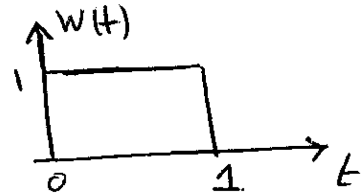
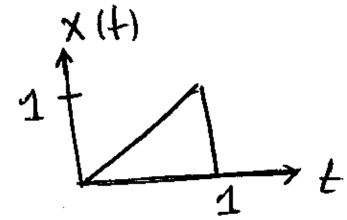
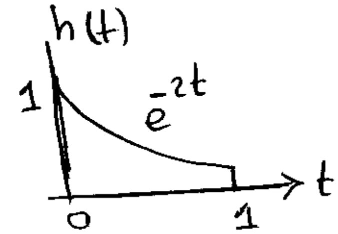
## Q1 Find the laplac transform for the followings signals




(a)

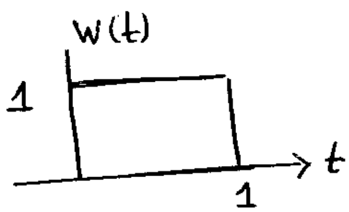



(b)




### Solution

(a)




$$\Longleftrightarrow W(s) = \int_0^1 (1)\, e^{-st}\, dt = -\frac{e^{-st}}{s}\Big|_0^1$$

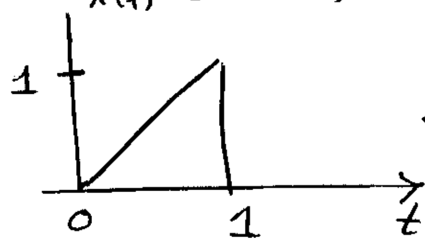
$$= -\frac{e^{-s(1)} - e^{-s(0)}}{s} = \frac{1 - e^{-s}}{s}$$

Using the table and properties

$$w(t) = u(t) - u(t-1) \Longleftrightarrow W(s) = \frac{1}{s} - \frac{1}{s} e^{-s} = \frac{1 - e^{-s}}{s}$$

(b)

$x(t) = t\,w(t)$ ↦ part (a)




$$\Longleftrightarrow X(s) = \int_0^1 t\, e^{-st}\, dt$$

$$u = t \qquad dv = e^{-st}\,dt$$

$$du = dt \qquad v = -\frac{e^{-st}}{s}$$

$$X(s) = -\frac{t e^{-st}}{s}\Bigg|_0^1 + \frac{1}{s}\int_0^1 e^{-st}\,dt$$

$$= -\frac{e^{-s}}{s} + \frac{1}{s}\,\frac{e^{-st}}{-s}\Bigg|_0^1 = -\frac{e^{-s}}{s} - \frac{e^{-s}-1}{s^2}$$

$$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$

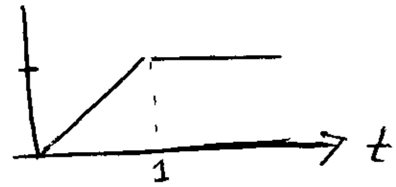
<u>using the table and properties</u>

① $x(t) = t\,w(t) \Longleftrightarrow X(s) = -\frac{d}{ds}\mathcal{L}[w(t)]$

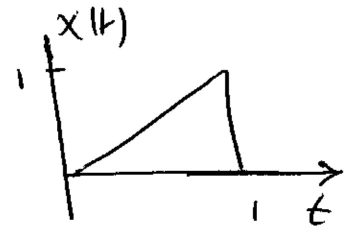
$$= -\frac{d}{ds}\left(\frac{1-e^{-s}}{s}\right)$$

$$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$

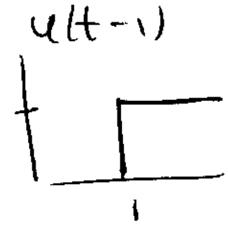
(2) since $\displaystyle\int_0^t w(t)\,dt =$




