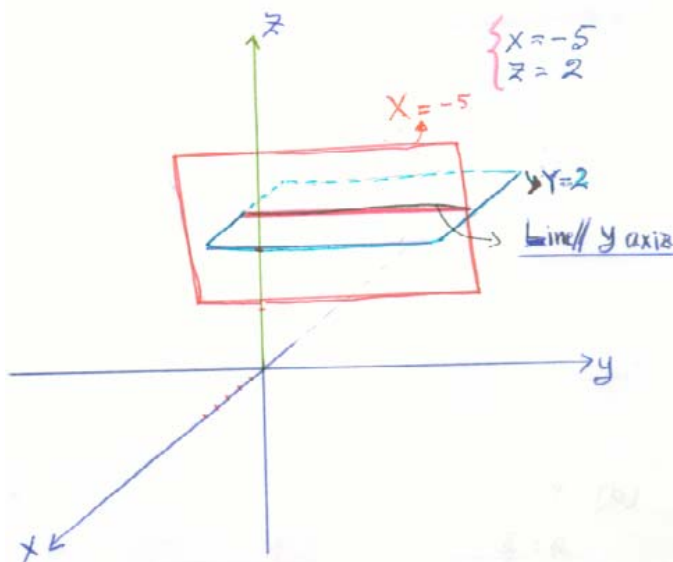


# EE 340 Electromagnetics Lab

## Problem Session #1

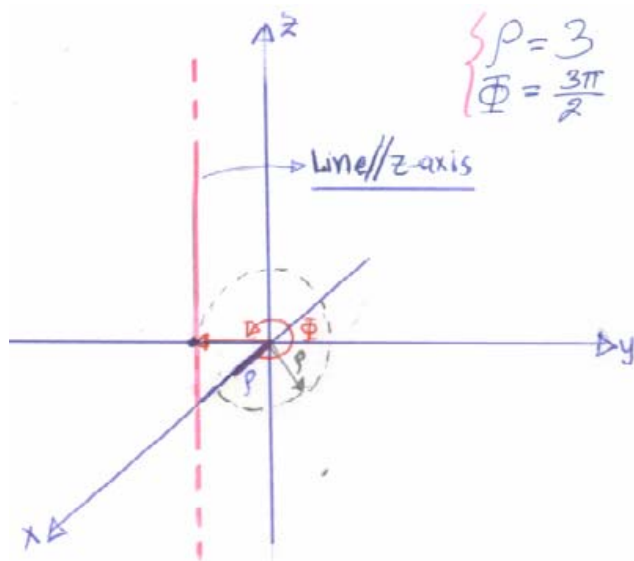
### **Part 1: Visualization of surfaces in 3D coordinate systems:**

(a)



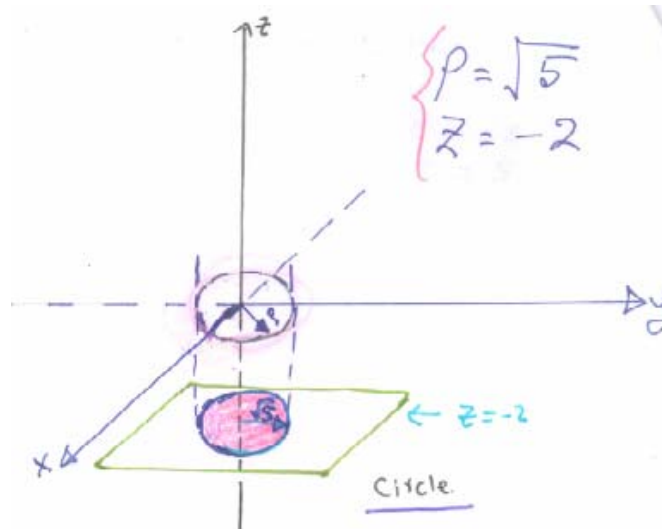
**Part 1: Visualization of surfaces in 3D coordinate systems:**

(b)



