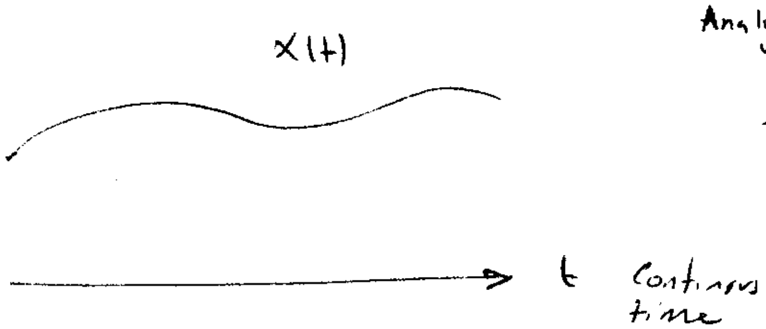
ch8

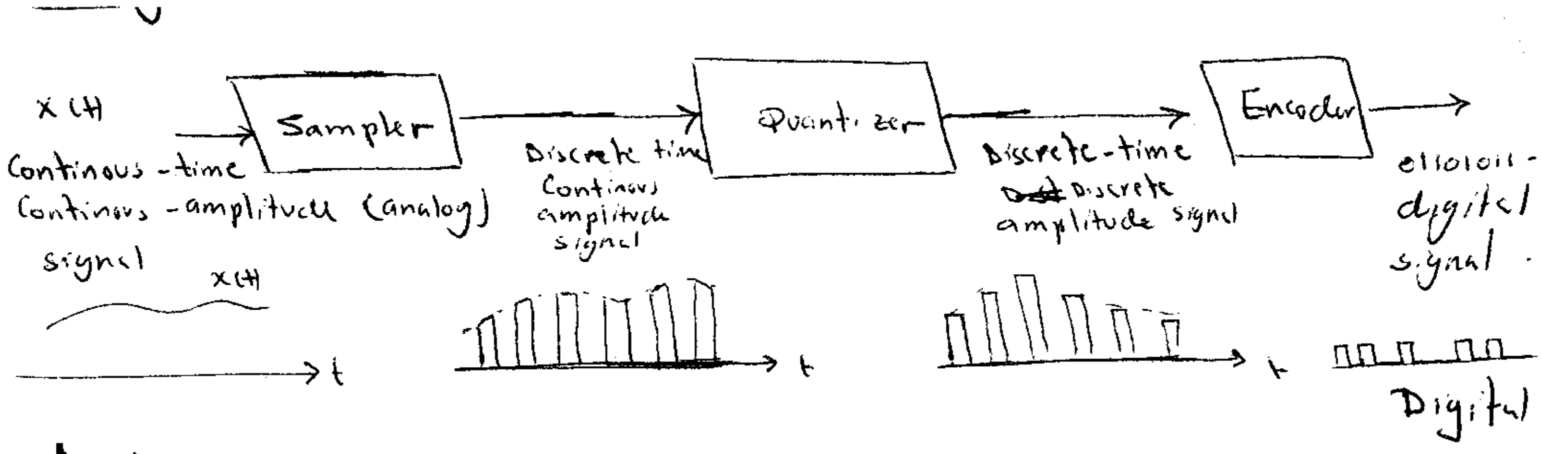
In previous chapters, we dealt with ~~Cont~~ Continuous-tim ~~singals~~ signals



Analyze → F.T
→ F.S (if periodical
→ L.T

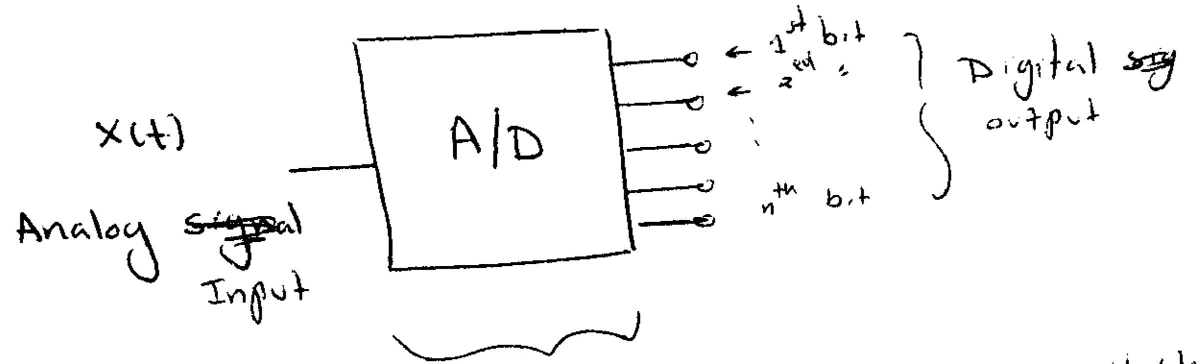
However with the advance of digital Computer and the Cost reduction in digital circuit, it becomes necessary to serach for a way to process the signal using digital Computer However digital Computer only accept 0 or 1, that mean the signal level is two states only 0 or 1 Therefore we Can not apply the signal x(t) (whic assumes infinte uncountable values at all time) direc to the digital circuit.

