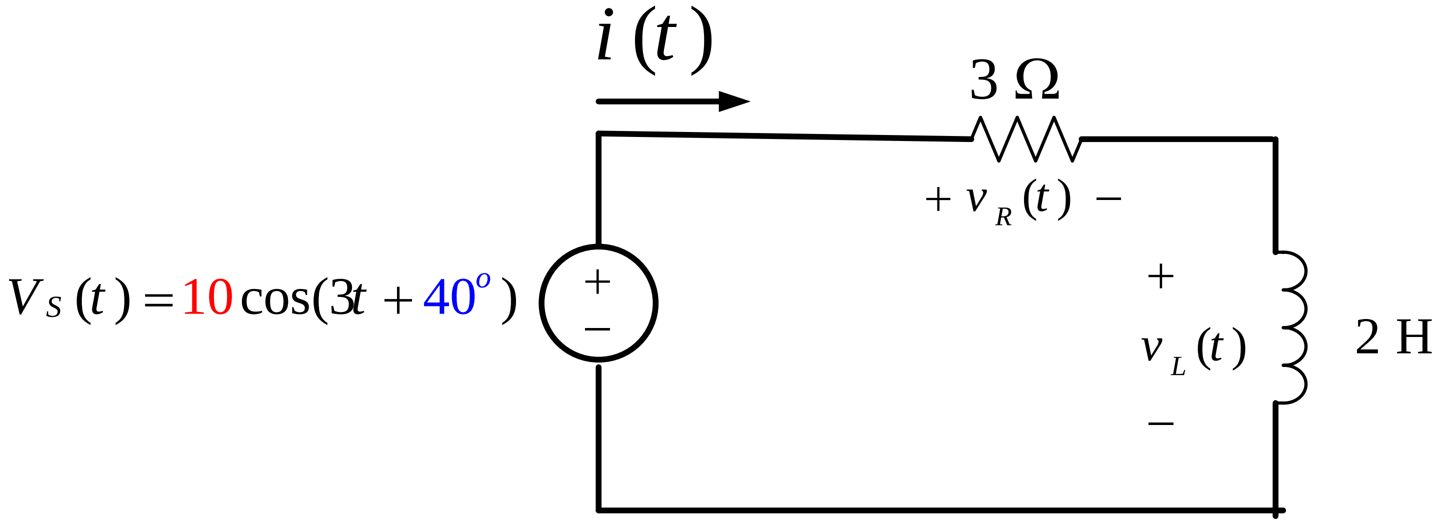
9.2 The Sinusoidal Response



KVL $$L\frac{di}{dt} + Ri = 10\cos(3t + 40^o)$$

Solution for i(t) should be a sinusoidal of frequency 3

$$i(t) = 1.58\cos(3t - 31.56^o)$$

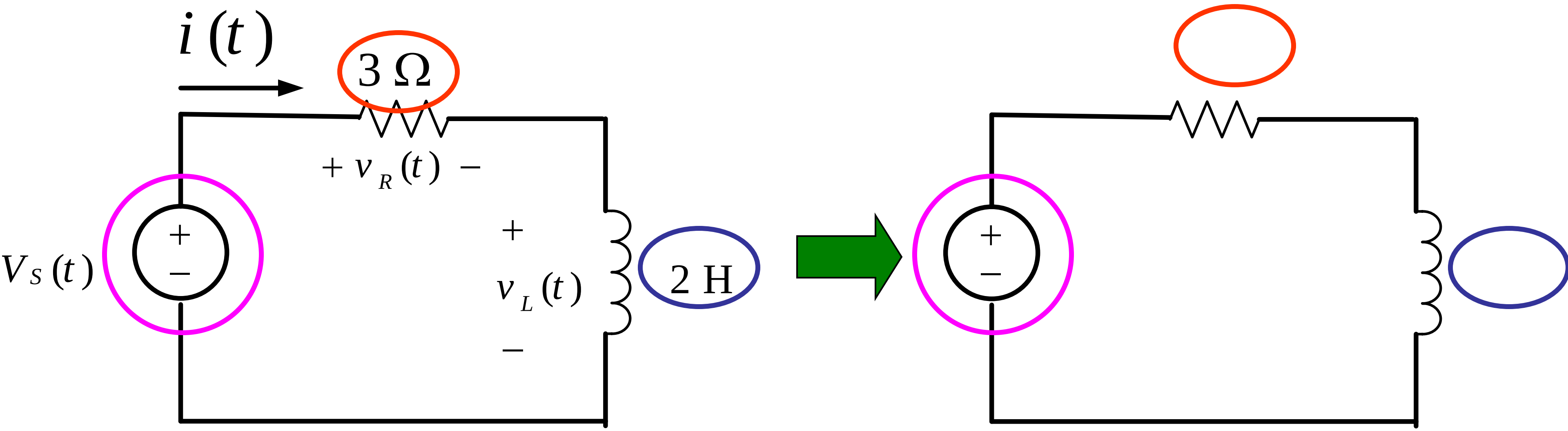
$$v_R(t) = 3.1\cos(3t - 31.56^o)$$

$$v_L(t) = 9.5\cos(3t - 58.43^o)$$

We notice that only the **amplitude** and **phase** change

In this chapter, we develop a technique for calculating the response directly without solving the differential equation

Time Domain | **Complex Domain**



Deferential Equation | **Algebraic Equation**

$$L\frac{di}{dt} + Ri = V_S(t)$$

9.3 The phasor

The **phasor** is a complex number that carries the **amplitude** and **phase** angle information of a sinusoidal function

The **phasor** concept is rooted in **Euler's** identity $e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$

Euler's identity relates the complex exponential function to the trigonometric function

We can think of the **cosine** function as the **real part** of the complex exponential and the **sine** function as the imaginary part

$$\cos(\theta) = \Re\{e^{j\theta}\} \qquad \sin(\theta) = \Im\{e^{j\theta}\}$$

Because we are going to use the cosine function on analyzing the sinusoidal steady-state we can apply

$$\cos(\theta) = \Re\{e^{j\theta}\}$$

$$e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta) \qquad \cos(\theta) = \Re\{e^{j\theta}\} \qquad \sin(\theta) = \Im\{e^{j\theta}\}$$

$$v = V_m \cos(\omega t + \phi) = V_m \Re\{e^{j(\omega t + \phi)}\} = V_m \Re\{e^{j\omega t} e^{j\phi}\}$$

We can move the coefficient V_m inside $\Rightarrow$ $v = \Re\{V_m e^{j\phi} e^{j\omega t}\}$

The quantity $V_m e^{j\phi}$ is a complex number define to be the **phasor** that carries the amplitude and phase angle of a given sinusoidal function

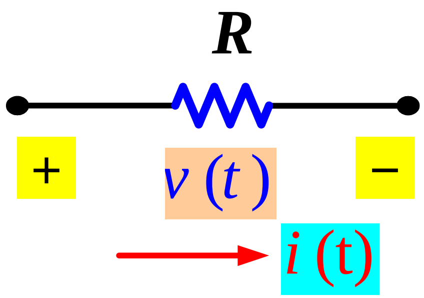
Phasor Transform

$$P\{V_m \cos(\omega t + \phi)\} = V_m e^{j\phi} = \mathbf{V}$$

Were the notation $P\{V_m \cos(\omega t + \phi)\}$

Is read " the phasor transform of $V_m \cos(\omega t + \phi)$

The V–I Relationship for a Resistor



$$v(t) = Ri(t)$$

Let the current through the resistor be a **sinusoidal** given as $i(t) = I_m \cos(\omega t + \theta_i)$

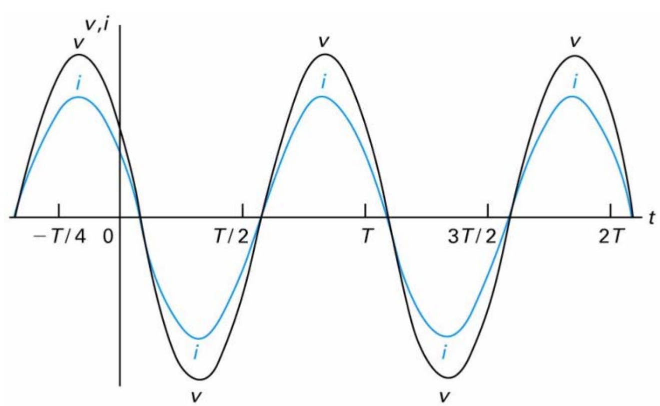
$$\Rightarrow \quad v(t) = Ri(t) = R\left[I_m \cos(\omega t + \theta_i)\right] = RI_m\left[\cos(\omega t + \theta_i)\right]$$

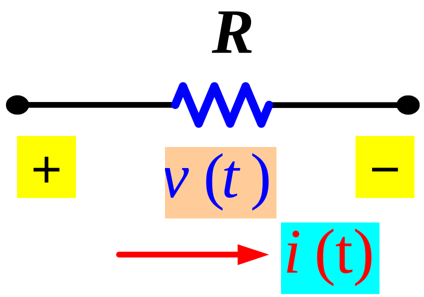
$$\Rightarrow \quad v(t) = RI_m\left[\cos(\omega t + \underbrace{\theta_i}_{\text{voltage phase}})\right]$$

Is also **sinusoidal** with

amplitude $V_m = RI_m$ And phase $\theta_v = \theta_i$

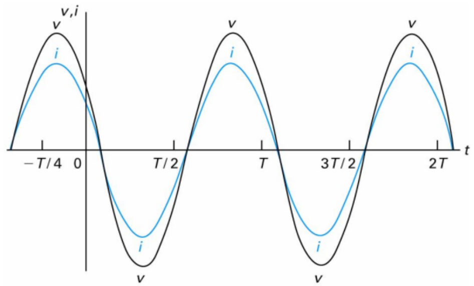
⇒ The sinusoidal voltage and current in a **resistor** are **in phase**





$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$v(t) = R I_m \left[\cos(\omega t + \theta_i)\right]$$