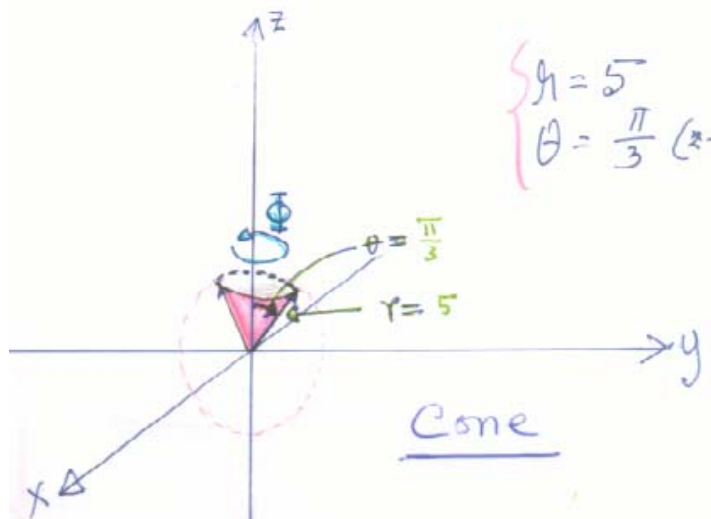
**Part 1: Visualization of surfaces in 3D coordinate systems:**

(c)



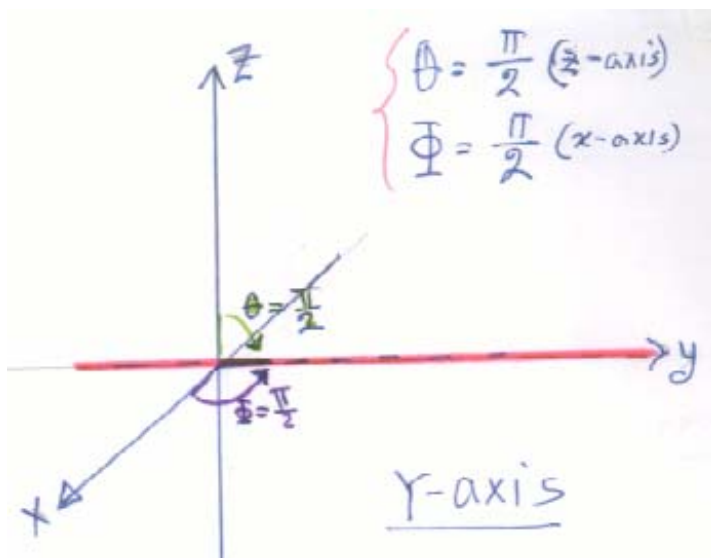
**Part 1: Visualization of surfaces in 3D coordinate systems:**

(d)



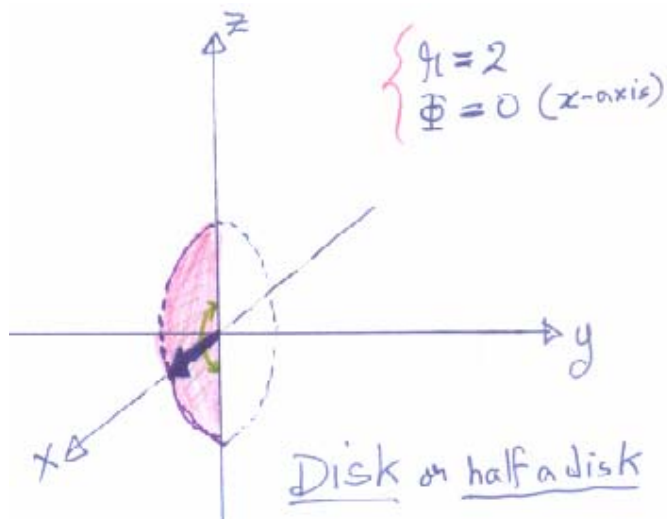
**Part 1: Visualization of surfaces in 3D coordinate systems:**

(e)



