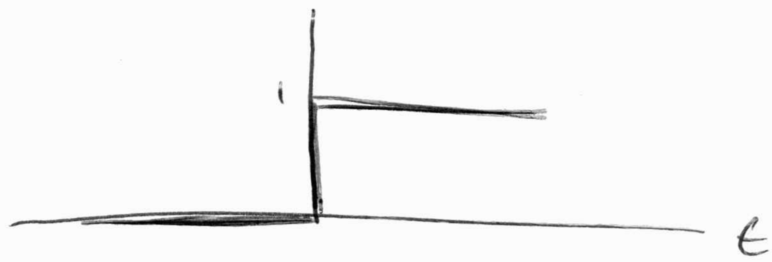
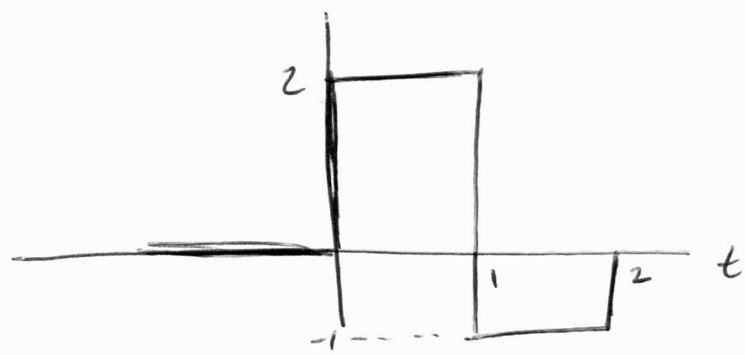
3.2

$h(t) = u(t)$



$x(t)$

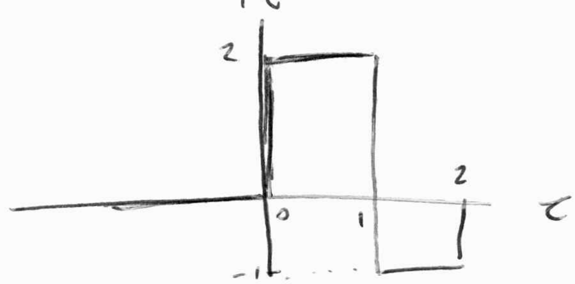


$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau)\, d\tau$$

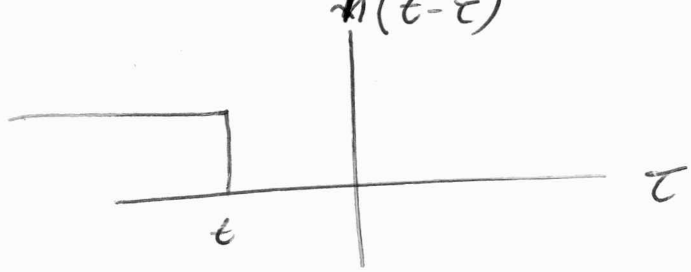
$$= \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

will do this.

