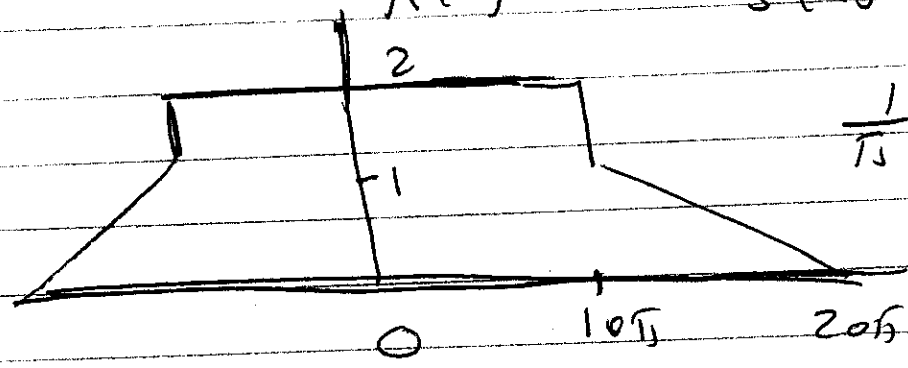
6.16

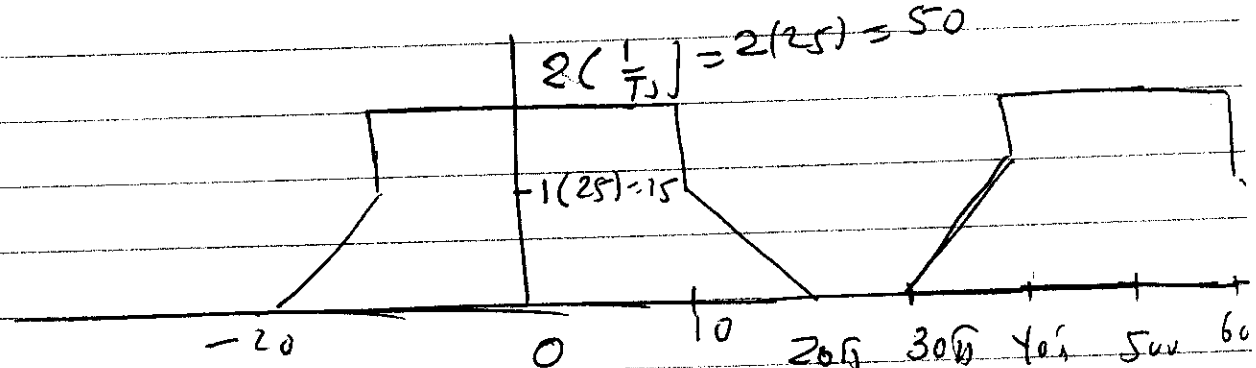
$\cancel{F_s(w) = \frac{1}{T_s}\sum F(w}$

$\frac{1}{T_s}\sum F(w-kw_s$



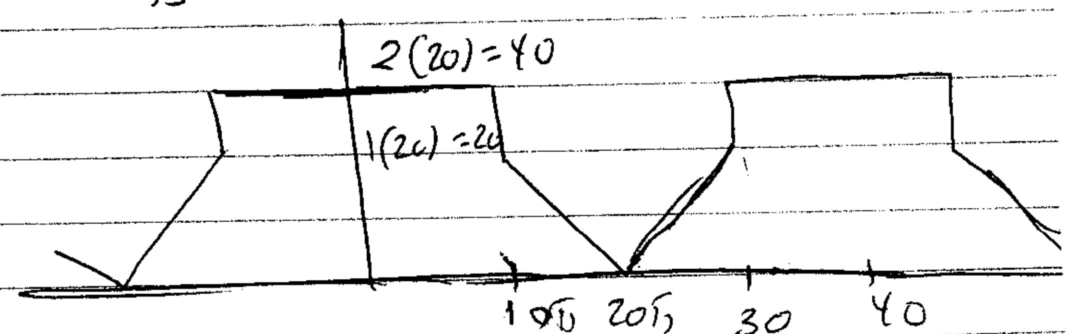
if idealy sampled at $T_s = 40$ ms

$\Rightarrow w_s = \frac{2\pi}{T_s} = 2\pi\left(\frac{1}{40\times10^{-3}}\right) = 2\pi(25) = 5$



if sampled at $T_s = 50$ ms

$\Rightarrow w_s = \frac{2\pi}{T_s} = 2\pi\left(\frac{1}{50\times10^{-3}}\right) = 2\pi(20) = 40\pi$