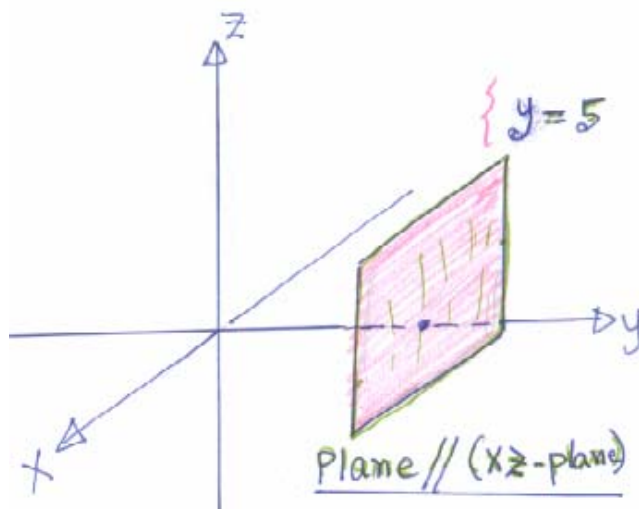
**Part 1: Visualization of surfaces in 3D coordinate systems:**

(f)



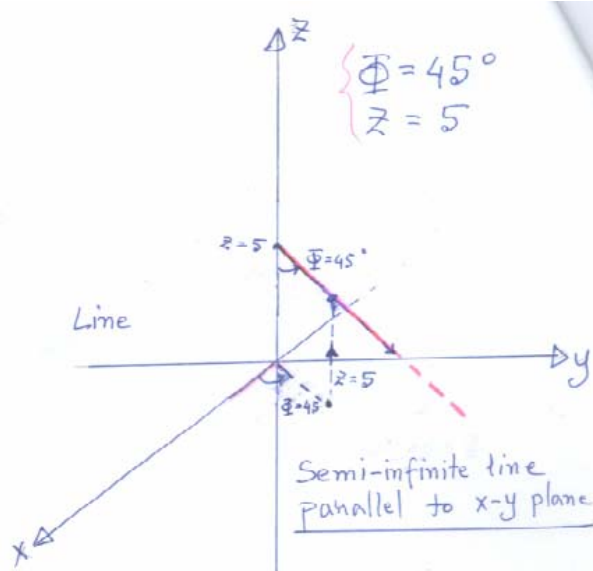
**Part 1: Visualization of surfaces in 3D coordinate systems:**

(g)



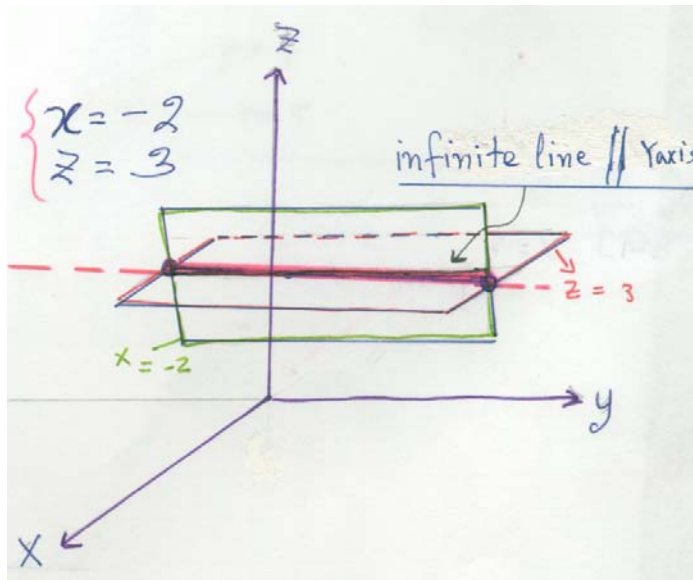
**Part 2: Visualization of surfaces in 3D coordinate systems:**

(a)