$x(\tau)$



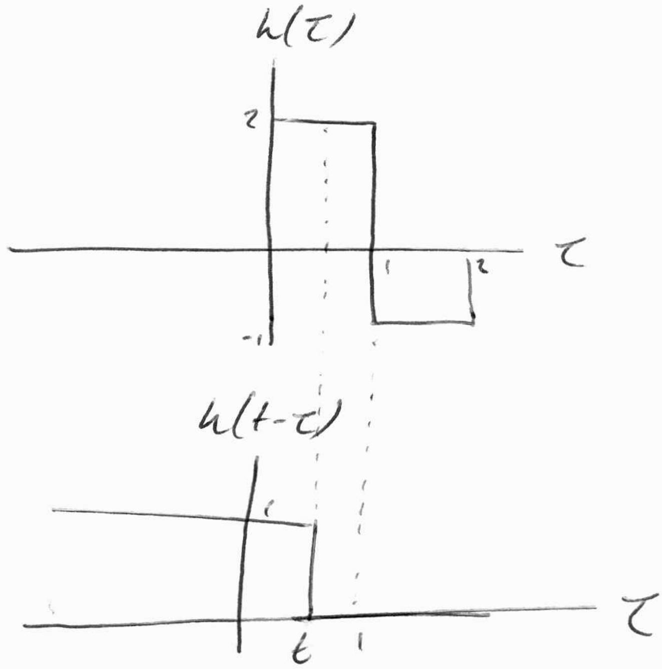
$h(t-\tau)$



for $t < 0$

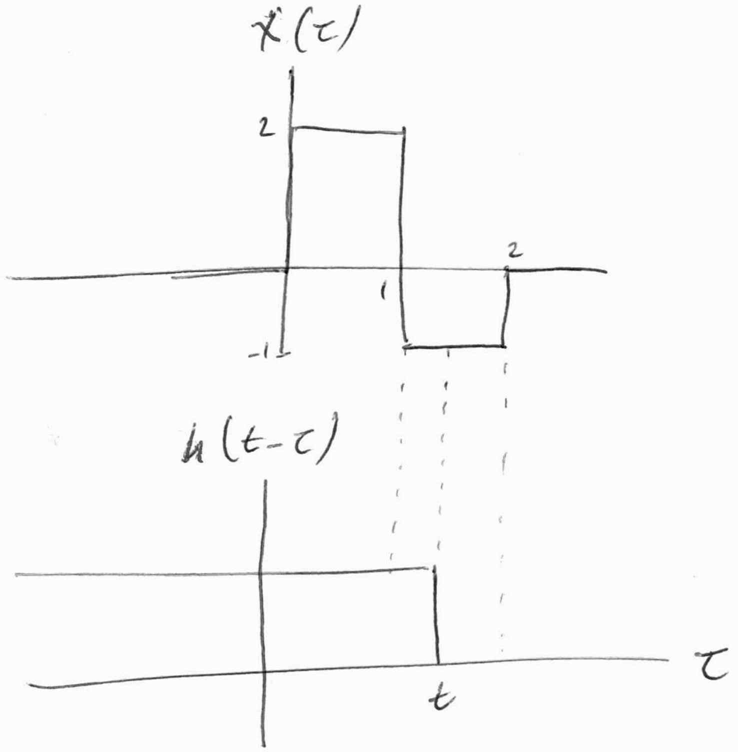
$y(t) = 0$

for ~~t~~ $0 \leq t < 1$



$$y(t) = \int_0^t (1)(2)\, d\tau = 2\tau \Big|_0^t = 2t - 0$$
$$= 2t$$

for $1 \leq t < 2$

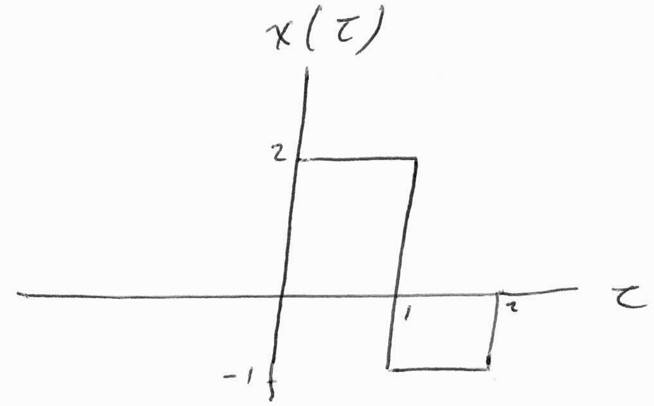


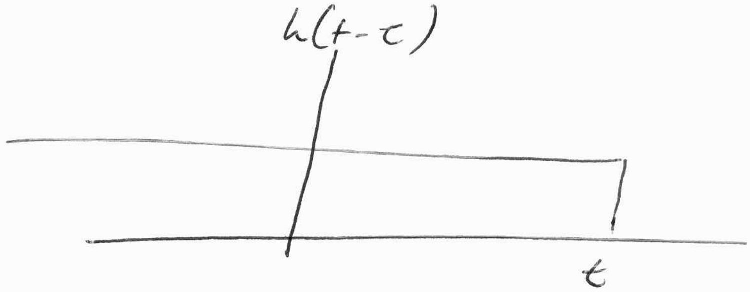
$$y(t) = \int_0^1 (1)(2)\, d\tau + \int_1^t (1)(-1)\, d\tau$$

