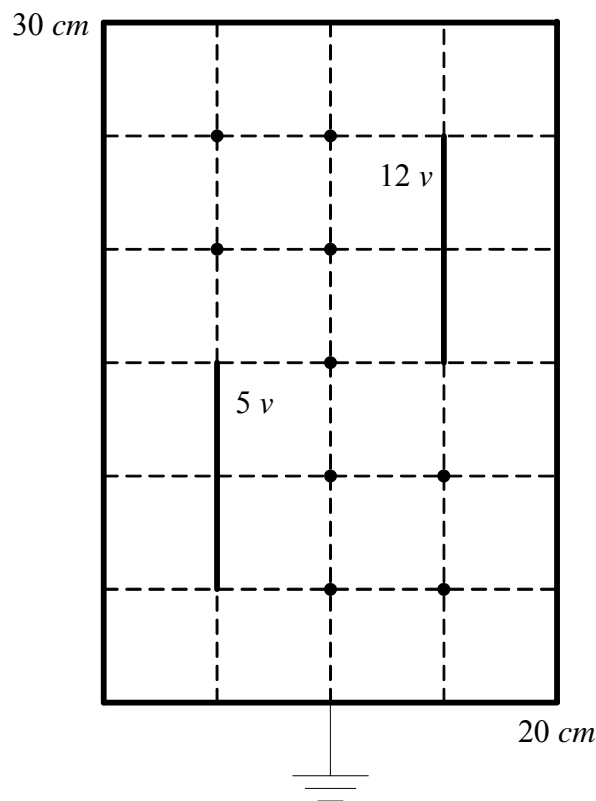4. Produce contour plots for the equi-potential lines inside the structure (If you are using MATLAB, there is a function for contouring).

Figure 3: The structure used in the experiment