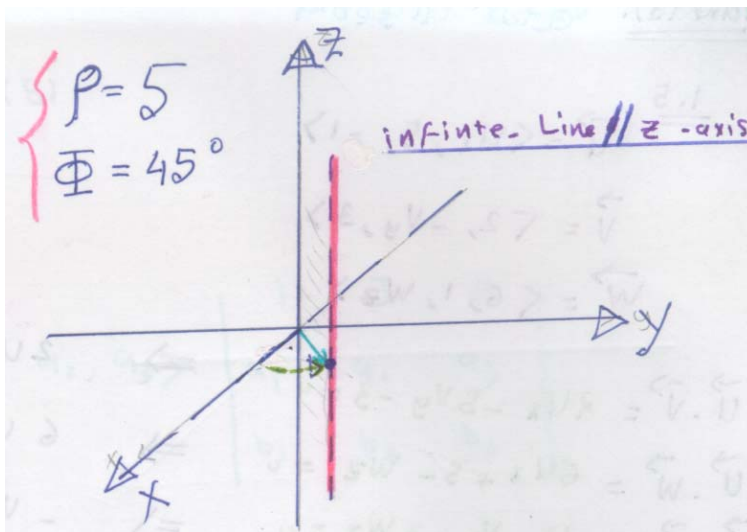
**Part 2: Visualization of surfaces in 3D coordinate systems:**

(b)



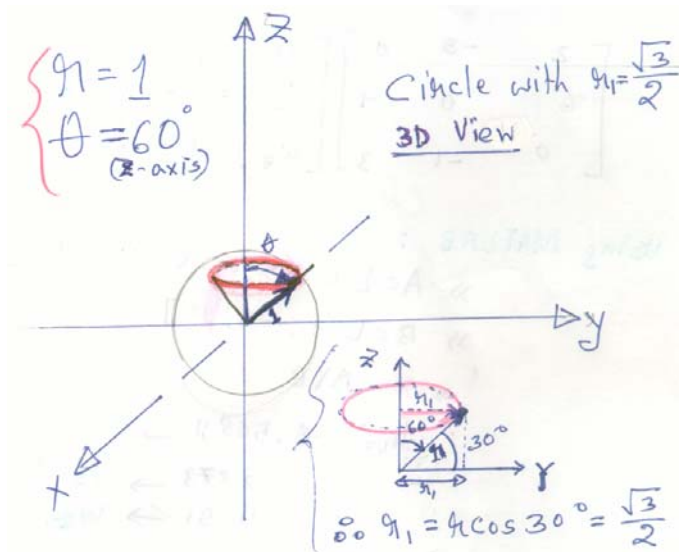
**Part 2: Visualization of surfaces in 3D coordinate systems:**

(c)



**Part 2: Visualization of surfaces in 3D coordinate systems:**

(d)



**Part(3): Vector Algebra**

**Problem 1.5**

For  $\vec{U} = U_x \vec{a}_x + 5 a_y - a_z$ ,  $\vec{V} = 2 a_x - V_y a_y + 3 a_z$  and  $\vec{W} = 6 a_x + a_y + W_z a_z$ , obtain  $U_x$ ,  $V_y$  and  $W_z$  such that  $\vec{U}$ ,  $\vec{V}$  and  $\vec{W}$  are mutually orthogonal ?

$$\begin{aligned} \vec{U} \cdot \vec{V} &= (U_x \vec{a}_x + 5 a_y - a_z) \cdot (2 a_x - V_y a_y + 3 a_z) \\ &= 2U_x - 5V_y - 3 = 0 \quad \dots\dots\dots (1) \\ \vec{U} \cdot \vec{W} &= (U_x \vec{a}_x + 5 a_y - a_z) \cdot (6 a_x + a_y + W_z a_z) \\ &= 6U_x + 5 - W_z = 0 \quad \dots\dots\dots (2) \\ \vec{V} \cdot \vec{W} &= (2 a_x - V_y a_y + 3 a_z) \cdot (6 a_x + a_y + W_z a_z) \\ &= 12 - V_y + 3W_z = 0 \quad \dots\dots\dots (3) \end{aligned}$$

$\left. \begin{array}{l} \text{eq. 1} * (-3) + \text{eq. 2} \Rightarrow 15V_y - W_z + 14 = 0 \quad \text{---(4)} \\ \text{eq. 3} * (15) + \text{eq. 4} \Rightarrow W_z = -97/22 \end{array} \right\}$

**Problem 1.10**

(a)  $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0 = \vec{B} \cdot (\vec{A} \times \vec{B})$

$$(\vec{A} \times \vec{B}) = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Remember (dot prod.)  
 $a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$   
 $a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$