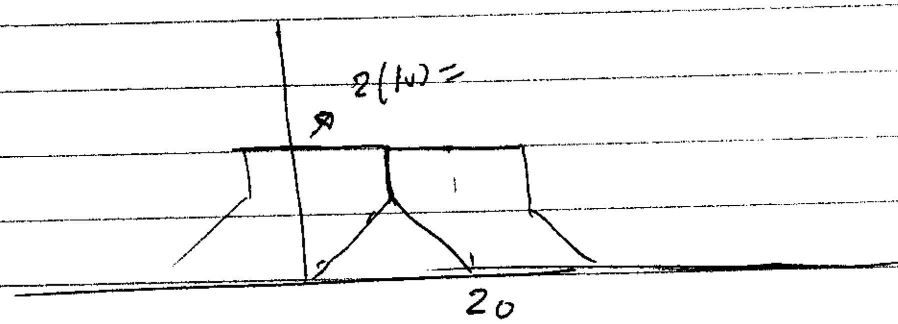


If $T_s = 100ms \Rightarrow \omega_s = 2\pi\left(\frac{1}{100 \times 10^{-3}}\right)$

$= 2\pi(10) = 20\pi$

$\omega_s < 2(20\pi) = 40\pi$

$\Rightarrow$ aliasing



8-12. Assume that a constant signal is sampled so that $x(nT) = a$ for all N. Show that, for $f_s >> f_3$, the interpolation formula for an RC low-pass reconstruction filter yields an output $y(nT)$ that is equal to the sample values $x(nT)$.

Section 8-3

8-13. Use z-transform pair 3 in Table 8-1 to establish z-transform pairs 4 and 5. (*Hint:* First write

$$\sum_{n=0}^{\infty} e^{-\alpha nT} z^{-n} = \frac{1}{1 - e^{-\alpha T} z^{-1}}$$

and differentiate both sides with respect to α.)

8-14. Use z-transform pair 3 in Table 8-1 to establish z-transform pairs 6 and 7. (*Hint:* Again first write

$$\sum_{n=0}^{\infty} e^{-\alpha nT} z^{-n} = \frac{1}{1 - e^{-\alpha T} z^{-1}}$$

Let $\alpha = jb$ and equate real and imaginary parts.)

8-15. Use z-transform pair 3 in Table 8-1 to establish z-transform pairs 8 and 9. (*Hint:* The procedure is the same as in Problem 8-14. What should you let α be now?)

8-16. Show that the z-transform of $a^n x(nT)$ is $X(z/a)$. Use this result to show that entry 3 in Table 8-1 follows from entry 2.

8-17. Show that the z-transform of $nx(nT)$ is given by $-z\, dX(z)/dz$. Use this result to show that entry 4 in Table 8-1 follows from entry 3.

8-18. The signal below is sampled at 25 samples per second. Determine the z-transform of $x(nT)$ for $0 \le n \le 8$.

$$x(t) = 5 \sin 20\pi t + 2\Pi\left(\frac{t - 0.14}{0.16}\right)$$

8-19. Determine the z-transform for the following sequences of samples:

(a) $x(nT) = \left(\frac{1}{5}\right)^n u(n)$

(b) $x(nT) = \left(-\frac{1}{5}\right)^n u(n)$

(c) $x(nT) = u(n) + \left(\frac{3}{4}\right)^n u(n - 4)$

(d) $x(nT) = 2u(n) - 2u(n - 8)$

8-20. Verify the results of Problem 8-19 using MATLAB.