$=$




$+$




$$\Rightarrow \int_0^t w(t)\,dt = x(t) + u(t-1)$$

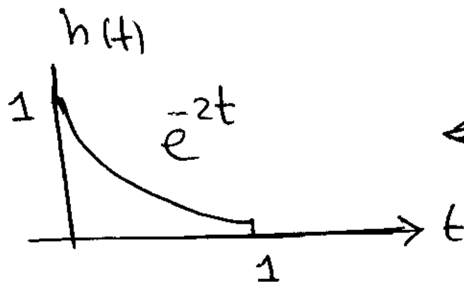
$$\Rightarrow x(t) = \int_0^t w(t)\,dt - u(t-1)$$

$$\Rightarrow X(s) = \frac{W(s)}{s} - \frac{1}{s}e^{-s}$$

$$= \frac{\frac{1-e^{-s}}{s}}{s} - \frac{1}{s}e^{-s}$$

$$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$

(c)




$$\Longleftrightarrow \quad H(s) = \int_0^1 e^{-2t} e^{-st} \, dt$$

$$= \int_0^1 e^{-(s+2)t} \, dt$$

$$= -\frac{e^{-(s+2)t}}{(s+2)} \Bigg|_0^1$$

$$= -\frac{1}{(s+2)} \left[ e^{-(s+2)(1)} - e^{0} \right]$$

$$= \frac{1 - e^{-(s+2)}}{(s+2)}$$

<u>Using the table and Properties</u>

since $\cancel{x}$ $h(t) = e^{-2t} w(t)$ $\Longleftrightarrow$ $H(s) = W(s+2)$

↳ part(a)

$$= \frac{1 - e^{-(s+2)}}{(s+2)}$$

**Q2** IF $X(s) = \dfrac{4}{(s+1)(s+2)^3}$ Find $x(t)$

**Solution** Since we have repeated root,

$$X(s) = \frac{A_1}{(s+1)} + \frac{B_0}{(s+2)^3} + \frac{B_1}{(s+2)^2} + \frac{B_2}{(s+2)}$$

Using Heaviside method,

$$A_1 = (s+1)X(s)\Big|_{s=-1} = \frac{4}{\cancel{(s+1)}(s+2)^3}\cancel{(s+1)}\Big|_{s=-1}$$

$$= \frac{4}{(s+2)^3}\Big|_{s=-1} = 4$$

$$B_0 = (s+2)^3 X(s)\Big|_{s=-2} = \frac{4}{(s+1)}\Big|_{s=-2} = -4$$

using $B_m = \dfrac{1}{(n-m)!}\dfrac{d^{(n-m)}}{ds^{(n-m)}}\left[(s+\alpha)^n X(s)\right]_{s=-\alpha}$

$$B_1 = \frac{d}{ds}\left[\frac{4}{(s+1)}\right]\Big|_{s=-2} = -\frac{4}{(s+1)^2}\Big|_{s=-2} = -4$$

$$B_2 = \frac{1}{2}\frac{d^2}{ds^2}\left[\frac{4}{(s+1)}\right]\Big|_{s=-2} = \frac{4}{(s+1)^3} = -4$$

$$\Rightarrow X(s) = \frac{4}{s+1} - \frac{4}{(s+2)^3} - \frac{4}{(s+2)^2} - \frac{4}{s+2}$$

$$4e^{-t}u(t) - 2t^2 e^{-2t}u(t) - 4te^{-2t}u(t) - 4e^{-2t}u(t)$$

Q3 If the input $x(t)$ and output $y(t)$ of a LTI system is described by the following differential equation,

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 4e^{-2t}$$

with $y(0) = 3$ and $\underbrace{\left.\frac{dy(t)}{dt}\right|_{t=0}}_{y'(0)} = 4$

Find $y(t)$ ?