$-a_x (A_y B_z - A_z B_y) - a_y (A_x B_z - A_z B_x) + a_z (A_x B_y - A_y B_x)$   
 $A \cdot (\vec{A} \times \vec{B}) = A_x A_y B_z - A_x A_z B_y - A_y A_x B_z + A_y A_z B_x + A_z A_x B_y - A_z A_y B_x = 0$   
 $B \cdot (\vec{A} \times \vec{B}) = A_y B_x B_z - A_z B_x B_y - A_x B_y B_z + A_z B_x B_y + A_x B_y B_z - A_y B_x B_z = 0$   
 So this statement is true.

**Part 3: Vector Algebra**

**Problem 1.5**

(b)  $(\vec{A} \cdot \vec{B})^2 + |\vec{A} \times \vec{B}|^2 = (AB)^2$   
 $|\vec{A} \times \vec{B}|^2 = (AB)^2 \sin^2 \theta$   $\rightarrow$  direction ignored.  
 $(\vec{A} \cdot \vec{B})^2 + |\vec{A} \times \vec{B}|^2 = (AB \cos \theta)^2 + (AB \sin \theta)^2 = (AB)^2 \cos^2 \theta + (AB)^2 \sin^2 \theta$   
 $= (AB)^2 (\cos^2 \theta + \sin^2 \theta) = (AB)^2 * 1 = \text{RHS}$

(c)  $\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$

Because  $A \cdot \vec{a}_x \in A_x$   
 $A \cdot \vec{a}_y \in A_y$   
 $A \cdot \vec{a}_z \in A_z$  } subs.  $A_x, A_y$  &  $A_z$  leads;

$$\vec{A} = (\vec{A} \cdot \vec{a}_x) \vec{a}_x + (\vec{A} \cdot \vec{a}_y) \vec{a}_y + (\vec{A} \cdot \vec{a}_z) \vec{a}_z$$

### Part 4: Coordinate Transformations

Problem 2.1 convert the following points to Cartesian coordinates:

$$X = \rho \cos \Theta \quad Y = \rho \sin \Theta \quad Z = Z$$

(a)  $P(5, 120^\circ, 0)$

$$X = 5 \cos 120 = -2.5 \quad Y = 5 \sin 120 = 4.33 \quad Z = 0$$

$$P(-2.5, 4.33, 0)$$

(b)  $P(1, 30^\circ, -10)$

$$X = 1 \cos 30 = 0.866 \quad Y = 1 \sin 30 = 0.5 \quad Z = -10$$

$$P(0.866, 0.5, -10)$$

(c)  $P(10, 3\pi/4, \pi/2)$

$$X = r \sin \Theta \cos \Phi \quad Y = r \sin \Theta \sin \Phi \quad Z = r \cos \Theta$$

$$X = 10 \sin 3\pi/4 \cos \pi/2 = 0$$

$$Y = 10 \sin 3\pi/4 \sin \pi/2 = 7.071$$

$$Z = 10 \cos 3\pi/4 = -7.071$$

$$P(0, 7.071, -7.071)$$

### Part 4: Coordinate Transformations

Problem 2.2 Express in Cylindrical and spherical coord

(a)  $P(1, -4, -3)$

$$\rho = \sqrt{1+16} = \sqrt{17}, \quad \Phi = \tan^{-1}(-4/1) = -75.94^\circ = 284.06^\circ, \quad \text{and } z = -3$$

$$P(\sqrt{17}, -75.94^\circ, -3) \longrightarrow \text{Cylindrical}$$

$$r = \sqrt{1+16+9} = \sqrt{26} \quad \text{as } r = \sqrt{x^2 + y^2 + z^2}$$

$$\Theta = \tan^{-1}(\sqrt{17}/-3) = -53.96^\circ$$

$$P(\sqrt{26}, -53.96^\circ, -75.94^\circ) \longrightarrow \text{Spherical}$$

(b)  $Q(3, 0, 5)$

$$\rho = \sqrt{9+0} = 3, \quad \Phi = \tan^{-1}(0/3) = 0^\circ \quad \text{and } z = 5$$

