5.4 SAMPLING CONTINUOUS-TIME SIGNALS

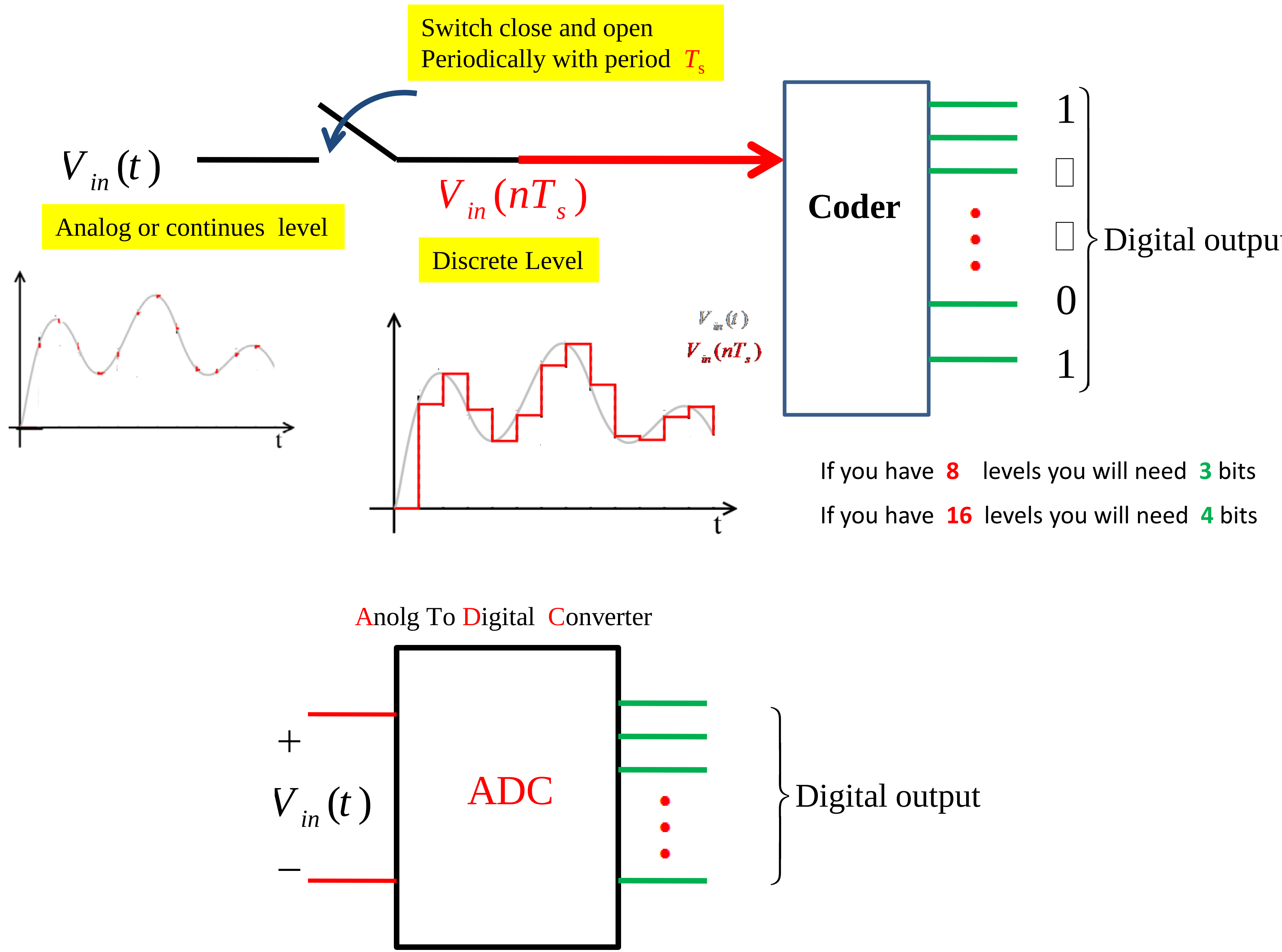
The sampling of continuous-time signals is an important topic

It is required by many important technologies such as:

Digital Communication Systems (Wireless Mobile Phones, Digital TV (Coming) , Digital Radio etc)

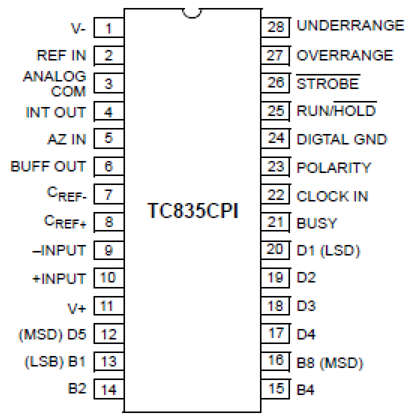
CD and DVD

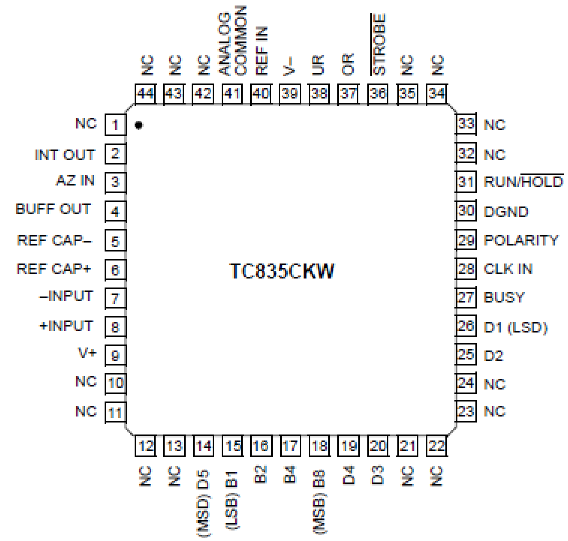
Digital Photos



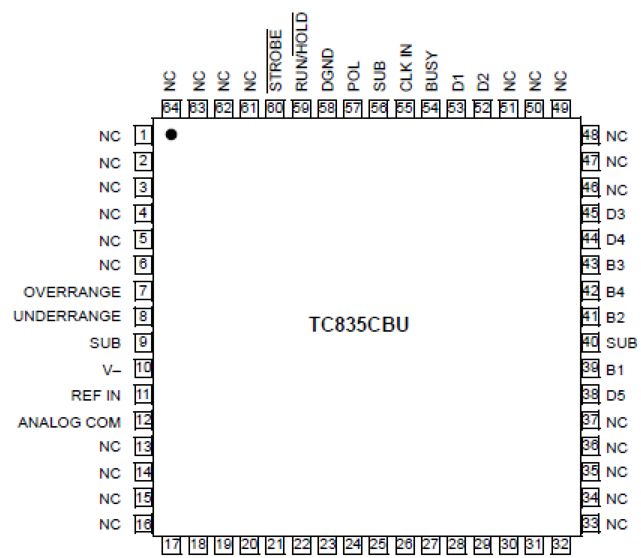
Switch close and open
Periodically with period T_s
$V_{in}(t)$
Analog or continues level
$V_{in}(nT_s)$
Discrete Level
t
$V_{in}(t)$
$V_{in}(nT_s)$
t
Coder
1
0
1
Digital output
If you have 8 levels you will need 3 bits
If you have 16 levels you will need 4 bits
Anolg To Digital Converter
+
$V_{in}(t)$
−
ADC
Digital output