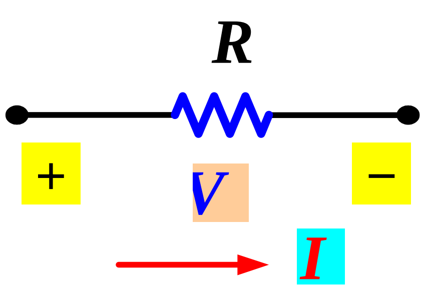


Now let us see the pharos domain representation or pharos transform of the **current** and **voltage**

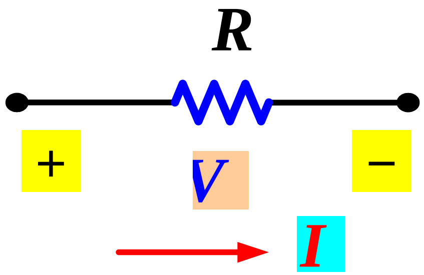
$$i(t) = I_m \cos(\omega t + \theta_i) \quad \xrightarrow{\text{Phasor Transform}} \quad \mathbf{I} = I_m e^{j\theta_i} = I_m \angle \theta_i$$

$$v(t) = R I_m \left[\cos(\omega t + \theta_i)\right] \quad \xrightarrow{\text{Phasor Transform}} \quad \mathbf{V} = R I_m e^{j\theta_i} = \underbrace{R I_m}_{V_m} \angle \underbrace{\theta_i}_{\theta_v} = R\mathbf{I}$$

Which is Ohm’s law on the **phasor** (**or complex**) domain

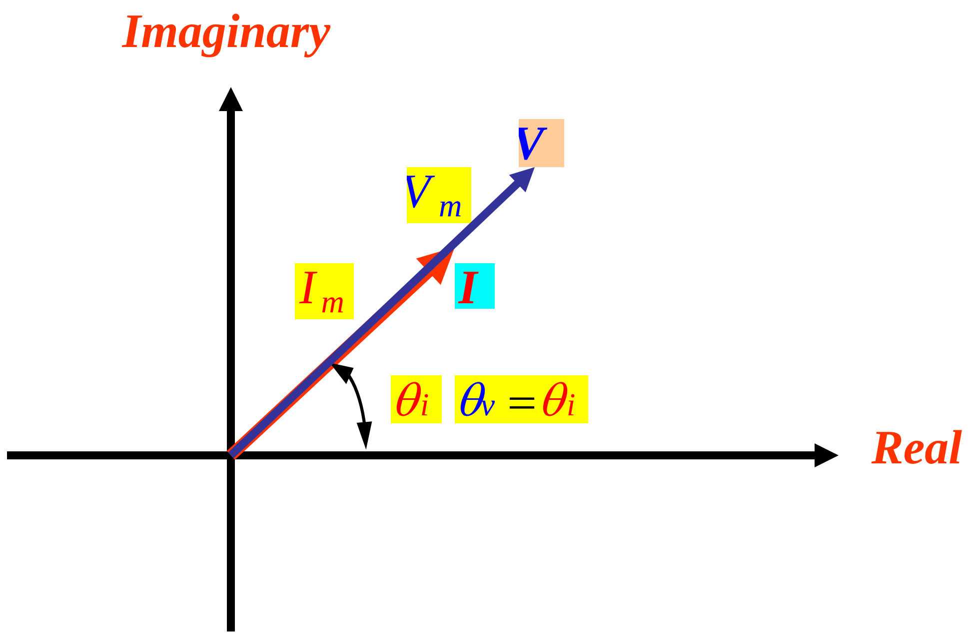


$$\mathbf{V} = R\mathbf{I}$$

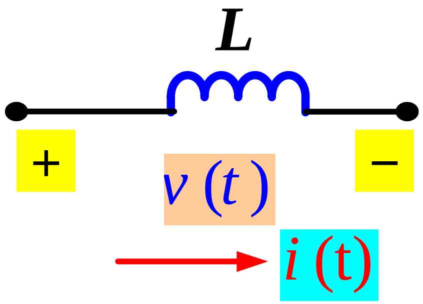


$V = RI$

The voltage and the current are in phase



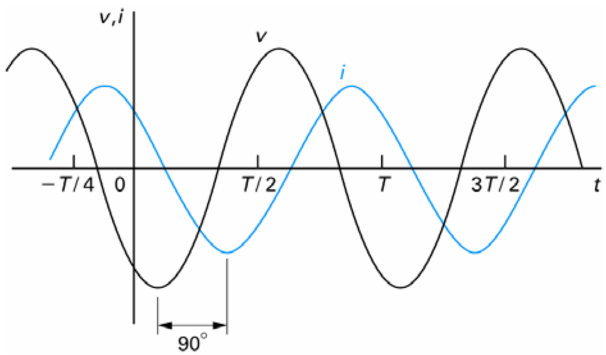
The V–I Relationship for an Inductor



$$v(t) = L\frac{di(t)}{dt}$$

Let the current through the resistor be a **sinusoidal** given as $i(t) = I_m \cos(\omega t + \theta_i)$

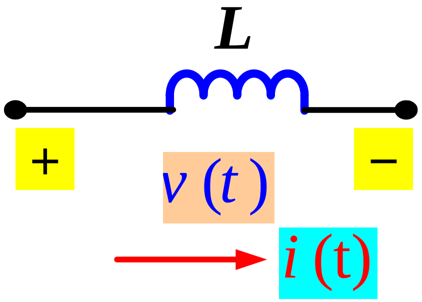
$$v(t) = L\frac{di(t)}{dt} = -\omega L I_m \sin(\omega t + \theta_i)$$



The sinusoidal voltage and current in an **inductor** are **out of phase** by **90º**

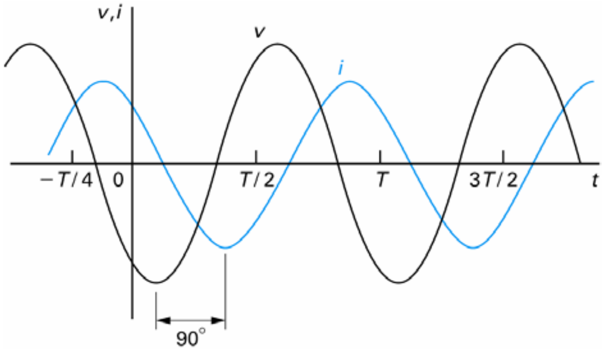
The voltage **lead** the current by **90º** or the current **lagging** the voltage by **90º**

You can express the voltage **leading** the current by **T/4 or 1/4f seconds** were **T** is the period and **f** is the frequency



$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$v(t) = -\omega L I_m \sin(\omega t + \theta_i)$$



Now we rewrite the sin function as a cosine function

(remember the phasor is defined in terms of a cosine function)

$$\Rightarrow \quad v(t) = -\omega L I_m \cos(\omega t + \theta_i - 90^o)$$

The pharos representation or transform of the **current** and **voltage**

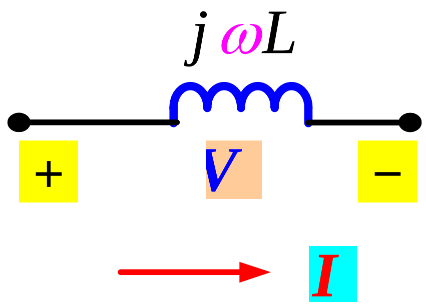
$$i(t) = I_m \cos(\omega t + \theta_i) \Rightarrow \mathbf{I} = I_m e^{j\theta_i} = I_m \angle \theta_i$$

$$v(t) = -\omega L I_m \cos(\omega t + \theta_i - 90^o) \Rightarrow \mathbf{V} = -\omega L I_m e^{j(\theta_i - 90^o)} = -\omega L I_m e^{j\theta_i} \underbrace{e^{-j\,90^o}}_{=-j} = j\,\omega L I_m e^{j\theta_i}$$

But since $\quad j = 1 e^{j\,90^o} = 1 \angle 90^o$

Therefore $\quad \mathbf{V} = j\,\omega L I_m e^{j\theta_i} = \omega L I_m e^{j\,90^o} e^{j\theta_i} = \omega L I_m e^{j(\theta_i + 90^o)} = \underbrace{\omega L I_m}_{V_m} \angle \underbrace{(\theta_i + 90^o)}_{\theta_v}$

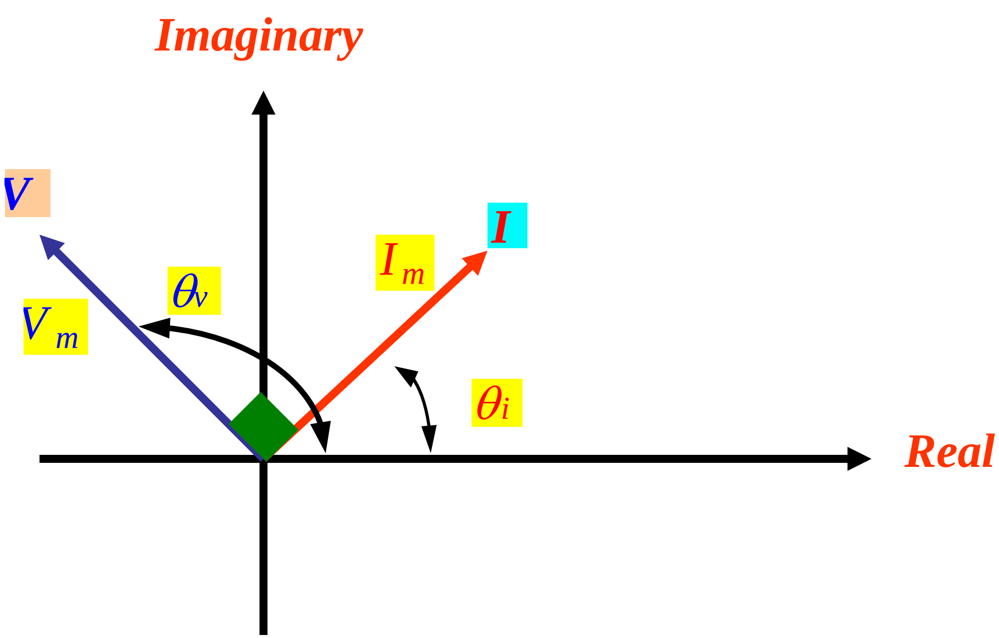
$$\Rightarrow \quad V_m = \omega L I_m \quad \text{and} \quad \theta_v = \theta_i + 90^o$$