$$Q(3, 0, 5) \longrightarrow \text{Cylindrical}$$

$$Q(3, 0^\circ, 5) \longrightarrow \text{Spherical}$$

(c)  $R(-2, 6, 0)$

$$\rho = \sqrt{4+36} = \sqrt{40}, \quad \Phi = \tan^{-1}(6/-2) = -71.565^\circ \quad \text{and } z = 0$$

$$P(\sqrt{40}, -71.565^\circ, 0)$$



### Part 4: Coordinate Transformations

2.3(a) / 
$$\begin{pmatrix} P_\rho \\ P_\phi \\ P_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y+z \\ 0 \\ 0 \end{pmatrix} \quad (\text{eq. 2.13})$$

#### Cylindrical

$$P_\rho = \cos\phi (y+z) = \cos\phi (\rho \sin\phi + z)$$

$$P_\phi = -\sin\phi (y+z) = -\sin\phi (\rho \sin\phi + z)$$

$$P_z = 0$$

$$\rightarrow \vec{P} = (\rho \sin\phi + z) (\cos\phi \vec{a}_\rho - \sin\phi \vec{a}_\phi)$$

#### Spherical

$$P_r = (y+z) (\sin\theta \cos\phi) = (r \sin\theta \sin\phi + r \cos\theta) (\sin\theta \cos\phi)$$

$$P_\theta = (y+z) (\cos\theta \cos\phi) \Rightarrow$$

$$P_\phi = (y+z) (-\sin\phi) \Rightarrow$$

$$\rightarrow \vec{P} = r (\sin\theta \sin\phi + \cos\theta) (\sin\theta \cos\phi \vec{a}_r + \cos\theta \cos\phi \vec{a}_\theta - \sin\phi \vec{a}_\phi) \quad (\text{eq. 2.27})$$

### Part 4: Coordinate Transformations

#### 2.3(b) / Cylindrical

$$\vec{Q} = (y \cos\phi + xz \sin\phi) \vec{a}_\rho + (-y \sin\phi + xz \cos\phi) \vec{a}_\phi + (x+y) \vec{a}_z$$

$$\text{sub. } x = \rho \cos\phi \quad y = \rho \sin\phi$$

$$\Rightarrow \vec{Q} = \frac{1}{2} \rho \sin 2\phi (1+z) \vec{a}_\rho + \rho (z \cos^2\phi - \sin^2\phi) \vec{a}_\phi + \rho (\cos\phi + \sin\phi) \vec{a}_z$$

#### Spherical

$$\vec{Q} = \left[ \frac{1}{2} r \sin 2\theta \sin^2\phi + \frac{1}{2} r^2 \sin^2\theta \cos\theta \sin^2\phi + \frac{1}{2} r (\cos\phi + \sin\phi) \cos 2\theta \right]$$

$$+ \left[ \frac{1}{4} r \sin 2\theta \sin 2\phi + \frac{1}{2} r^2 \cos^2\theta \sin\theta \sin 2\phi - \frac{r}{2} (\cos\phi + \sin\phi) \sin^2\theta \right] \vec{a}_\theta$$

$$+ (-r \sin\theta \sin^2\phi + \frac{1}{2} r^2 \sin 2\theta \cos^2\phi) \vec{a}_\phi$$

**Part 4: Coordinate Transformations** (Cylindrical case)

$$\frac{2.3}{(c)} T = \left[ \frac{x^2}{x^2+y^2} - y^2 \right] ax + \left[ \frac{xy}{x^2+y^2} + xy \right] ay + az$$

$$\begin{bmatrix} A\rho \\ A\theta \\ Az \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ax \\ ay \\ az \end{bmatrix}$$

$$\begin{bmatrix} A\rho \\ A\theta \\ A\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix}$$

$$\begin{aligned} A\rho &= \cos\phi \left[ \frac{x^2}{x^2+y^2} - y^2 \right] + \sin\phi \left[ \frac{xy}{x^2+y^2} + xy \right] + 0 \\ &= \cos^3\phi - \rho^2 \sin^2\phi \cos\phi + \sin^2\phi \cos\phi + \rho^2 \sin^2\phi \cos\phi = \cos\phi \end{aligned}$$

