

8-19

$$a) X(z) = \frac{1}{1 - 0.2z^{-1}}$$

$$b) X(z) = \frac{1}{1 + 0.2z^{-1}}$$

$$c) x(nT) = u(n) + \left(\frac{3}{4}\right)^n u(n-4) \\ = u(n) + \left(\frac{3}{4}\right)^4 \left(\frac{3}{4}\right)^{n-4} u(n-4)$$

$$X(z) = \frac{1}{1 - z^{-1}} + \left(\frac{3}{4}\right)^4 \frac{z^{-4}}{1 - \frac{3}{4}z^{-1}}$$

$$d) X(z) = \frac{2}{1 - z^{-1}} - \frac{2z^{-8}}{1 - z^{-1}}$$

8-29

a)

$$\begin{array}{r}
 1 - 0.3z^{-1} + 0.3z^{-2} - 0.5z^{-3} \quad \left| \begin{array}{l} 1 + 0.3z^{-1} - 0.21z^{-2} + 0.347z^{-3} + \dots \\ 1 \\ \hline 0.3z^{-1} - 0.3z^{-2} + 0.5z^{-3} \\ \hline 0.3z^{-1} - 0.09z^{-2} + 0.09z^{-3} - 0.15z^{-4} \\ \hline -0.21z^{-2} + 0.41z^{-3} + 0.15z^{-4} \\ \hline -0.21z^{-2} + 0.063z^{-3} - 0.063z^{-4} + 0.105z^{-5} \\ \hline 0.347z^{-3} + 0.213z^{-4} - 0.105z^{-5} \end{array} \right.
 \end{array}$$

$$x(0) = 1, \quad x(T) = 0.3, \quad x(2T) = -0.21, \quad x(3T) = 0.347$$

$$b) X(z) = \frac{1 - 0.7z^{-1}}{1 + z^{-1} + 0.25z^{-2}}$$

$$\begin{array}{r}
 1 + z^{-1} + 0.25z^{-2} \quad \left| \begin{array}{l} 1 - 1.7z^{-1} + 1.45z^{-2} - 1.025z^{-3} + \dots \\ 1 - 0.7z^{-1} \\ \hline -1.7z^{-1} - 0.25z^{-2} \\ \hline -1.7z^{-1} - 1.7z^{-2} - 0.425z^{-3} \\ \hline 1.45z^{-2} + 0.425z^{-3} \\ \hline 1.45z^{-2} + 1.45z^{-3} + 0.3625z^{-4} \\ \hline -1.025z^{-3} - 0.3625z^{-4} \end{array} \right.
 \end{array}$$

$$x(0) = 1, \quad x(T) = -1.7, \quad x(2T) = 1.45, \quad x(3T) = -1.025$$

$$c) X(z) = \frac{z^{-1} + 0.4z^{-2} + 0.04z^{-3}}{1 - 0.8z^{-2} + 0.16z^{-4}}$$

$$\begin{array}{r}
 z^{-1} + 0.4z^{-2} + 0.84z^{-3} + 0.32z^{-4} + \dots \\
 \hline
 1 - 0.8z^{-2} + 0.16z^{-4} \quad \left| \begin{array}{l} z^{-1} + 0.4z^{-2} + 0.04z^{-3} \\ z^{-1} - 0.8z^{-3} + 0.16z^{-5} \end{array} \right. \\
 \hline
 0.4z^{-2} + 0.84z^{-3} - 0.16z^{-5} \\
 \hline
 0.4z^{-2} - 0.32z^{-4} + 0.064z^{-6} \\
 \hline
 0.84z^{-3} + 0.32z^{-4} - 0.16z^{-5} - 0.064z^{-6} \\
 \hline
 0.84z^{-3} - 0.672z^{-5} + 0.1344z^{-7} \\
 \hline
 0.32z^{-4} + 0.512z^{-5} - 0.064z^{-6} - 0.1344z^{-7}
 \end{array}$$

$$x(0) = 0, x(T) = 1, x(2T) = 0.4, x(3T) = 0.84, x(4T) = 0.32.$$

8-33

$$a) X(z) = \frac{z^3}{(z-1)(z+0.5)(z-0.2)}$$

$$\frac{X(z)}{z} = \frac{z^2}{(z-1)(z+0.5)(z-0.2)} = \frac{A_1}{z-1} + \frac{A_2}{z+0.5} + \frac{A_3}{z-0.2}$$

$$= \frac{0.833}{z-1} + \frac{0.238}{z+0.5} - \frac{0.167}{z-0.2}$$

$$X(z) = \frac{0.833}{1-z^{-1}} + \frac{0.238}{1+0.5z^{-1}} - \frac{0.167}{1-0.2z^{-1}}$$

$$x(nT) = 0.833u(n) + 0.238(-0.5)^n u(n) - 0.167(0.2)^n u(n)$$

$$b) X(z) = \frac{z^3 - 0.5z^2}{(z-1)(z-0.5)(z-0.2)}$$

We can repeat the previous procedure to $X(z)$
or we can use the result obtained in part
a) along with delay operations.

$$X(z) = (1 - 0.5z^{-1}) X_a(z)$$

$$\therefore x(nT) = 0.833 \left[u(n) - 0.5u(n-1) \right] + 0.238 \left[(-0.5)^n u(n) - 0.5(-0.5)^{n-1} u(n-1) \right] - 0.167 \left[(0.2)^n u(n) - (0.2)^{n-1} u(n-1) \right]$$

8-49

$$y(nT) = h(nT) * x(nT)$$

$$Y(z) = H(z) X(z)$$

$$H(z) = 2 \left[z^{-4} + z^{-5} + z^{-6} + z^{-7} + z^{-8} \right]$$

$$X(z) = 2 \left[z^{-2} + z^{-3} + z^{-4} + z^{-5} \right]$$

$$Y(z) = 4 \left[z^{-6} + 2z^{-7} + 3z^{-8} + 4z^{-9} + 4z^{-10} + 3z^{-11} + 2z^{-12} + z^{-13} \right]$$

$$\therefore x(6T) = 4, x(7T) = 8, x(8T) = 12, x(9T) = 16, \dots$$