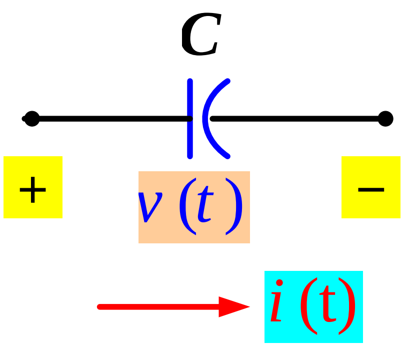
$$V = j\omega L I$$

$V_m = \omega L I_m$ and $\theta_v = \theta_i + 90^o$

The voltage **lead** the current by **90º** or the current **lagging** the voltage by **90º**



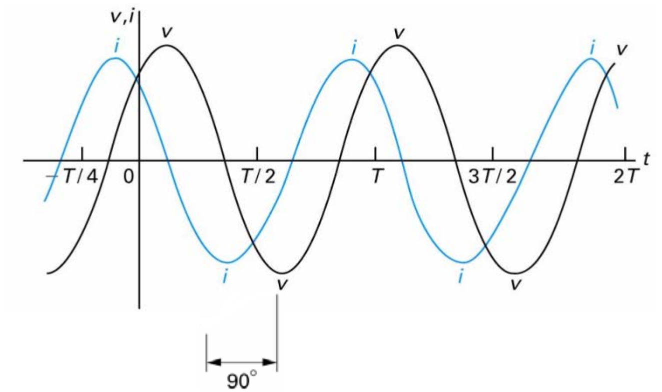
The V–I Relationship for a Capacitor



$$i(t) = C\frac{dv(t)}{dt}$$

Let the voltage across the capacitor be a **sinusoidal** given as $v(t) = V_m \cos(\omega t + \theta_v)$

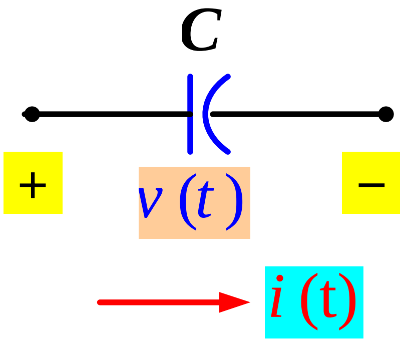
$$i(t) = C\frac{dv(t)}{dt} = -\omega C V_m \sin(\omega t + \theta_v)$$



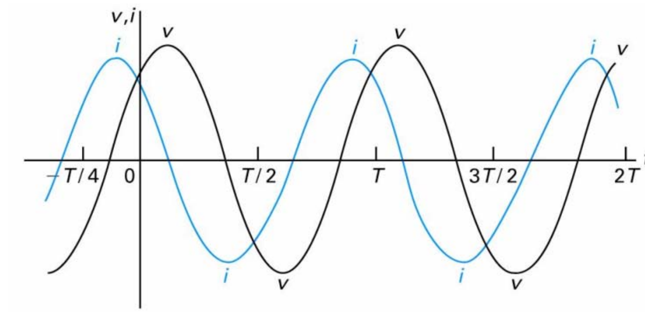
The sinusoidal voltage and current in an **inductor** are **out of phase** by **90º**

The voltage **lag** the current by **90º** or the current **leading** the voltage by **90º**

The V–I Relationship for a Capacitor



$$v(t) = V_m \cos(\omega t + \theta_v) \qquad i(t) = -\omega C V_m \sin(\omega t + \theta_v)$$



The pharos representation or transform of the **voltage** and **current**

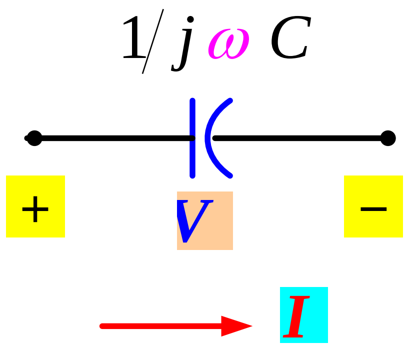
$$v(t) = V_m \cos(\omega t + \theta_v) \Rightarrow \mathbf{V} = V_m e^{j\theta_v} = V_m \angle \theta_v$$

$$i(t) = -\omega C V_m \sin(\omega t + \theta_v) = -\omega C V_m \cos(\omega t + \theta_v - 90^o) \Rightarrow \mathbf{I} = -\omega L V_m e^{j(\theta_v - 90^o)}$$

$$\Rightarrow \mathbf{I} = -\omega C V_m e^{j\theta_v} \underbrace{e^{j\,-90^o}}_{-j} = j\omega C V_m e^{j\theta_v} = j\omega C\mathbf{V}$$

$$\Rightarrow \mathbf{V} = \frac{\mathbf{I}}{j\omega C} = \frac{I_m e^{j\theta_i}}{\underbrace{1e^{j90^o}}_{j}\omega C} = \frac{I_m e^{j(\theta_i - 90^o)}}{\omega C} = \underbrace{\frac{I_m}{\omega C}}_{V_m} \angle \underbrace{(\theta_i - 90^o)}_{\theta_v}$$

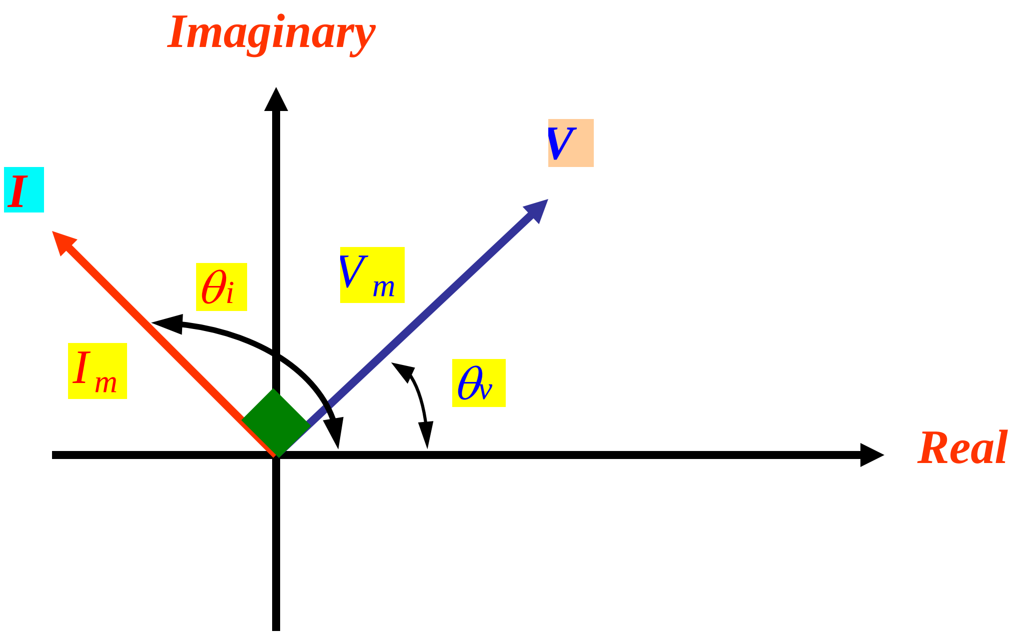
$$\Rightarrow V_m = \frac{I_m}{\omega C} \quad \text{and} \quad \theta_v = \theta_i - 90^o$$



$$V = \frac{I}{j\omega C}$$

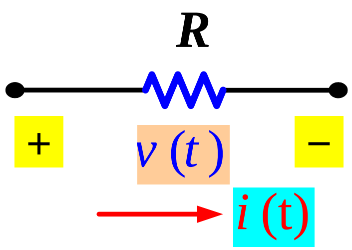
$$V_m = \frac{I_m}{\omega C} \quad \text{and} \quad \theta_v = \theta_i - 90^o$$

The voltage **lag** the current by **90º** or the current **lead** the voltage by **90º**