Commercial type ADC or A/D

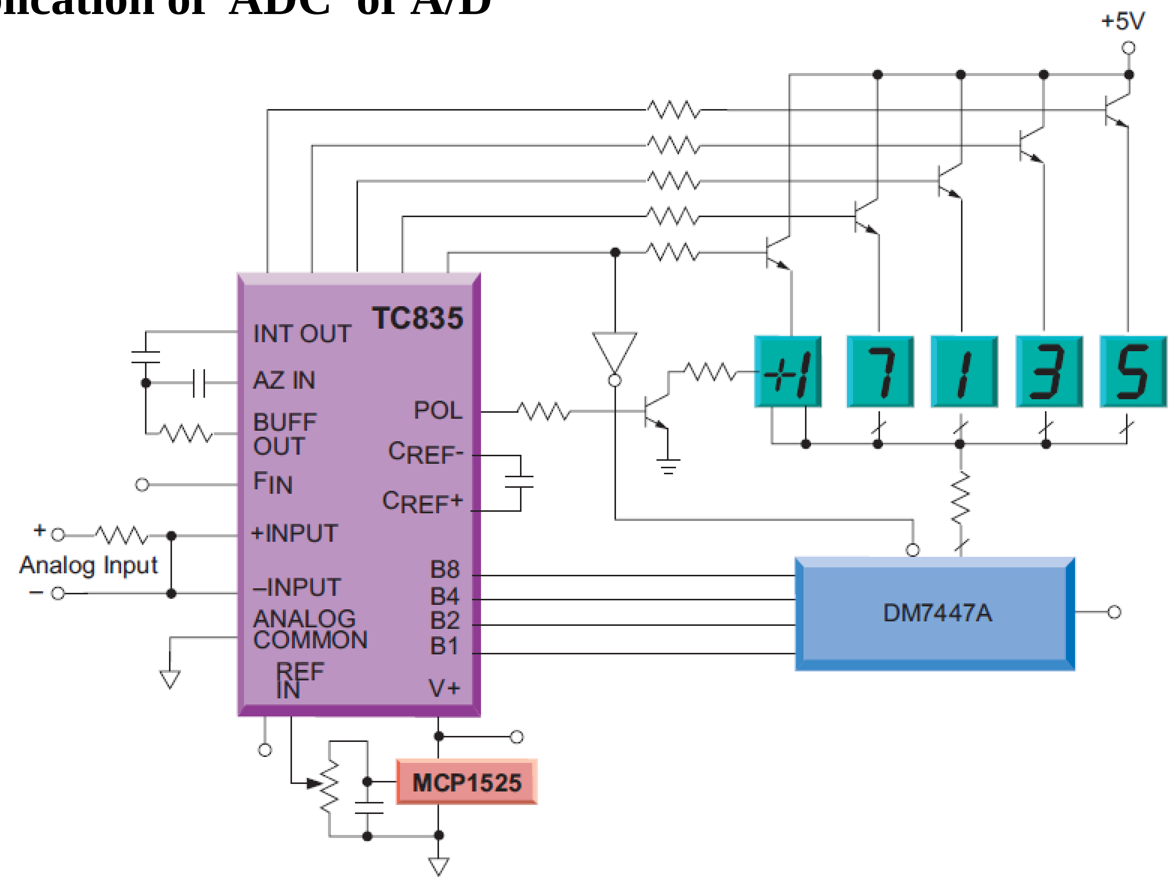




64-Pin MQFP



Some application of ADC or A/D



Recall Fourier Transform of periodical signal

$$\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \overset{\mathscr{F}}{\longleftrightarrow} 2\pi \sum_{k=-\infty}^{\infty} C_k \delta(\omega - k\omega_0)$$

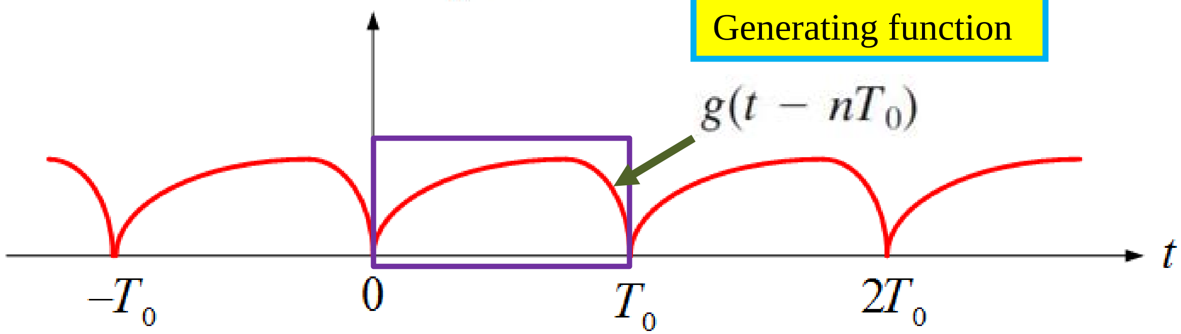
$$f(t) =$$

$$\sum_{n=-\infty}^{\infty} g(t - nT_0) \longleftrightarrow 2\pi \sum_{n=-\infty}^{\infty} \frac{G(n\omega_0)}{T_0} \delta(\omega - n\omega_0)$$