$$= 2\tau\Big|_0^1 + (-\tau)\Big|_1^t$$

$$= 2 + (-t) + 1 = 3 - t$$

for $t \geq 2$



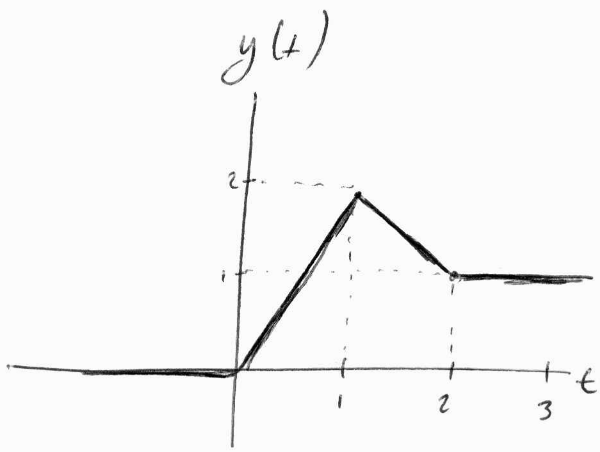


$$y(t) = \int_0^1 (1)(2)\,d\tau + \int_1^2 (1)(-1)\,d\tau$$

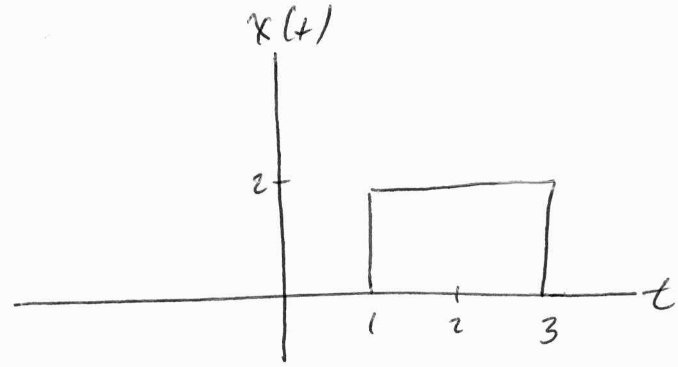
$$= 2 - 1 = 1$$

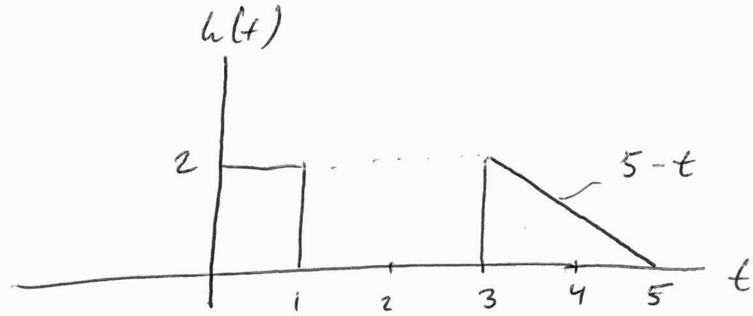
So,

$$y(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 \leq t < 1 \\ 3-t & 1 \leq t < 2 \\ 1 & 2 \leq t \end{cases}$$