Time-Domain

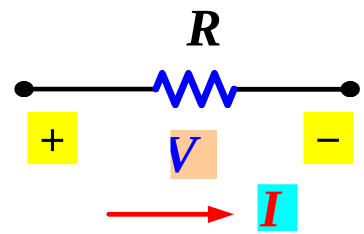
Phasor (Complex or Frequency) Domain



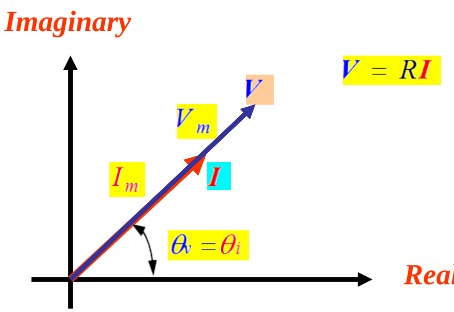
$v(t) = R\,i(t)$

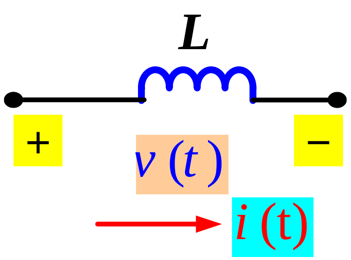
$i(t) = I_m \cos(\omega t + \theta_i)$

$v(t) = R I_m [\cos(\omega t + \theta_i)]$



$\mathbf{V} = R\mathbf{I}$

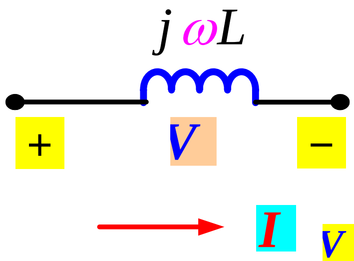




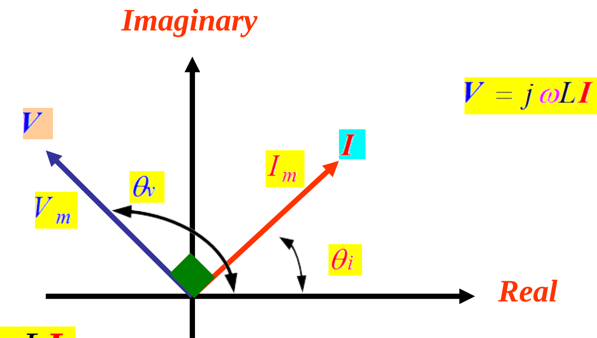
$i(t) = I_m \cos(\omega t + \theta_i)$

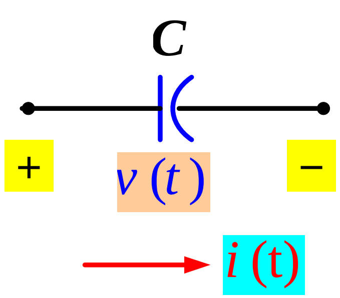
$v(t) = L \dfrac{di(t)}{dt}$

$v(t) = -\omega L I_m \sin(\omega t + \theta_i)$

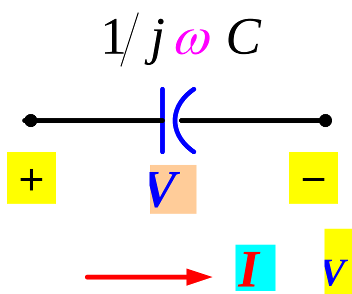


$\mathbf{V} = j\omega L\mathbf{I}$

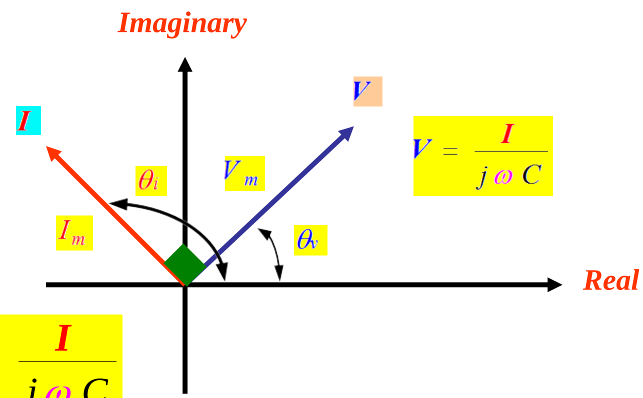




$i(t) = C \dfrac{dv(t)}{dt}$



$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$



$v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = -\omega C V_m \sin(\omega t + \theta_v)$

Impedance and Reactance

The relation between the **voltage** and **current** on the phasor domain (complex or frequency) for the three elements R, L, and C we have

$$V = RI \qquad V = j\omega LI \qquad V = \frac{I}{j\omega C} = \frac{1}{j\omega C}I$$

When we compare the relation between the **voltage** and **current** , we note that they are all of form:

$V = ZI$ Which the state that the phasor voltage is some complex constant (Z) times the phasor current

This resemble (شبه) Ohm's law were the complex constant (Z) is called "**Impedance**" (أعاقه)

Recall on Ohm's law previously defined , the proportionality content R was real and called "**Resistant**" (مقاومه)

Solving for (Z) we have $Z = \frac{V}{I}$

The **Impedance** of a resistor is $Z_R = R$

The **Impedance** of an indictor is $Z_L = j\omega L$

The **Impedance** of a capacitor is $Z_C = \frac{1}{j\omega C}$

In all cases the impedance is measured in Ohm's Ω

$V = RI$ $V = j\omega L I$ $V = \frac{1}{j\omega C} I$

Impedance $Z = \frac{V}{I}$

The **Impedance** of a resistor is $Z_R = R$

The **Impedance** of an indictor is $Z_L = j\omega L$

The **Impedance** of a capacitor is $Z_C = \frac{1}{j\omega C}$

In all cases the impedance is measured in Ohm's Ω

The imaginary part of the impedance is called "**reactance**"

The **reactance** of a resistor is $X_R = 0$

The **reactance** of an inductor is $X_L = \omega L$

The **reactance** of a capacitor is $X_C = \frac{-1}{\omega C}$

We note the "**reactance**" is associated with energy storage elements like the **inductor** and **capacitor**

Note that the impedance in general (***exception is the resistor***) is a function of frequency

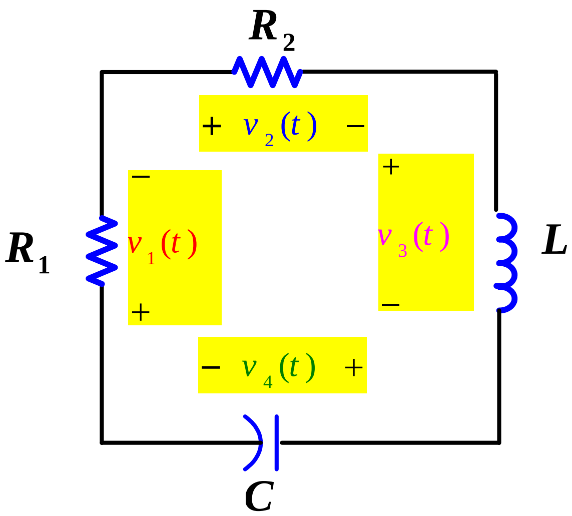
At $\omega = 0$ (DC), we have the following

$Z_L = j\omega L = j(0) L = \mathbf{0}$ → **short**

$Z_C = \frac{1}{j\omega C} = \frac{1}{j(0) C} = \infty$ → **open**

9.5 Kirchhoff's Laws in the Frequency Domain (Phasor or Complex Domain)

Consider the following circuit



Phasor Transformation

$$v_1(t) = V_1\cos(\omega t + \theta_1) \;\Rightarrow\; \boldsymbol{V}_1 = V_1 e^{j\theta_1}$$

$$v_2(t) = V_2\cos(\omega t + \theta_2) \;\Rightarrow\; \boldsymbol{V}_2 = V_2 e^{j\theta_2}$$

$$v_3(t) = V_3\cos(\omega t + \theta_3) \;\Rightarrow\; \boldsymbol{V}_3 = V_3 e^{j\theta_3}$$

$$v_4(t) = V_4\cos(\omega t + \theta_4) \;\Rightarrow\; \boldsymbol{V}_4 = V_4 e^{j\theta_4}$$

KVL $\Rightarrow$ $v_1(t) + v_2(t) + v_3(t) + v_4(t) = 0$

$$\Rightarrow V_1\cos(\omega t + \theta_1) + V_2\cos(\omega t + \theta_2) + V_3\cos(\omega t + \theta_3) + V_4\cos(\omega t + \theta_4) = 0$$

Using Euler Identity we have $\Re\{V_1 e^{j\theta_1} e^{j\omega t}\} + \Re\{V_2 e^{j\theta_2} e^{j\omega t}\} + \Re\{V_3 e^{j\theta_3} e^{j\omega t}\} + \Re\{V_4 e^{j\theta_4} e^{j\omega t}\} = 0$

Which can be written as $\Re\{V_1 e^{j\theta_1} e^{j\omega t} + V_2 e^{j\theta_2} e^{j\omega t} + V_3 e^{j\theta_3} e^{j\omega t} + V_4 e^{j\theta_4} e^{j\omega t}\} = 0$

Factoring $e^{j\omega t}$ $\Rightarrow$ $\Re\{(\underbrace{V_1 e^{j\theta_1}}_{\boldsymbol{V}_1} + \underbrace{V_2 e^{j\theta_2}}_{\boldsymbol{V}_2} + \underbrace{V_3 e^{j\theta_3}}_{\boldsymbol{V}_3} + \underbrace{V_4 e^{j\theta_4}}_{\boldsymbol{V}_4}) e^{j\omega t}\} = 0$ $\Rightarrow$ $\boldsymbol{V}_1 + \boldsymbol{V}_2 + \boldsymbol{V}_3 + \boldsymbol{V}_4 = 0$

Phasor

Can not be zero

KVL on the phasor domain

So in general $\boldsymbol{V}_1 + \boldsymbol{V}_2 + \cdots + \boldsymbol{V}_n = 0$

Kirchhoff's Current Law

A similar derivation applies to a set of **sinusoidal** current summing at a node

$$i_1(t) = I_1 \cos(\omega t + \theta_1) \quad i_2(t) = I_2 \cos(\omega t + \theta_2) \quad \cdots \quad i_n(t) = I_n \cos(\omega t + \theta_n)$$

Phaser Transformation

$$\boldsymbol{I}_1 = I_1 e^{j\theta_1} \qquad \boldsymbol{I}_2 = I_2 e^{j\theta_2} \qquad \boldsymbol{I}_n = I_n e^{j\theta_n}$$

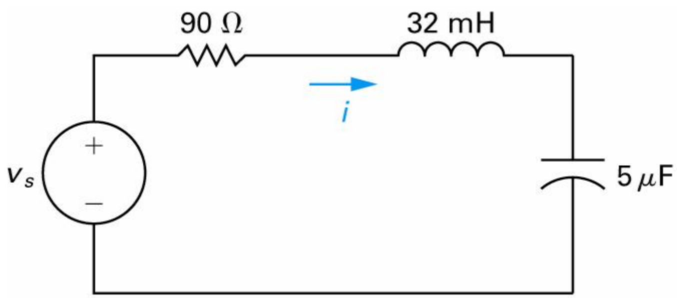
KCL → $i_1(t) + i_2(t) + \cdots + i_n(t) = 0$

→ $\boldsymbol{I}_1 + \boldsymbol{I}_2 + \cdots + \boldsymbol{I}_\text{n} = 0$

KCL on the phasor domain

9.6 Series, Parallel, and Delta-to Wye Simplifications

Example 9.6 for the circuit shown below the source voltage is sinusoidal



$$v_s(t) = 750\cos(5000t + 30^o)$$

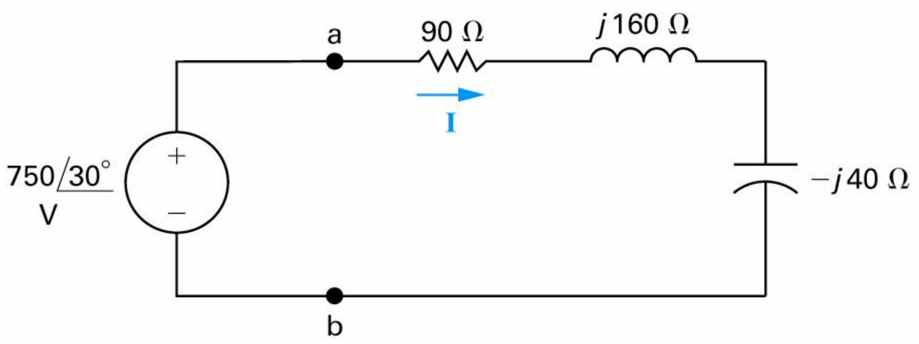
(a) Construct the frequency-domain (phasor, complex) equivalent circuit ?