Solution Taking laplace transform of both side,

$$s^2Y(s) - sy(0) - y'(0) + 3[sY(s) - y(0)]$$

$$+ 2Y(s) = \frac{4}{s+2}$$

substitute for the initial conditions and rearrange,

$$(s^2+3s+2)Y(s) = 3s+4+9 + \frac{4}{s+2}$$

$$= \frac{3s^2+19s+30}{s+2}$$

$$\Rightarrow Y(s) = \frac{3s^2+19s+30}{(s+2)(s^2+3s+2)} = \frac{3s^2+19s+30}{(s+2)(s+2)(s+1)}$$

$$= \frac{3s^2+19s+30}{(s+2)^2(s+1)} = \frac{A_1}{s+1} + \frac{B_0}{(s+2)^2} + \frac{B_1}{s+2}$$

(Problem P3 solution Continue)

$$A_1 = (s+1)Y(s)\Big|_{s=-1} = \frac{3s^2+19s+30}{(s+2)^2}\Big|_{s=-1} = 14$$

$$B_0 = (s+2)^2\,Y(s)\Big|_{s=-2} = \frac{3s^2+19s+30}{(s+1)}\Big|_{s=-2} = -4$$

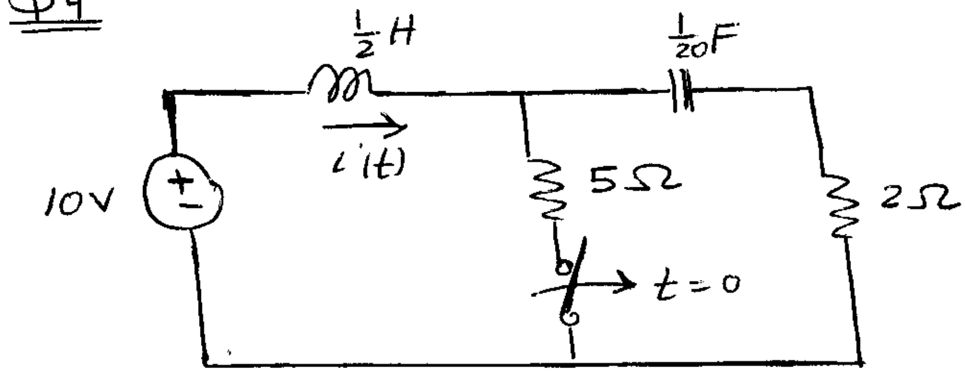
$$B_1 = \frac{d}{ds}\left[\frac{3s^2+19s+30}{(s+1)}\right]\Bigg|_{s=-2} = -11$$

$$\Rightarrow Y(s) = \frac{14}{s+1} + \frac{(-4)}{(s+2)^2} + \frac{(-11)}{(s+2)}$$

$$\Rightarrow y(t) = 14e^{-t}u(t) - 4te^{-2t}u(t) - 11e^{-2t}u(t)$$

$$= (14e^{-t} - 4te^{-2t} - 11e^{-2t})\,u(t)$$

## Q4



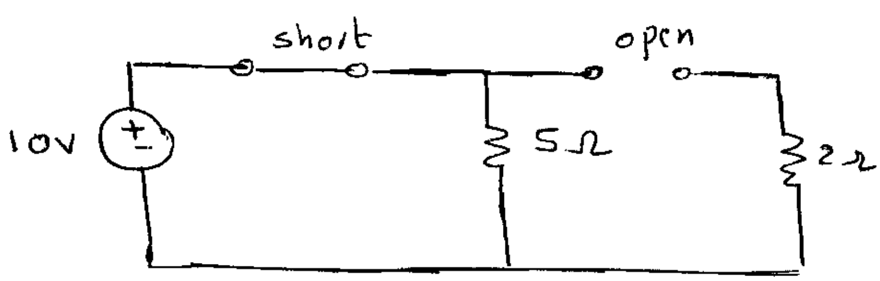

For the circuit shown above, the switch is in closed position for a long time.

