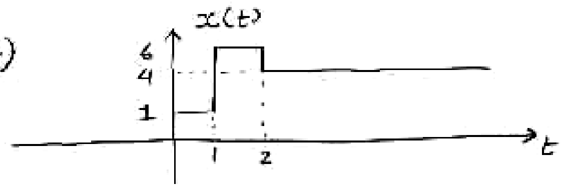
1.38

a)



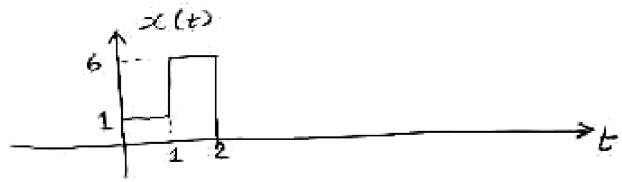
The area under $x^2(t)$ from $-\infty$ to $+\infty$ is infinite.

$\Rightarrow E = \infty$

$$P = \lim_{T \to \infty} \frac{1 + 36 + 16 \cdot \infty}{2T} = \frac{16 \cdot \infty}{2 \cdot \infty} = 8\ W$$

(Power signal).

b)



Area under $x^2(t) = 1 \times 1 + 36 \times 1 = 37$

$E = 37\ J$ (Energy signal).

c) $E = \lim_{T \to \infty} \int_0^T e^{-10t} \, dt = \left. \frac{e^{-10t}}{-10} \right|_0^\infty = \frac{1}{10} \text{ J}$

(Energy signal).

d) $E = \lim_{T \to \infty} \int_0^T (e^{-5t} + 1)^2 \, dt = \lim_{T \to \infty} \left. \left(\frac{e^{-10t}}{-10} + \frac{2e^{-5t}}{-5} + t \right) \right|_0^T$

$= \infty$

$P = \lim_{T \to \infty} \frac{1}{2T} \left[\frac{e^{-10t}}{-10} + \frac{2e^{-5t}}{-5} + t \right]_0^T$

$= \frac{1}{2} \text{ W}$

(Power signal).

e) Power signal. Jusification, similar to d). $P = \frac{1}{2} \text{ W}$

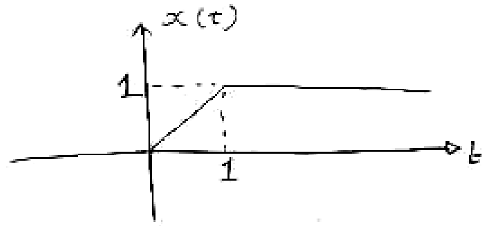
f) $E = \infty$ (by inspection).

$P = \lim_{T \to \infty} \frac{1}{2T} \int_0^T t \, dt = \lim_{T \to \infty} \frac{1}{2T} \left(\left. \frac{t^2}{2} \right|_0^T \right)$

$= \lim_{T \to \infty} \frac{T}{4} = \infty$

(neither).

g)

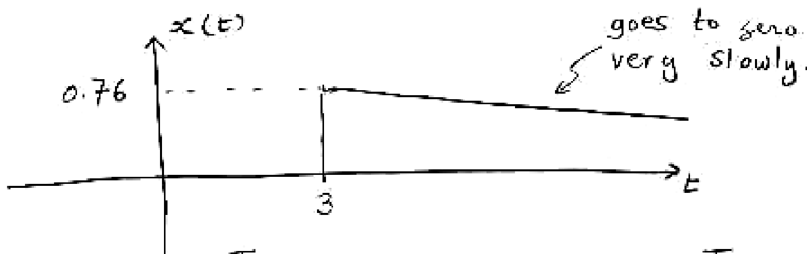


$E = \infty$ by inpection.

$$P = \lim_{T \to \infty} \frac{1}{2T} \left[\int_0^1 t^2 dt + \int_1^T 1^2 dt \right] = \frac{1}{2} W$$

(Power signal).

h)



$$E = \lim_{T \to \infty} \int_3^T t^{-1/2} dt = \lim_{T \to \infty} \left. \frac{t^{1/2}}{1/2} \right|_3^T = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \frac{(T^{1/2} - 3^{1/2})}{1/2} = 0$$

(neither)

2-1

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 5 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

2-2

a) First order b) Differentiate once ⇒ First Order

c) Zeroth order d) Differentiate once ⇒ second order

e) Second order.

2-3

a), b), c) and e) represent fixed systems.

d) does not represent a fixed system, because of the presence of a time-varying coefficient.

2-4

c) and e) are nonlinear. The remaining systems are linear.

For system c) ⇒

$$4(\alpha_1 y_1) + 10\alpha_1 = \frac{d(\alpha_1 x_1)}{dt} + 5(\alpha_1 x_1)$$

$$+$$

$$4(\alpha_2 y_2) + 10\alpha_2 = \frac{d(\alpha_2 x_2)}{dt} + 5(\alpha_2 x_2)$$

$$4\left[\alpha_1 y_1 + \alpha_2 y_2\right] + 10\left[\alpha_1 + \alpha_2\right] = \frac{d}{dt}\left[\alpha_1 x_1 + \alpha_2 x_2\right] + 5\left[\alpha_1 x_1 + \alpha_2 x_2\right]$$

which is <u>not</u> the same as the original relation.

It can be shown that a similar procedure applied to system e) does not result in the original relation.

For system a) $\Rightarrow$

$$2\frac{d(\alpha_1 y_1)}{dt} + 3(\alpha_1 y_1) = \frac{d^2(\alpha_1 x_1)}{dt^2} + \alpha_1 x_1$$

$+$

$$2\frac{d(\alpha_2 y_2)}{dt} + 3(\alpha_2 y_2) = \frac{d^2(\alpha_2 x_2)}{dt^2} + \alpha_2 x_2$$

$$2\frac{d}{dt}\left[\alpha_1 y_1 + \alpha_2 y_2\right] + 3\left[\alpha_1 y_1 + \alpha_2 y_2\right]$$

$$= \frac{d^2\left[\alpha_1 x_1 + \alpha_2 x_2\right]}{dt^2} + \left[\alpha_1 x_1 + \alpha_2 x_2\right]$$

Which is the same as the original relation.

2 - 6

a) $y(t) = 10\,x(t+2) + 5$ ①

For the input x_1, the output is y_1
For " " x_2, " " " y_2

$\therefore\ y_1(t) = 10\,x_1(t+2) + 5$ ②

$y_2(t) = 10\,x_2(t+2) + 5$ ③

Adding $y_1(t) + y_2(t) = 10\left[x_1(t+2) + x_2(t+2)\right] + 10$

$\Rightarrow y(t) = 10\,x(t) + 10$ ④

Where $y(t)$ is the combined output $= y_1 + y_2$
& $x(t)$ " " " input $= x_1 + x_2$

Since eqn. ④ is <u>different</u> from the original relationship [eqn. ①] $\Rightarrow$ Relationship is **nonlinear**

b) $y(t) = 10\,x(t+2) + 5$ represents a non-causal system. To see this

let $t = 0 \Rightarrow$

$y(0) = 10\,x(2) + 5$, which means the output y at $t = 0$ depends on the input x at $t = 2$ (y depends on **future** values of x)