**Remember:**  $x = \rho \cos\phi$  ;  $y = \rho \sin\phi$  ;  $z = z$

$$\begin{aligned} A\theta &= \left[ \frac{x^2}{x^2+y^2} - y^2 \right] (-\sin\phi) + \left[ \frac{xy}{x^2+y^2} + xy \right] (\cos\phi) + 0 \\ &= -\sin\phi \cos^2\phi + \rho^2 \sin^3\phi + \sin\phi \cos^2\phi + \rho^2 \sin\phi \cos^2\phi = \rho^2 \sin\phi \end{aligned}$$

$$Az = 1 \quad \text{So, } T = \cos\phi \bar{a}_\rho + \rho^2 \sin\phi \bar{a}_\theta + \bar{a}_z$$

**Part 4: Coordinate Transformations** (Spherical case)

$$\begin{aligned} Ar &= \left[ \frac{x^2}{x^2+y^2} - y^2 \right] \sin\theta \cos\phi + \left[ \frac{xy}{x^2+y^2} + xy \right] \sin\theta \sin\phi + \cos\theta \\ &= \sin\theta \cos^3\phi - r^2 \sin^3\theta \sin^2\phi \cos\phi + \cos\phi \sin^2\phi \sin\theta + r^2 \sin^3\theta \cos\phi \sin^2\phi \\ &\quad + \cos\theta \\ &= \sin\theta \cos\phi (\cos^2\phi + \sin^2\phi + r^2 \sin^2\theta \sin^2\phi - r^2 \sin^2\phi \sin^2\theta) + \cos\theta \\ &= \sin\theta \cos\phi + \cos\theta \end{aligned}$$

**Remember:**  $x = r \sin\theta \cos\phi$

$$\begin{aligned} A\theta &= \left[ \frac{x^2}{x^2+y^2} - y^2 \right] \cos\theta \cos\phi + \left[ \frac{xy}{x^2+y^2} + xy \right] \cos\theta \sin\phi - \sin\theta \\ &= (\cos^2\phi - r^2 \sin^2\theta \cos^2\phi) (\cos\theta \cos\phi) + (\cos\phi \sin\phi + r^2 \sin^2\theta \cos\phi \sin\phi) (\cos\theta \sin\phi) - \sin\theta \\ &= r^2 \sin^2\theta \cos\phi \cos\theta (\cos^2\theta + \sin^2\phi) + \cos\theta \cos\phi (\sin^2\phi - \cos^2\phi) + \cos\theta \\ &= \cos\theta \cos^3\phi - r^2 \sin^2\theta \sin^2\phi \cos\theta \cos\phi + \cos\theta \sin^2\phi \cos\phi + r^2 \sin^2\theta \cos\phi \sin^2\phi \cos\theta - \sin\theta \\ &= \cos\theta \cos\phi - \sin\theta \end{aligned}$$

**Part 4: Coordinate Transformations** (Spherical case)

$$A_{\phi} = \left[ \frac{x^2}{x^2 + y^2} - y^2 \right] (-\sin \phi) + \left[ \frac{xy}{x^2 + y^2} + xy \right] \cos \phi + 0$$

$$= (\cos^2 \phi - r^2 \sin^2 \theta \cos^2 \phi) (-\sin \phi) + (\cos \phi \sin \phi + r^2 \sin^2 \theta \cos \phi \sin \phi) \cos \phi + 0$$

$$= -\sin \Phi \cos^2 \Phi + r^2 \sin^2 \theta \sin^3 \Phi + \cos^2 \Phi \sin \Phi + r^2 \sin^2 \theta \cos^2 \Phi \sin \Phi$$

$$= r^2 \sin^2 \theta \sin \phi$$

$$T = (\sin \theta \cos \Phi + \cos \theta) \bar{a}_{\theta} + (\cos \theta \cos \Phi - \sin \theta) \bar{a}_{\phi} + r^2 \sin^2 \theta \sin \Phi \bar{a}_{\phi}$$