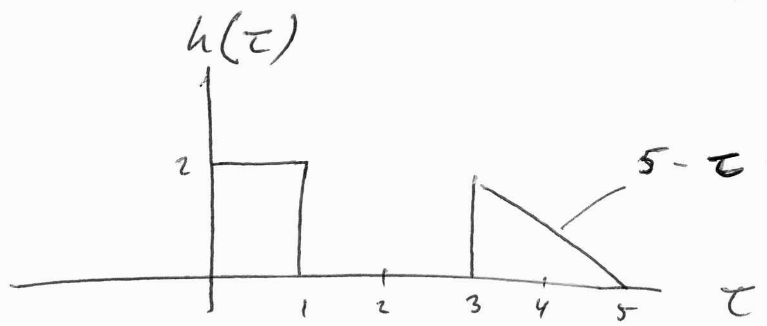
3.4 a

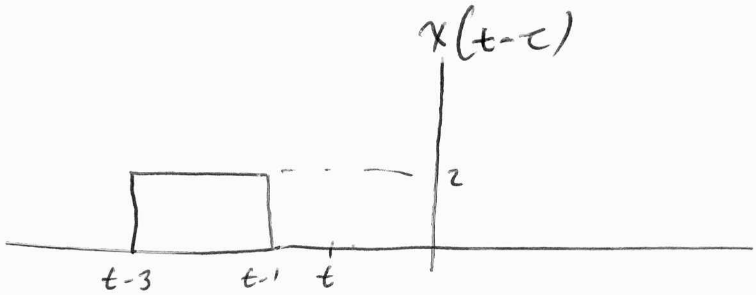




for $t < 1$

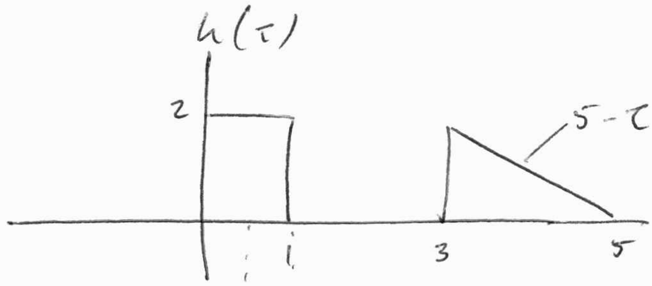
τ not t

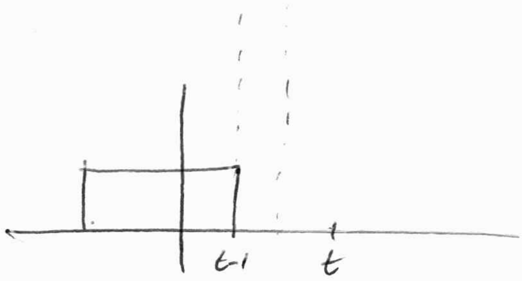




$y(t) = 0$

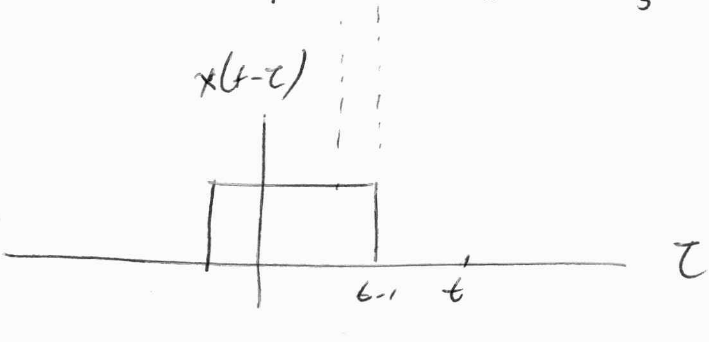
for $1 \le t < 2 \Rightarrow$ (i.e., $0 < (t-1) < 1$)





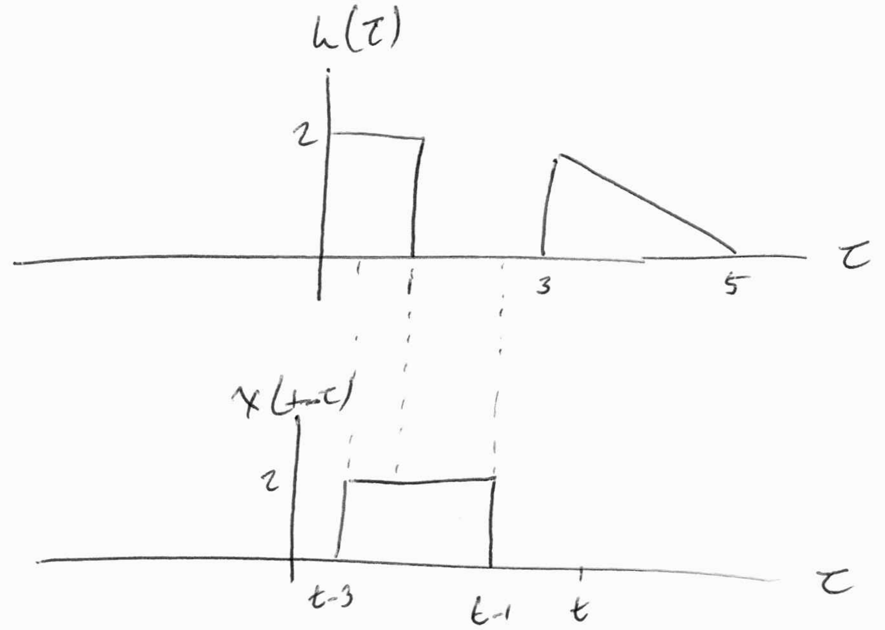
$$y(t) = \int_0^{t-1} (2)(2)\, d\tau = 4\tau \Big|_0^{t-1} = 4t - 4$$

for $2 \le t < 3 \Rightarrow$ (i.e., $1 < (t-1) < 2$)