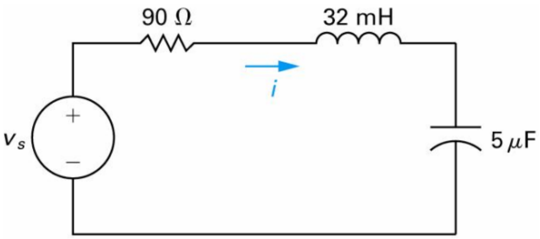
(b) Calculte the steady state current *i*(t) ?

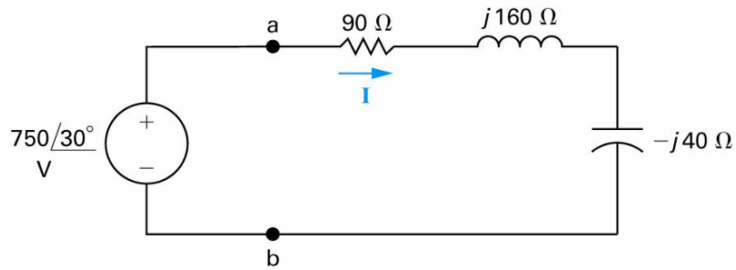
The source voltage pahsor transformation or equivalent ➡ $V_s = 750\,e^{j\,30^o} = 750\angle 30^o$

The **Impedance** of the indictor is $Z_L = j\,\omega L \; = j\,(5000)(32 \text{ X } 10^{-3}) = j\,160\ \Omega$

The **Impedance** of the capacitor is $Z_C = \dfrac{1}{j\,\omega\, C} = \dfrac{1}{j\,(5000)\,(5 \text{ X } 10^{-6})} = -j\,40\ \Omega$







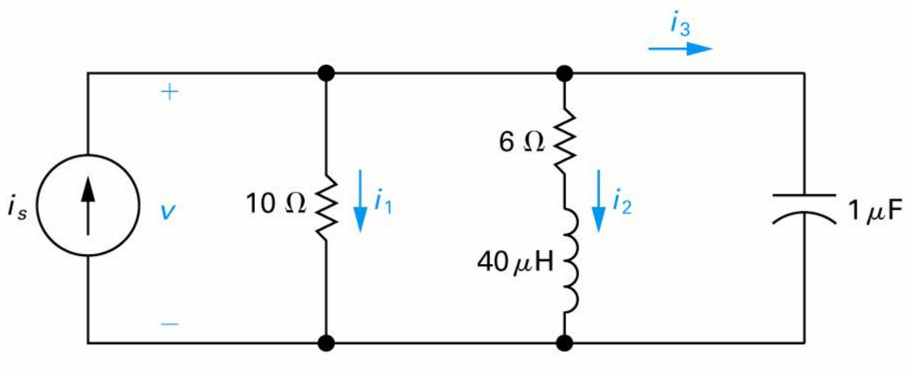
$$v_s(t) = 750\cos(5000t + 30^o)$$

To Calculate the phasor current ***I***

$$I = \frac{\mathbf{V}_s}{Z_{ab}} = \frac{750e^{j30^o}}{90 + j160 - j40} = \frac{750e^{j30^o}}{90 + j120} = \frac{750\angle 30^o}{150\angle 53.13^o} = 5\angle -23.13^o \text{ A}$$

$$i(t) = 5\cos(5000t - 23.13^o) \text{ A}$$

Example 9.7 Combining Impedances in series and in Parallel

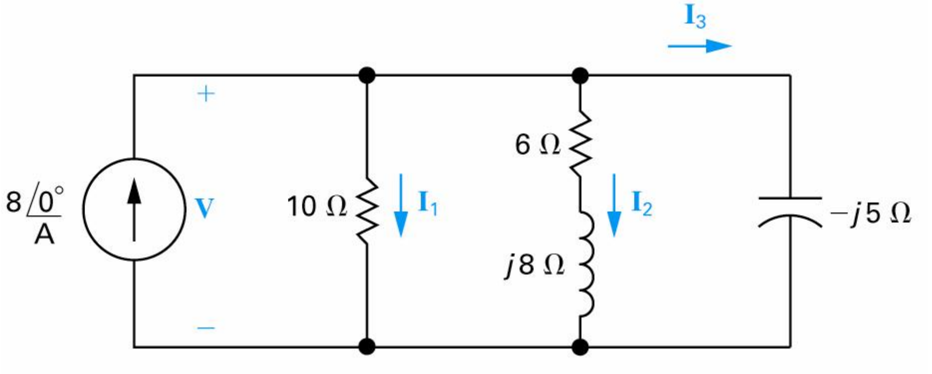


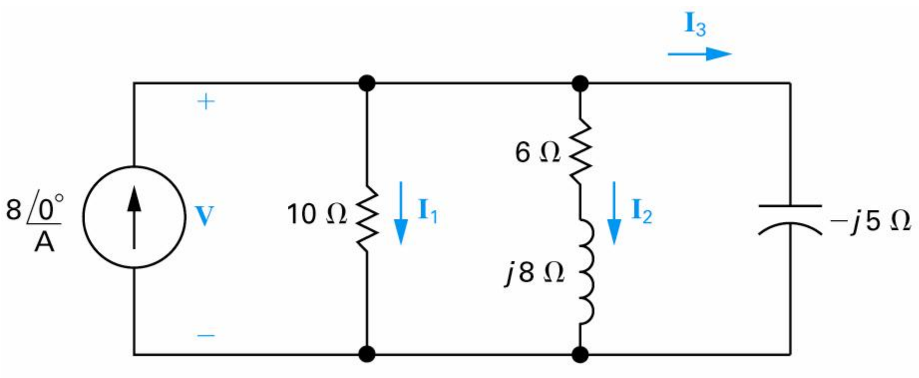
$$i_s(t) = 8\cos(200{,}000t) \quad \text{A}$$

(a) Construct the frequency-domain (phasor, complex) equivalent circuit ?

(b) Find the steady state expressions for v, i_1, i_2, and i_3 ? ?

(a)





$$Y_1 = \frac{1}{10} = 0.1 \text{ S}$$

$$Y_2 = \frac{1}{6+j8} = \frac{6-j8}{100} = 0.06 - j0.08 \text{ S}$$

$$Y_3 = \frac{1}{-j5} = j0.2 \text{ S}$$

The admittance of the three branches is $Y = Y_1 + Y_2 + Y_3 = 0.16 + j0.12 = 0.2 \angle 36.87° \text{ S}$

$$Z = \frac{1}{Y} = 5 \angle -36.87° \ \Omega \quad \mathbf{V} = Z\mathbf{I} = 40 \angle -36.87° \text{ V} \Rightarrow v = 40\cos(200{,}000t - 36.87°) \text{ V}$$

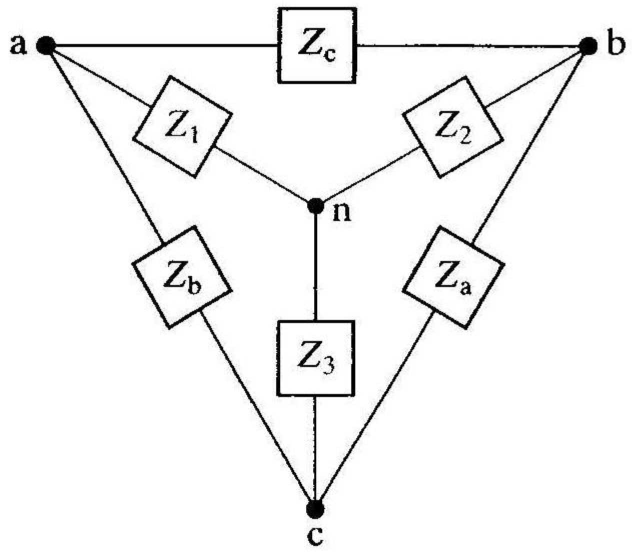
$$\mathbf{I}_1 = \frac{40 \angle -36.87°}{10} = 4 \angle -36.87° = 3.2 - j2.4 \text{ A} \Rightarrow i_1 = 4\cos(200{,}000t - 36.87°) \text{ A}$$

$$\mathbf{I}_2 = \frac{40 \angle -36.87°}{6+j8} = 4 \angle -90° = -j4 \text{ A} \Rightarrow i_2 = 4\cos(200{,}000t - 90°) \text{ A}$$

$$\mathbf{I}_3 = \frac{40 \angle -36.87°}{5 \angle -90°} = 8 \angle 53.13° = 4.8 + j6.4 \text{ A} \Rightarrow i_3 = 8\cos(200{,}000t + 53.13°) \text{ A}$$

We check the computations $\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 3.2 - j2.4 - j4 + 4.8 + j6.4 \boxed{= 8 + j0} = \mathbf{I}$

Delta-to Wye Transformations



Δ to Y

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

Y to Δ

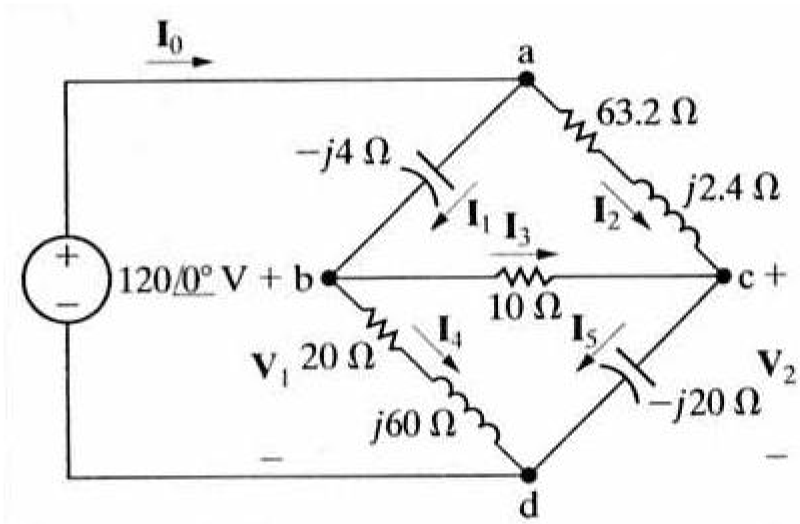
$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

