2.29

a) $a(t) = \int_{-\infty}^{t} h(\lambda) d\lambda$

$$= \int_{-\infty}^{t} \left[\delta(\lambda) - \frac{R}{L} e^{-(R/L)\lambda} u(\lambda) \right] d\lambda$$

$\therefore$ For $t < 0 \implies a(t) = 0$

For $t > 0 \implies a(t) = 1 - \frac{R}{L} \int_{0}^{t} e^{-(R/L)\lambda} d\lambda$

$$= 1 - \frac{R}{L} \left. \frac{e^{-(R/L)\lambda}}{-R/L} \right|_{0}^{t}$$

$$= 1 + \left(e^{-\frac{R}{L}t} - 1 \right)$$

$$= e^{-\frac{R}{L}t}$$

$\therefore$ $a(t) = e^{-\frac{R}{L}t} u(t)$, for all t.

b) $y_R(t) = \int_{-\infty}^{t} a(\lambda) d\lambda = \int_{-\infty}^{t} e^{-\frac{R}{L}\lambda} u(\lambda) d\lambda$

$\therefore$ $y_R(t) = 0$ for $t < 0$

And for $t > 0 \implies \int_{0}^{t} e^{-\frac{R}{L}\lambda} d\lambda = -\frac{L}{R} \left(e^{-\frac{R}{L}t} - 1 \right)$

$\therefore$ $y_R(t) = -\frac{L}{R} \left(e^{-\frac{R}{L}t} - 1 \right) u(t)$ for all t.

3-2

$$I_2 = \int_{T_0} \cos m\omega_0 t \cos n\omega_0 t \, dt = \int_{T_0} \left(\frac{e^{jm\omega_0 t} + e^{-jm\omega_0 t}}{2}\right)\left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}\right) dt$$

$$= \frac{1}{4} \int_{T_0} \left(e^{j(m+n)\omega_0 t} + e^{j(n-m)\omega_0 t} + e^{j(m-n)\omega_0 t} + e^{-j(m+n)\omega_0 t}\right) dt$$

Using $\displaystyle\int_{T_0} e^{jk\omega_0 t} dt = \begin{cases} T_0 & , k = 0 \\ 0 & , k \neq 0 \end{cases}$

$$\therefore I_2 \overset{m \neq n}{=} \frac{1}{4}(0 + 0 + 0 + 0) = 0$$

$$I_2 \overset{m = n \neq 0}{=} \frac{1}{4}(0 + T_0 + T_0 + 0) = \frac{T_0}{2}$$

$$\therefore I_2 = \begin{cases} 0 & , m \neq n \\ \dfrac{T_0}{2} & , m = n \neq 0 \end{cases}$$

$$I_3 = \int_{T_0} \sin m\omega_0 t \cos n\omega_0 t \, dt = 0 \quad , \text{all } m, n$$

can be shown to be valid using the same method used in evaluating I_2, above.

(3-2 continued)

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

Multiply both sides by $\sin m\omega_0 t$ and integrate over a period $\Rightarrow$

$$\int_{T_0} x(t) \sin m\omega_0 t \, dt = \int_{T_0} \left[a_0 \sin m\omega_0 t + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t \sin m\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \sin m\omega_0 t \right] dt$$

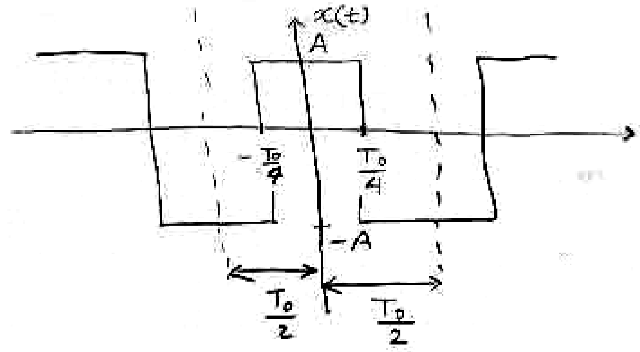
$$= a_0 \int_{T_0} \sin m\omega_0 t \, dt + \sum_{n=1}^{\infty} a_n \int_{T_0} \cos n\omega_0 t \sin m\omega_0 t \, dt + \sum_{n=1}^{\infty} b_n \int_{T_0} \sin n\omega_0 t \sin m\omega_0 t \, dt$$

$$= 0 + \sum_{n=1}^{\infty} a_n (0) + \sum_{n=1}^{\infty} b_n \int_{T_0} \sin n\omega_0 t \sin m\omega_0 t \, dt$$

$= b_m \frac{T_0}{2}$ (only one term of the infinite sum does not equal zero, which corresponds to $n = m$).

$$\therefore b_m = \frac{2}{T_0} \int_{T_0} x(t) \sin m\omega_0 t \, dt \Rightarrow b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t \, dt$$

3.4



$a_0 = 0$

$b_n = 0$ ($x(t)$ is an even function) for all n.

$$a_n = \frac{2}{T_0}\left[\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} A\cos n\omega_0 t\,dt - \int_{\frac{T_0}{4}}^{\frac{3T_0}{4}} A\cos n\omega_0 t\,dt\right]$$

$$= \frac{2A}{T_0 n\omega_0}\left[\sin n\omega_0 \Big|_{-\frac{T_0}{4}}^{\frac{T_0}{4}} - \sin n\omega_0 \Big|_{\frac{T_0}{4}}^{\frac{3T_0}{4}}\right]$$

$$= \frac{2A}{n2\pi}\left[\sin n\frac{\pi}{2} - \sin\left(-\frac{n\pi}{2}\right) - \sin\frac{3n\pi}{2} + \sin\frac{n\pi}{2}\right]$$

$$= \frac{A}{n\pi}\left[3\sin\frac{n\pi}{2} - \sin\frac{3n\pi}{2}\right]$$

$$a_n = \begin{cases} 0 & , n = 2, 4, 6, \cdots \\ \frac{4A}{n\pi} & , n = 1, 5, 9, \cdots \\ -\frac{4A}{n\pi} & , n = 3, 7, 11, \cdots \end{cases}$$