$$y(t) = \int_0^{1} (2)(2)\, d\tau = 4$$

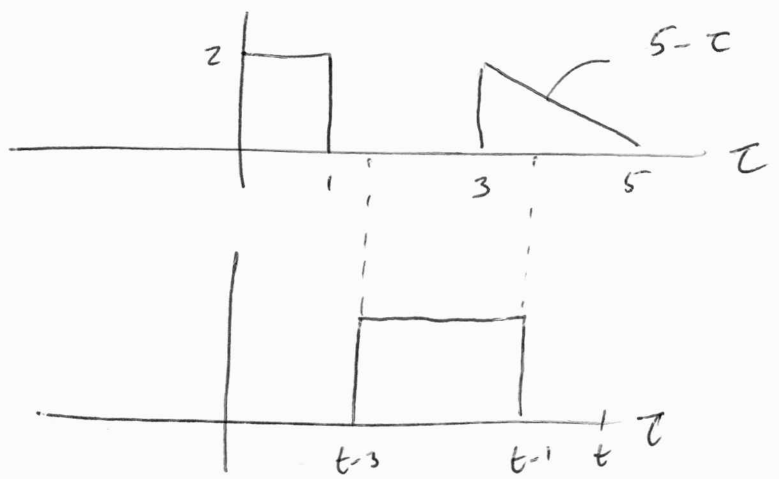
for $3 \leq t < 4 \Rightarrow$ (i.e., $2 < (t-1) < 3$



$$y(t) = \int_{t-3}^{1} (2)(2)\, d\tau = 4\tau \Big|_{t-3}^{1} = 4 - 4(t-3)$$

$$= -4t + 16$$

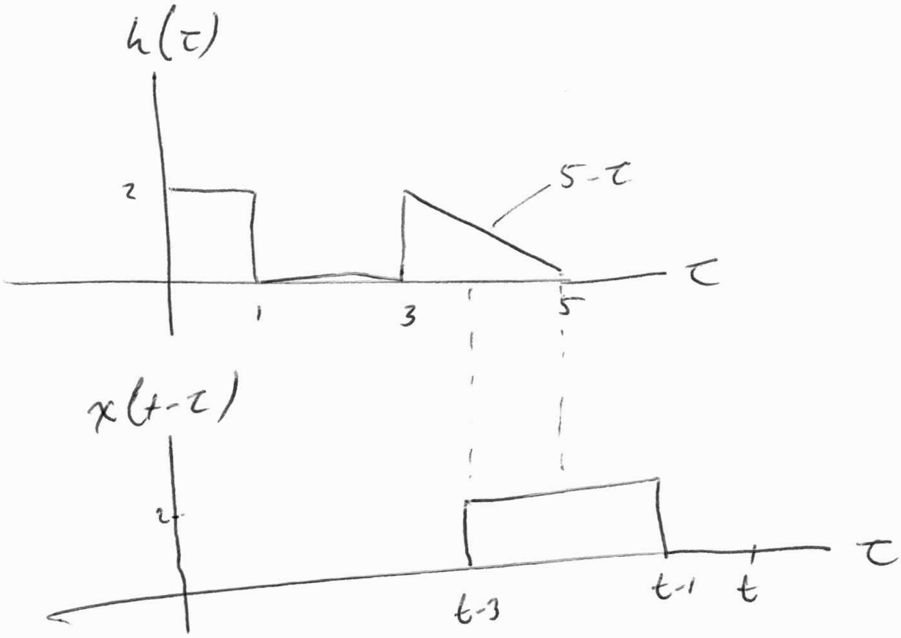
for $4 \leq t < 6 \Rightarrow$ (i.e., $3 \leq (t-1) < 5$)



$$y(t) = \int_{3}^{t-1} (2)(5-\tau)\, d\tau = \int_{3}^{t-1} 10 - 2\tau\, d\tau = 10\tau - \frac{\cancel{2}\tau^2}{\cancel{2}} \Big|_{3}^{t-1}$$

$$= 10(t-1) - \frac{\cancel{2}(t-1)^2}{\cancel{2}} - 30 + 9$$

for $6 \le t < 8 \Rightarrow$ (i.e., $5 \le (t-1) < 7$



$$y(t) = \int_{t-3}^{5} (2)(5-\tau)\, d\tau = \int_{t-3}^{5} 10 - 2\tau \; d\tau$$