At $t = 0$, the switch is opened, find the inductor current $i(t)$ for $t \geq 0$

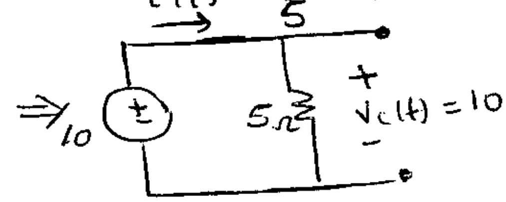
### solution

① when the ~~swit~~ switch is closed for a long time

the capacitor become open and the inductor short

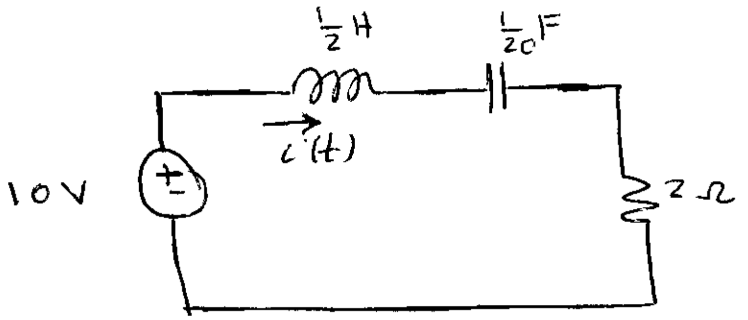



$i(t) = \frac{10}{5} = \cancel{5A}\ 2A$



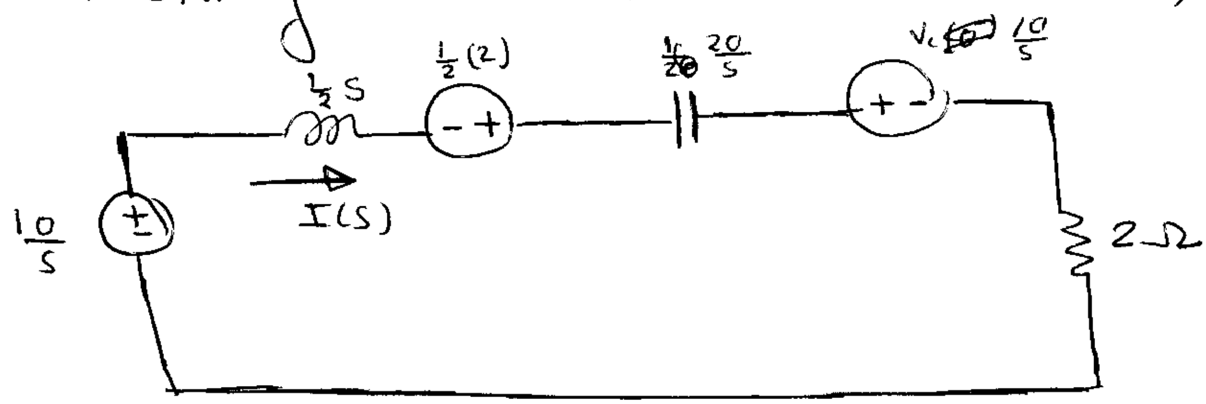

$i(0^-) = 2A$ $\qquad$ $V_c(0^-) = 10$

(2) Now when the switch open, we will have




$i(0^-) = 2A \quad V_c(0^-) = 10V$

Transfering the circuit to the s-domain,




KUL $\quad -\frac{10}{s} + \frac{1}{2}sI(s) - 1 + \frac{20}{s}I(s) + \frac{10}{s} + 2I(s) = 0$

solving for I(s), $\quad I(s) = \dfrac{2s}{s^2+4s+40}$

$$= \frac{2s}{(s-(-2+j6))(s-(-2-j6))}$$

using partial fraction,

$$i(t) = e^{-2t}\left[2\cos 6t - \frac{2}{3}\sin 6t\right]u(t)$$