$$C_n = \frac{G(n\omega_0)}{T_0}$$

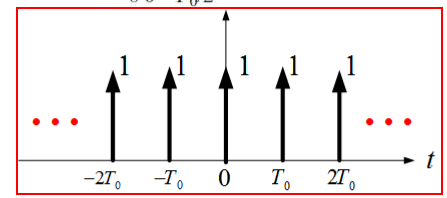
Fourier Transform of periodical signal

$$f(t) = \sum_{n=-\infty}^{\infty} g(t - nT_0)$$



$$\sum_{-\infty}^{\infty} \delta(t - nT_0) = \sum_{-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)dt = \frac{1}{T_0}$$



$$f(t) = \sum_{n=-\infty}^{\infty} g(t - nT_0) = \sum_{n=-\infty}^{\infty} g(t)*\delta(t - nT_0) = g(t)*\sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

$$f(t) = g(t)*\sum_{n=-\infty}^{\infty} \frac{1}{T_0} e^{jn\omega_0 t}$$

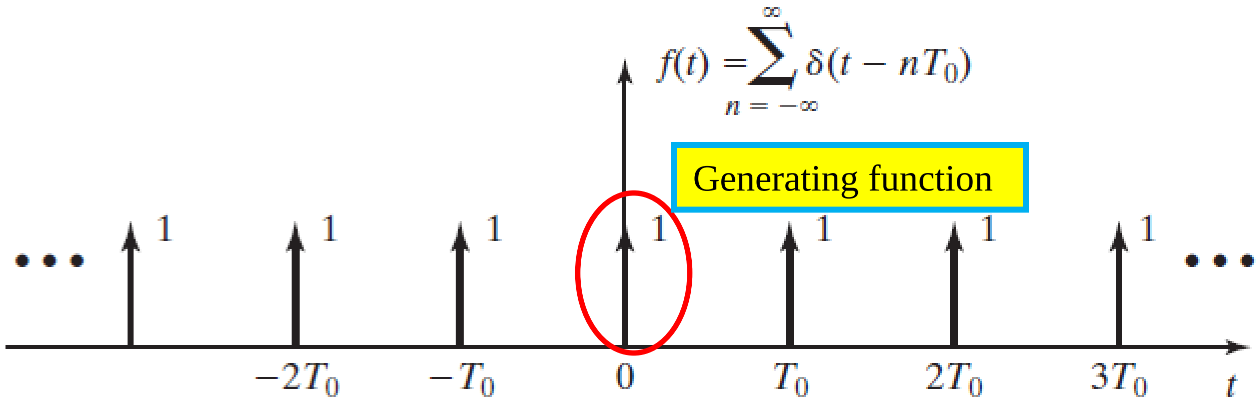
$$f(t) = g(t)*\sum_{n=-\infty}^{\infty} \frac{1}{T_0} e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} \frac{1}{T_0} g(t)*e^{jn\omega_0 t} \Longleftrightarrow \sum_{n=-\infty}^{\infty} \frac{1}{T_0} G(\omega)\bullet 2\pi\delta(\omega - n\omega_0)$$

Fourier Series
expnasion of
train impulses

Since $G(\omega)\bullet 2\pi\delta(\omega - n\omega_0) = 2\pi G(n\omega_0)\delta(\omega - n\omega_0)$

$$f(t) = \sum_{n=-\infty}^{\infty} g(t - nT_0) \longleftrightarrow 2\pi \sum_{n=-\infty}^{\infty} \frac{G(n\omega_0)}{T_0} \delta(\omega - n\omega_0)$$