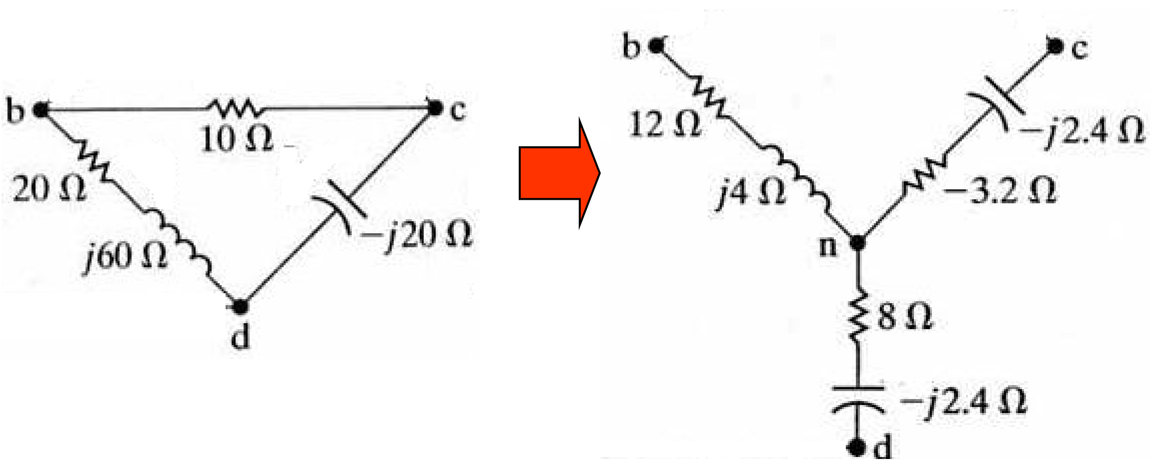
$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

Example 9.8

Use a Δ-to-Y impedance transformation to find $\mathbf{I}_0$, $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, $\mathbf{I}_4$, $\mathbf{I}_5$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the circuit



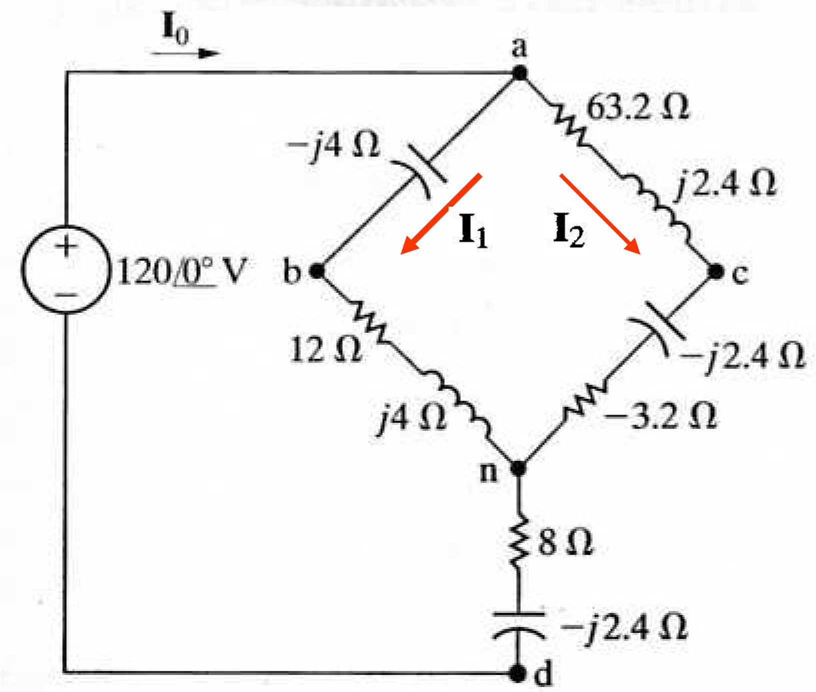


$$Z_1 = \frac{(20 + j60)(10)}{30 + j40} = 12 + j4\ \Omega$$

$$Z_2 = \frac{10(-j20)}{30 + j40} = -3.2 - j2.4\ \Omega$$

$$Z_3 = \frac{(20 + j60)(-j20)}{30 + j40} = 8 - j24\ \Omega$$

$\mathbf{I}_0 = \ = 4 \angle 53.13^\circ \ = 2.4 + j3.2 \text{ A}$



$$\mathbf{V}_{nd} = (8 - j24)\mathbf{I}_0 = 96 - j32 \text{ V}.$$

$$\mathbf{V} = \mathbf{V}_{an} + \mathbf{V}_{nd}$$

$$\mathbf{V}_{an} = 120 - 96 + j32 = 24 + j32 \text{ V}.$$

$$\mathbf{I}_{abn} = \frac{24 + j32}{12} = 2 + j\frac{8}{3} \text{ A},$$

$$\mathbf{I}_{acn} = \frac{24 + j32}{60} = \frac{4}{10} + j\frac{8}{15} \text{ A}$$

the branch currents

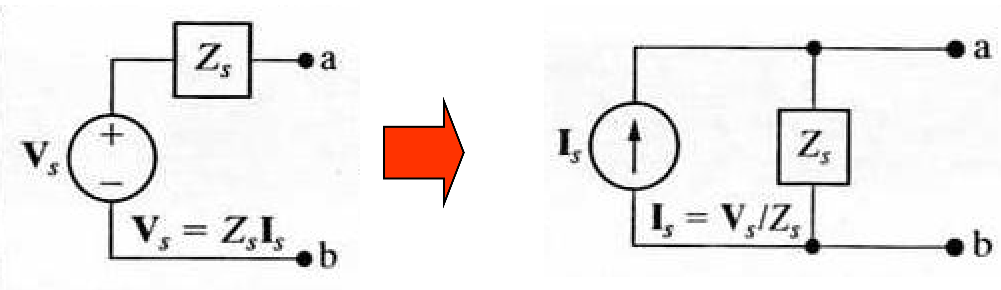
$$\mathbf{I}_1 = \mathbf{I}_{abn} = 2 + j\frac{8}{3} \text{ A}$$

$$\mathbf{I}_2 = \mathbf{I}_{acn} = \frac{4}{10} + j\frac{8}{15} \text{ A}$$

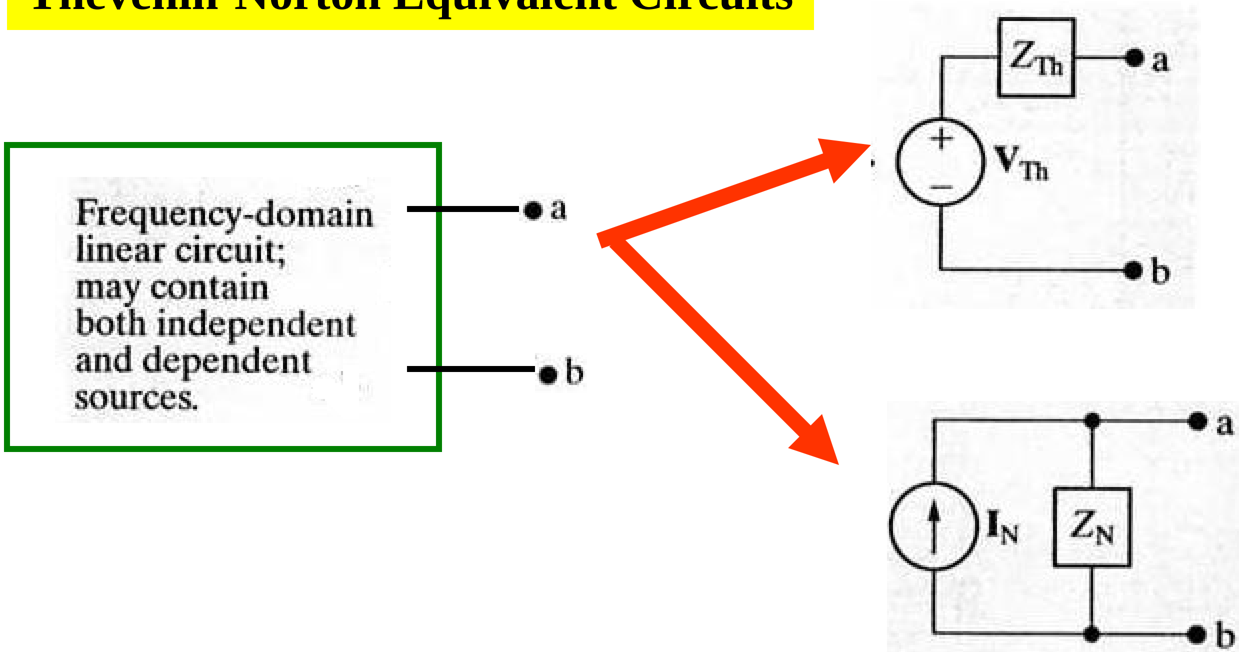
check the calculations $\quad \mathbf{I}_1 + \mathbf{I}_2 = 2.4 + j3.2 = \mathbf{I}_0$