$$= 10\tau - \tau^2 \Big|_{t-3}^{5} = 50 - 25 - 10(t-3) + (t-3)^2$$

---

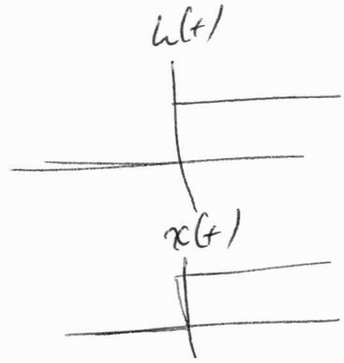
for $8 \le t$

$y(t) = 0$

3.8 a

i) $h(t) = u(t)$

$x(t) = u(t)$



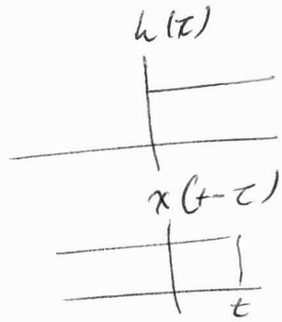
<u>for $t < 0$</u>

$y(t) = 0$

<u>for $t > 0$</u>

$$y(t) = \int_0^t 1 \, d\tau$$

$$= \tau \Big|_0^t = t$$



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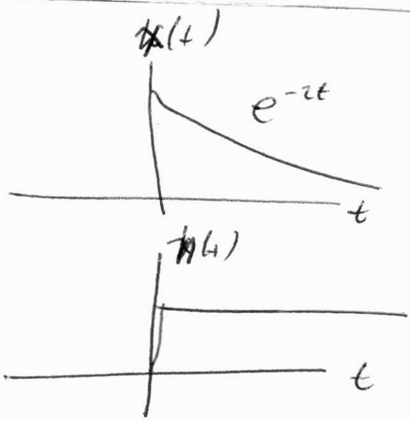
ii) $h(t) = u(t)$

$x(t) = e^{-2t} u(t)$



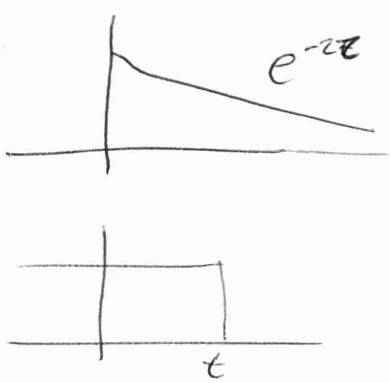
<u>for $t < 0$</u>

$y(t) = 0$

<u>for $t > 0$</u>

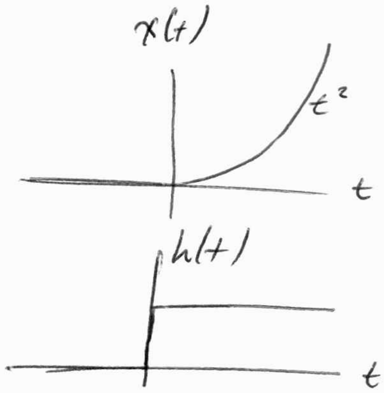
$$y(t) = \int_0^t e^{-2\tau} d\tau = -\frac{1}{2} e^{-2\tau} \Big|_0^t$$

$$= -\frac{1}{2} e^{-2t} + \frac{1}{2}$$



iii)

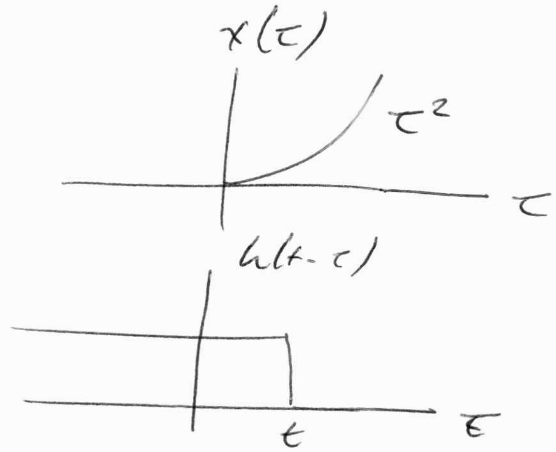
$$h(t) = u(t)$$
$$x(t) = t^2 u(t)$$



<u>for $t < 0$</u>

$$y(t) = 0$$

<u>for $t > 0$</u>



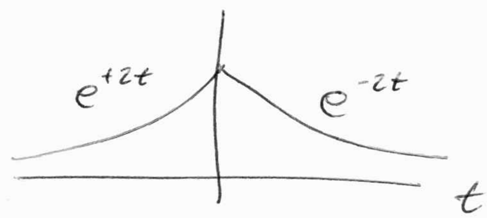
$$y(t) = \int_0^t \tau^2 \, d\tau$$

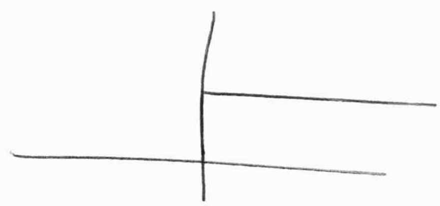
$$= \frac{\tau^3}{3}\Big|_0^t = \frac{t^3}{3}$$