EXAMPLE 5.14 The frequency spectrum of a periodic impulse signal

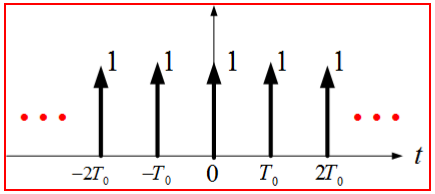


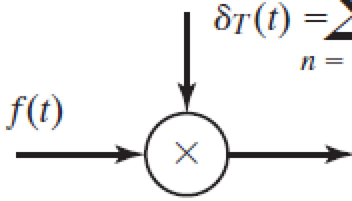
$$f(t) = \sum_{n=-\infty}^{\infty} g(t - nT_0) \longleftrightarrow 2\pi \sum_{n=-\infty}^{\infty} \frac{G(n\omega_0)}{T_0} \delta(\omega - n\omega_0)$$

The generating function is $g(t) = \delta(t) \Leftrightarrow 1 = G(\omega)$

$$f(t) = \sum_{n=-\infty}^{\infty} \delta(\omega - nT_0) \Leftrightarrow 2\pi \sum_{n=-\infty}^{\infty} \frac{1}{T_0} \delta(\omega - n\omega_0) = \frac{2\pi}{T_0} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) = \omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

Impulse Sampling

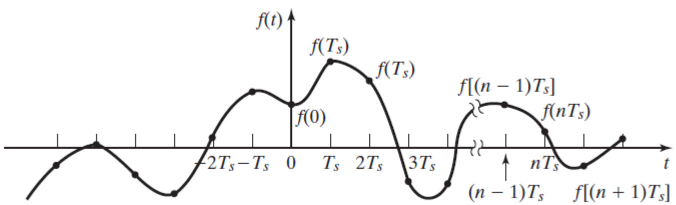


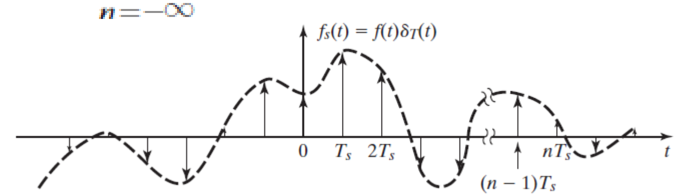


$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$f_s(t) = f(t)\delta_T(t) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_S)$$

$$= \sum_{n=-\infty}^{\infty} f(nT_S)\delta(t - nT_S)$$





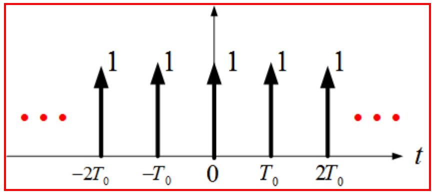
Since

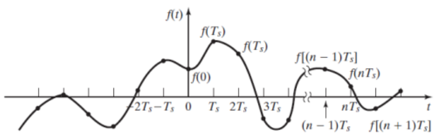
$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_S) \overset{\mathscr{F}}{\longleftrightarrow} \omega_S \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_S)$$

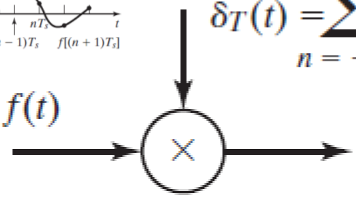
$$f_s(t) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_S) \Longleftrightarrow F_s(\omega) = \frac{1}{2\pi} F(\omega) * \left[\omega_s \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_S) \right]$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} F(\omega) * \delta(\omega - k\omega_S)$$

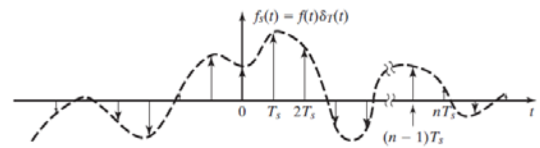
$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} F(\omega - k\omega_S)$$







$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



$$f_s(t) = f(t)\delta_T(t) = f(t)\sum_{n=-\infty}^{\infty} \delta(t - nT_S) = \sum_{n=-\infty}^{\infty} f(nT_S)\delta(t - nT_S)$$

$$F_s(\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} F(\omega - k\omega_S)$$