8-21. Determine the z-transform of the following sequences of samples:

(a) $x(nT) = \left(\frac{2}{3}\right)^n u(n - 4)$

(b) $x(nT) = \left(\frac{2}{3}\right)^{n-4} u(n - 4)$

8-22. Verify the results of Problem 8-21 using MATLAB.

8-19

(a) The Z-trans of $x(nT) = (1/5)^n u(n)$

$$X(z)=\sum_{n=0}^{\infty}\left(\frac{1}{5}\right)^n z^{-n}=\sum_{n=0}^{\infty}\left(\frac{1}{5}z^{-1}\right)^n$$

which is

$$X(z)=\frac{1}{1-\frac{1}{5}z^{-1}},\quad |z|>\frac{1}{5}$$

(b) The z-transform of $x(nT)=\left(-\frac{1}{5}\right)^n u(n)$ is

$$X(z)=\sum_{n=0}^{\infty}\left(-\frac{1}{5}\right)^n z^{-n}=\sum_{n=0}^{\infty}\left(-\frac{1}{5}z^{-1}\right)^n$$

which is

$$X(z)=\frac{1}{1+\frac{1}{5}z^{-1}},\quad |z|>\frac{1}{5}$$

(c) The z-transform of $x(nT)=u(n)+\left(\frac{3}{4}\right)^n u(n-4)$ is

$$X(z)=\sum_{n=0}^{\infty}z^{-n}+\sum_{n=4}^{\infty}\left(\frac{3}{4}\right)^n z^{-n}$$

which can be written

$$X(z)=\sum_{n=0}^{\infty}(z^{-1})^n+\sum_{n=4}^{\infty}\left(\frac{3}{4}z^{-1}\right)^n$$

With the change of index $k=n-4$ in the second sum, this becomes

$$X(z)=\frac{1}{1-z^{-1}}+\sum_{k=0}^{\infty}\left(\frac{3}{4}z^{-1}\right)^{k+4}$$

This gives

$$X(z)=\frac{1}{1-z^{-1}}+\frac{\left(\frac{3}{4}z^{-1}\right)^4}{1-\frac{3}{4}z^{-1}},\quad |z|>1$$

(d) This part of the problem is solved using the same approach as was used in part (c). The result is the following MATLAB script.

```
syms n z                     % Make n and z symbolic
xn = 2;                      % Define x(n) for first term
x1z = ztrans(xn,n,z);        % z-transform
x2z = x1z*z^(-8);            % Define second term
xz = x1z+x2z;                % Combine terms
xz                           % Display result

xz =

2*z/(z-1)+2/z^7/(z-1)
```

We see that the result is in agreement with Problem 8-19(d).

Problem 8-21

(a) The z-transform of $x(nT)=\left(\frac{2}{3}\right)^n u(n-4)$ is

$$X(z)=\sum_{n=4}^{\infty}\left(\frac{2}{3}\right)^n z^{-n}=\sum_{n=4}^{\infty}\left(\frac{2}{3}z^{-1}\right)^n$$

With the change of index $k=n-4$ we have

$$X(z)=\sum_{k=0}^{\infty}\left(\frac{2}{3}z^{-1}\right)^{k+4}$$

Thus

$$X(z)=\frac{\left(\frac{2}{3}\right)^4 z^{-4}}{1-\frac{2}{3}z^{-1}},\quad |z|>\frac{2}{3}$$

(b) The z-transform of $x(nT)=\left(\frac{2}{3}\right)^{n-4} u(n-4)$ is

$$X(z)=\sum_{n=4}^{\infty}\left(\frac{2}{3}\right)^{n-4} z^{-n}$$

We once again use the change of index $k=n-4$. This gives

$$X(z)=\sum_{k=0}^{\infty}\left(\frac{2}{3}\right)^k z^{-(k+4)}=z^{-4}\sum_{k=0}^{\infty}\left(\frac{2}{3}z^{-1}\right)^k$$