---

iii)

$$h(t) = u(t)$$
$$x(t) = \begin{cases} e^{-2t} & t \geq 0 \\ e^{2t} & t < 0 \end{cases}$$

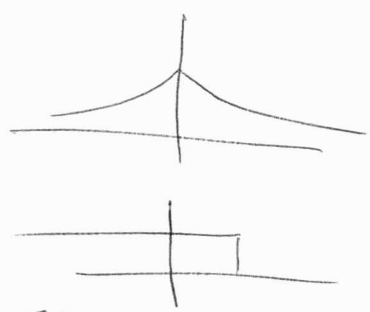




<u>for $t < 0$</u>

$$y(t) = \int_{-\infty}^t e^{2\tau} \, d\tau = \frac{1}{2} e^{2\tau}\Big|_{-\infty}^t = \frac{1}{2} e^{2t} - 0$$

<u>for $t > 0$</u>



$$y(t) = \int_{-\infty}^0 e^{2\tau} d\tau + \int_0^t e^{-2\tau} \, d\tau$$

$$= \frac{1}{2} e^{2\tau}\Big|_{-\infty}^0 - \frac{1}{2} e^{-2\tau}\Big|_0^t = \frac{1}{2} - \frac{1}{2} e^{-2t} + \frac{1}{2} = 1 - \frac{1}{2} e^{-2t}$$

Problem 3.15:

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

. For each of the following cases, find $h(t)$:

a) $y(t) = x(t-5)$

$$= \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

This is true only if $h(t) = \delta(t-5)$

b) $y(t) = \int_{-\infty}^{t} x(\tau - 5)\, d\tau$

$$= \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

This is true only if $h(t) = u(t-5)$

c) $y(t) = \int_{-\infty}^{t} \left[\int_{-\infty}^{\sigma} x(\tau - 5)\, d\tau \right] d\sigma$

$$= \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

This is true only if $h(t) = t\, u(t-5)$

Problem 3.20:

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = \cos(t)\, x(t)$$

a)

$a_1 x_1(t) \longrightarrow a_1 y_1(t) = a_1 \cos t\, x_1(t)$

$a_2 x_2(t) \longrightarrow a_2 y_2(t) = a_2 \cos t\, x_2(t)$

$a_1 x_2(t) + a_2 x_2(t) \longrightarrow \cos t\, [a_1 x_1(t) + a_2 x_2(t)]$

$= a_1 \cos t\, x_1(t) + a_2 \cos t\, x_2(t)$

$= a_1 y_1(t) + a_2 y_2(t)$

$\therefore$ The system is linear

b)

$x(t) \longrightarrow y(t) = \cos t\, x(t)$

$y_d(t) = \cos t\, x(t-t_0)$

$x(t-t_0) \longrightarrow y_d(t) = \cos t\, x(t-t_0)$

$y(t-t_0) = \cos(t-t_0)\, x(t-t_0)$

$\therefore y(t-t_0) \neq y_d(t)$

System is time-varying

c) $y(t) = \cos t\, \delta(t)$

$= \cos(0)\, \delta(t) = \delta(t)$

d) $y(t) = \cos t \; \delta\left(t - \frac{\pi}{2}\right)$

$$= \cos\left(\frac{\pi}{2}\right) \delta\left(t - \frac{\pi}{2}\right)$$

$$= 0 \times \delta\left(t - \frac{\pi}{2}\right)$$

$$= 0$$

3.22.