9.7 Source Transformations and Thevenin-Norton Equivalent Circuits

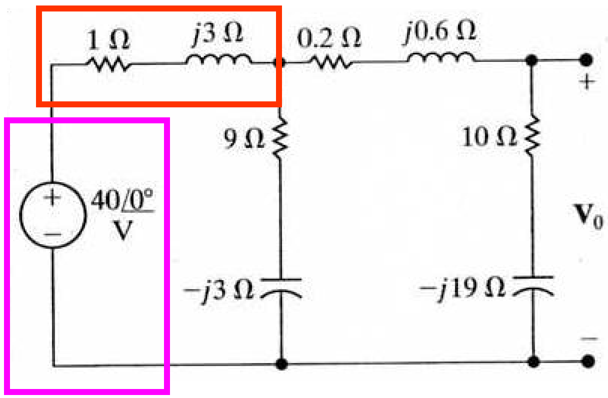
Source Transformations

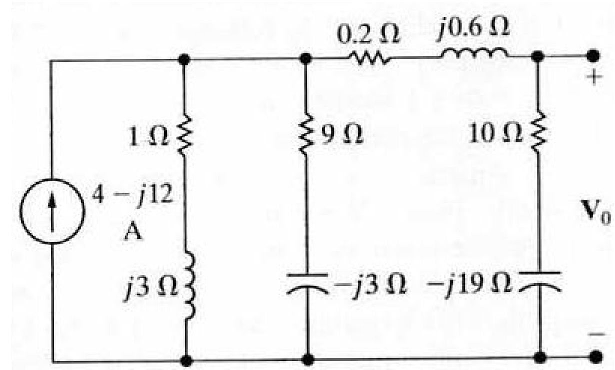


Thevenin-Norton Equivalent Circuits

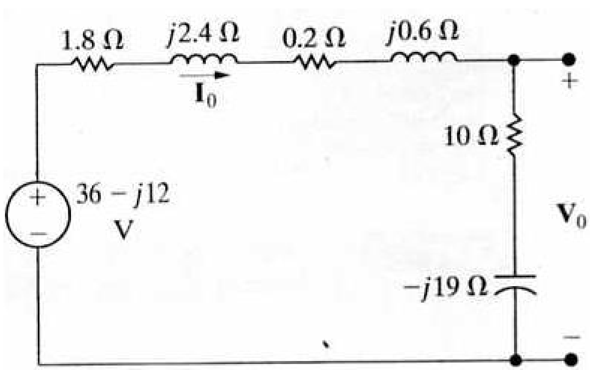


Example 9.9 Use the concept of source transformation to find the phasor voltage V_0 in the circuit shown





$$Z = \frac{(1 + j3)(9 - j3)}{10} = 1.8 + j2.4\ \Omega,$$

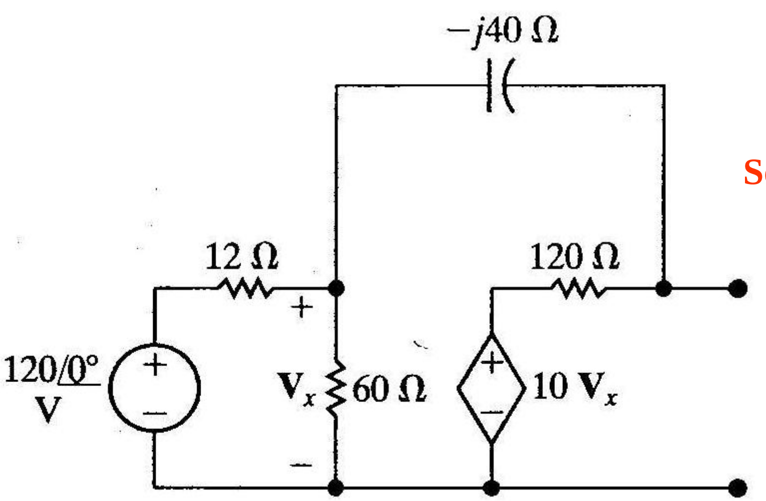


$$\mathbf{I}_0 = \frac{36 - j12}{12 - j16} = \frac{12(3 - j1)}{4(3 - j4)}$$

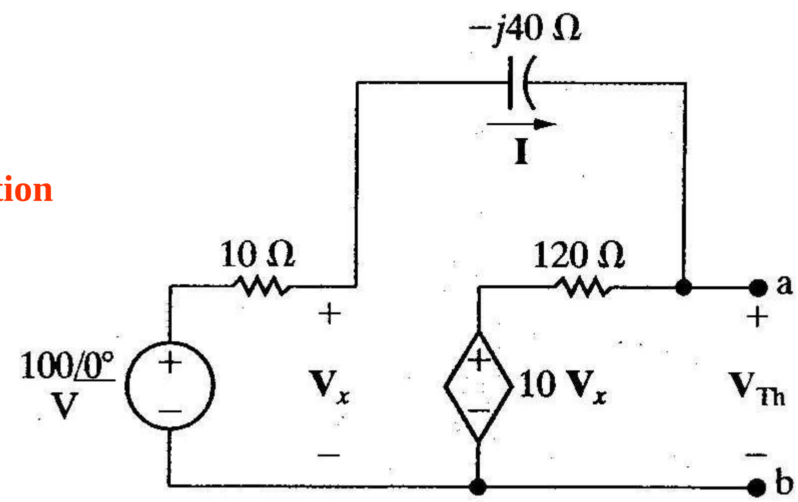
$$= \frac{39 + j27}{25} = 1.56 + j1.08\ \text{A}.$$

$$\mathbf{V}_0 = (1.56 + j1.08)(10 - j19) = 36.12 - j18.84\ \text{V}$$

Example 9.10 Find the Thévenin equivalent circuit with respect to terminals a,b for the circuit shown



Source Transformation



$$100 = 10\mathbf{I} - j40\mathbf{I} + 120\mathbf{I} + 10\mathbf{V}_x \quad = (130 - j40)\mathbf{I} + 10\mathbf{V}_x$$

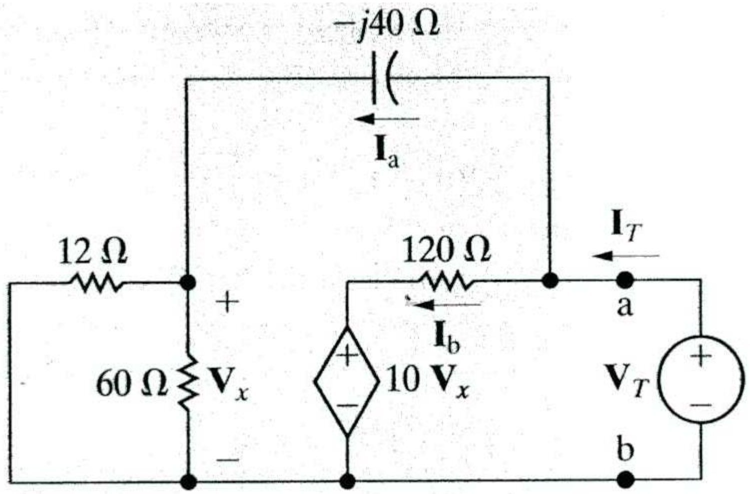
Since $\mathbf{V}_x = 100 - 10\mathbf{I}$ then $\mathbf{I} = 18\,\angle{-126.87^\circ}$ A

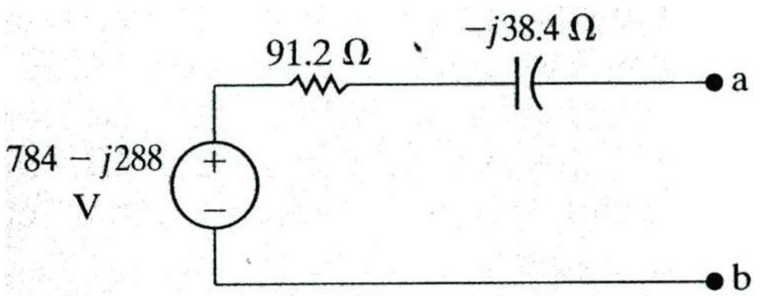
$$\mathbf{V}_x = 100 - 180\,\angle{-126.87^\circ} = 208 + j144 \text{ V}$$

$$\mathbf{V}_{Th} = 10\mathbf{V}_x + 120\mathbf{I} \quad \boxed{= 835.22\,\angle{-20.17^\circ}\ \text{V}}$$

Next we find the Thevenin Impedance

Thevenin Impedance





$$Z_{Th} = \frac{\mathbf{V}_T}{\mathbf{I}_T}$$

Find $\mathbf{I}_T$ interms of $\mathbf{V}_T$ then form the ratio $\frac{\mathbf{V}_T}{\mathbf{I}_T}$

$$\mathbf{I}_T = \mathbf{I}_a + \mathbf{I}_b$$

Find $\mathbf{I}_a$ and $\mathbf{I}_b$ interms of $\mathbf{V}_T$

$$\mathbf{I}_a = \frac{\mathbf{V}_T}{10 - j40} \qquad \mathbf{V}_x = 10\mathbf{I}_a \qquad \mathbf{I}_b = \frac{\mathbf{V}_T - 10\mathbf{V}_x}{120} = \frac{-\mathbf{V}_T(9 + j4)}{120(1 - j4)}$$

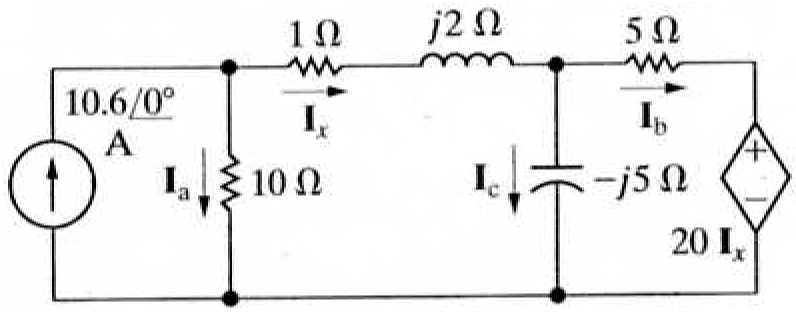
$12\Omega \| 60\Omega$

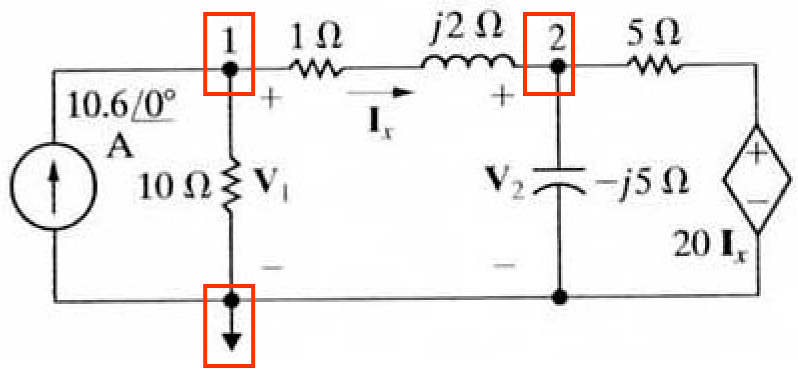
$$\mathbf{I}_T = \mathbf{I}_a + \mathbf{I}_b = \frac{\mathbf{V}_T}{10 - j40}\left(1 - \frac{9 + j4}{12}\right) = \frac{\mathbf{V}_T(3 - j4)}{12(10 - j40)}$$

$$Z_{Th} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 91.2 - j38.4\ \Omega$$

