

EE 207-03 – Fall 2010
Quiz 5

Section	SER	ID	NAME	KEY
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Q1 Let the Input / output of Linear Time Invariant (LTI) system is shown below :



$$\frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}$$

$$y(0^-) = 0 \quad x(0^-) = 2$$

If $x(t) = 5u(t)$, find $y(t)$?

Q2 Let the signal $x(t) = 5 + \cos(20\pi t)$, be ideally sampled at 30 samples / sec (30 Hz)

(a) What is the Nyquist rate ?

(b) Is there Aliasing, explain ?

(c) plot the spectrum of the sampled signal $X_s(f)$ for $|f| \leq 45$ HZ

$$\cos(2\pi f_0 t) \Leftrightarrow \frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)]$$

$$\underline{\text{Q1}} \quad \frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}$$

Taking Laplace of both side and since all initial conditions for $y(t)$ is zero, we have

$$s^2 Y(s) + 3sY(s) + 2Y(s) = sX(s) - x(0)$$

since $x(0) = 2$ $x(t) = 5u(t) \leftrightarrow X(s) = \frac{5}{s}$

$$\Rightarrow Y(s) = \frac{s\left(\frac{5}{s}\right) - 2}{s^2 + 3s + 2}$$

$$= \frac{3}{(s+2)(s+1)}$$

using partial fraction,

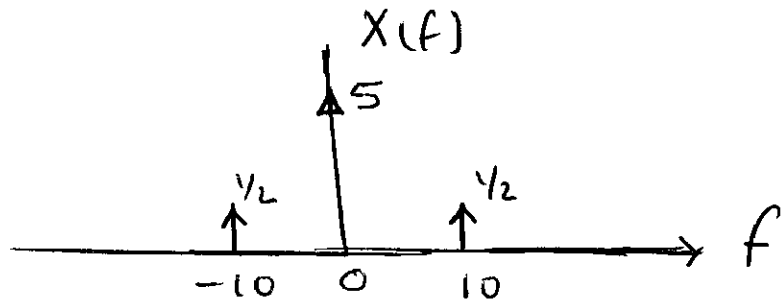
$$Y(s) = -\frac{3}{(s+2)} + \frac{3}{(s+1)}$$

$$\Rightarrow y(t) = 3(e^{-t} - e^{-2t})u(t)$$

Q2

$$x(t) = 5 + \cos(20t)$$

$$\Rightarrow X(f) = 5\delta(f) + \frac{1}{2}\delta(f-10) + \frac{1}{2}\delta(f+10)$$



- (a) Maximum frequency in $x(t) = 10 \text{ Hz}$
 \Rightarrow Nyquist rate $= 2(10) = 20 \text{ Hz}$

- (b) NO Aliasing
Since $f_s = 30 >$ Nyquist rate $= 20$