$$y(t) = \int_{0}^{\infty} e^{-\tau} x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\; x(t-\tau)\, d\tau$$

$$= e^{-t} u(t) * x(t)$$

(a) $$h(t) = e^{-t} u(t) * \delta(t)$$

$$= e^{-t} u(t)$$

(b) Causal, since $h(t) = 0 \quad t < 0$

(c) $x(t) = u(t+1)$

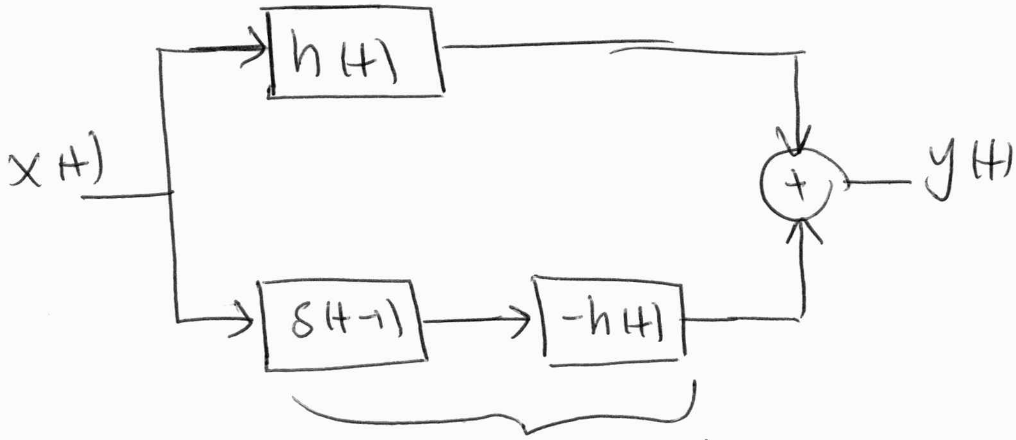
$$y(t) = h(t) * x(t)$$

$$= e^{-t} u(t) * u(t+1)$$

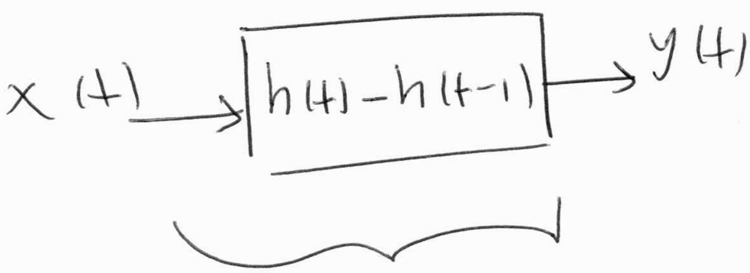
$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau) \cdot \underbrace{u(t+1-\tau)}_{=1} d\tau \qquad t+1-\tau \geq 0 \Rightarrow \lambda \leq t+1$$

$$= \int_{0}^{t+1} e^{-\tau} (1)\, d\tau = 1 - e^{-(t+1)}$$

(d)



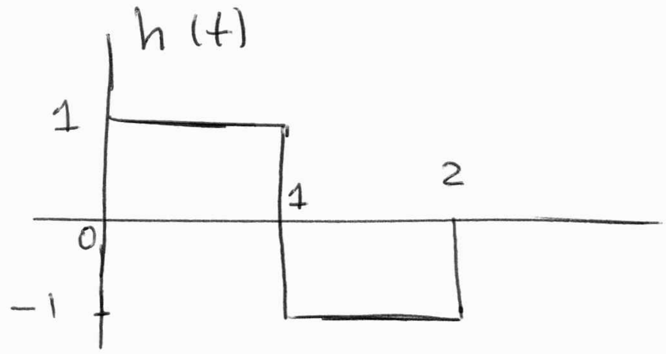
$$\delta(t-1) * (-h(t))$$
$$= -h(t-1)$$



$$h_{total}(t) = h(t) - h(t-1)$$
$$= e^{-t}u(t) - e^{-(t-1)}u(t-1)$$

3.25

$$h(t) = u(t) - 2u(t-1) + u(t-2)$$



(a) Since $h(t) = 0 \quad t < 0 \Rightarrow$ Causal

(b) Stable (BIBO) $\Rightarrow \int_{-\infty}^{\infty} |h(t)|\,dt < \infty$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{0}^{1} |1|\,dt + \int_{1}^{2} |-1|\,dt$$

$$= 1 + 1 = 2 < \infty$$

$\Rightarrow$ stable (BIBO).

(c) $y(t) = x(t) * h(t)$

$$= [\delta(t-1) - 2\delta(t-2)] * h(t)$$

$$= \delta(t-1) * h(t) - 2\,\delta(t-2) * h(t)$$

$$= h(t-1) - 2h(t-2)$$