9.8 The Node-Voltage Method

Example 9.11 Use the node-voltage method to find the branch currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ in the circuit shown





KCL at node 1

$$-10.6 + \frac{\mathbf{V}_1}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j2} = 0$$

$$\mathbf{V}_1(1.1 + j0.2) - \mathbf{V}_2 = 10.6 + j21.2 \qquad (1)$$

KCL at node 2

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{1 + j2} + \frac{\mathbf{V}_2}{-j5} + \frac{\mathbf{V}_2 - 20\mathbf{I}_x}{5} = 0$$

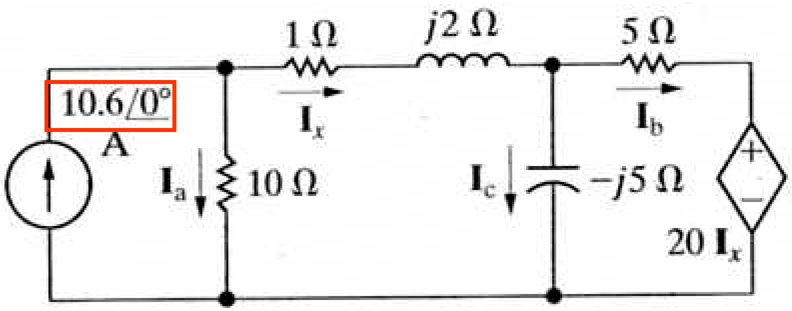
Since $\mathbf{I}_x = \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j2}$

$$-5\mathbf{V}_1 + (4.8 + j0.6)\mathbf{V}_2 = 0 \qquad (2)$$

Two Equations and Two Unknown , solving

$$\mathbf{V}_1 = 68.40 - j16.80 \text{ V}$$

$$\mathbf{V}_2 = 68 - j26 \text{ V}$$



$$\mathbf{V}_1 = 68.40 - j16.80 \text{ V}$$

$$\mathbf{V}_2 = 68 - j26 \text{ V}$$

$$\mathbf{I}_a = \frac{\mathbf{V}_1}{10} = 6.84 - j1.68 \text{ A}$$

$$\mathbf{I}_x = \frac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j2} = 3.76 + j1.68 \text{ A}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_2 - 20\mathbf{I}_x}{5} = -1.44 - j11.92 \text{ A}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_2}{-j5} = 6.84 - j1.68 + 3.76 + j1.68$$

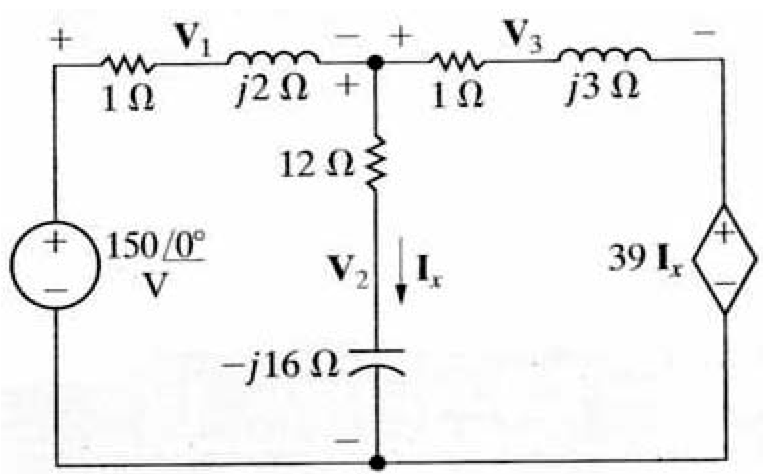
To Check the work

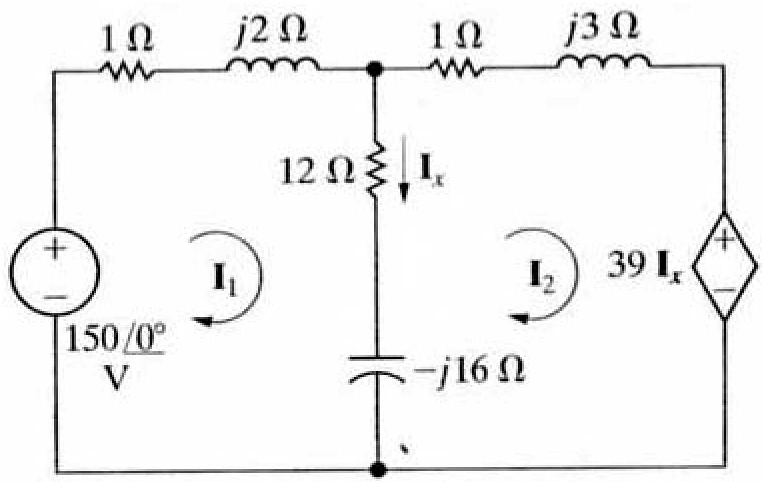
$$\mathbf{I}_a + \mathbf{I}_x = 6.84 - j1.68 + 3.76 + j1.68 = 10.6 \text{ A}$$

$$\mathbf{I}_x = \mathbf{I}_b + \mathbf{I}_c = -1.44 - j11.92 + 5.2 + j13.6 = 3.76 + j1.68 \text{ A}$$

9.9 The Mesh-Current Method

Example 9.12 Use the mesh-current method to find the voltages $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the circuit shown





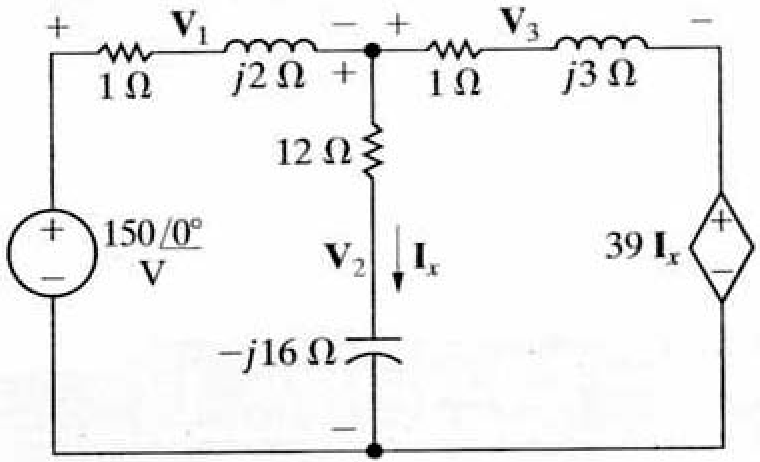
KVL at mesh 1 $150 = (1 + j2)\mathbf{I}_1 + (12 - j16)(\mathbf{I}_1 - \mathbf{I}_2)$ → $150 = (13 - j14)\mathbf{I}_1 - (12 - j16)\mathbf{I}_2$ **(1)**

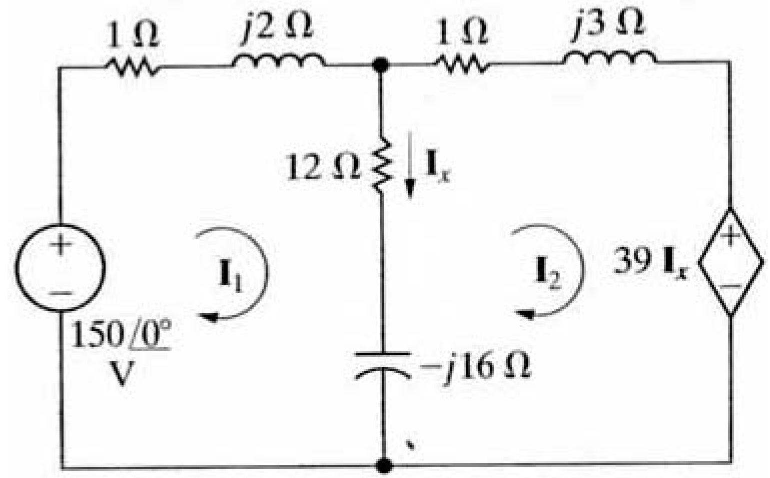
KVL at mesh 2 $0 = (12 - j16)(\mathbf{I}_2 - \mathbf{I}_1) + (1 + j3)\mathbf{I}_2 + 39\mathbf{I}_x$ **Since** $\mathbf{I}_x = \mathbf{I}_1 - \mathbf{I}_2$

→ $0 = (27 + j16)\mathbf{I}_1 - (26 + j13)\mathbf{I}_2$ **(2)**

Two Equations and Two Unknown , solving

$\mathbf{I}_1 = -26 - j52$ A $\quad \mathbf{I}_2 = -24 - j58$ A $\quad \mathbf{I}_x = \mathbf{I}_1 - \mathbf{I}_2 = -2 + j6$ A





$\mathbf{I}_1 = -26 - j52$ A $\quad \mathbf{I}_2 = -24 - j58$ A $\quad \mathbf{I}_x = -2 + j6$ A

$$\mathbf{V}_1 = (1 + j2)\mathbf{I}_1 = 78 - j104 \text{ V}$$

$$\mathbf{V}_2 = (12 - j16)\mathbf{I}_x = 72 + j104 \text{ V}$$

$$\mathbf{V}_3 = (1 + j3)\mathbf{I}_2 = 150 - j130 \text{ V}$$

9.12 The Phasor Diagram

the phasor quantities $10\angle 30^\circ$, $12\angle 150^\circ$, $5\angle -45^\circ$, and $8\angle -170^\circ$