We ~~x~~ thus have to do pre processing on x(t) before we ~~cat~~ appley it to the digital Circuit or Computer as follows:



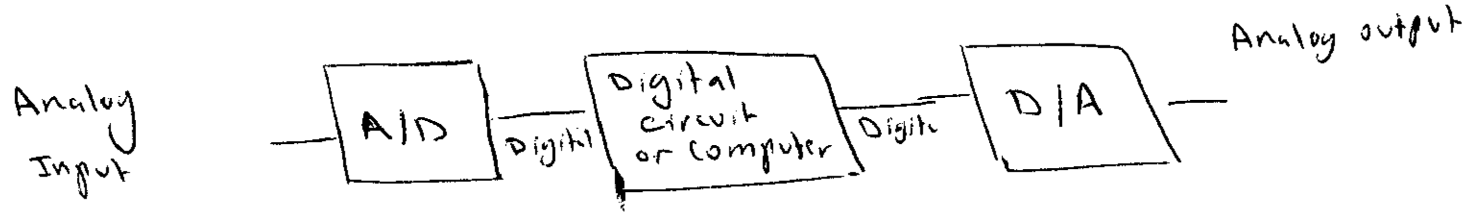
Analog

The total block's shown above ~~is~~ called "~~An~~ "Analog-to-Digital" Converter and for short A/D



This chip is commercially available

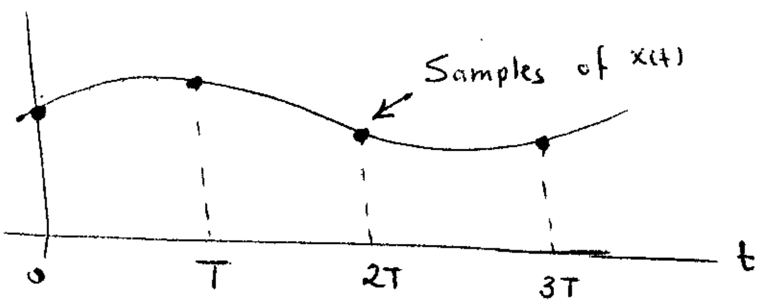
many Cap stone project use this chip

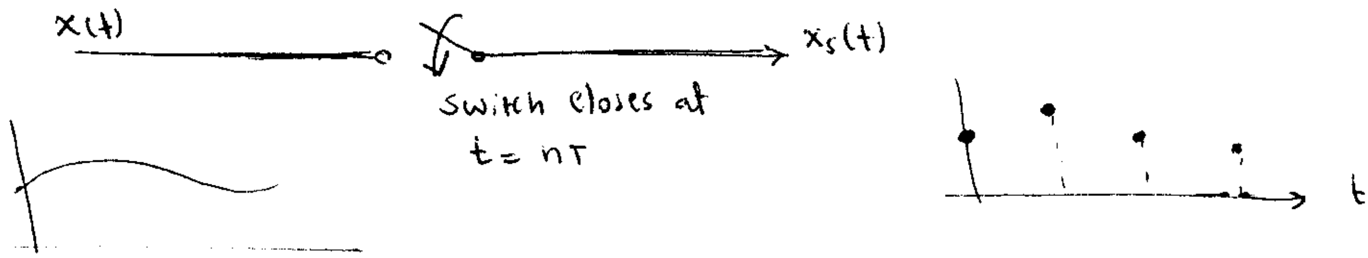


<u>where</u> D/A is the digital to analog Converter which takes digital output and Convert it to analog or Continous signals.

We will discuss each part separtly as shown next

Sampling: To sample a Continous - time signal $x(t)$ is to represent $x(t)$ at a discrete number of points, $t=nT$ where T is the sampling period



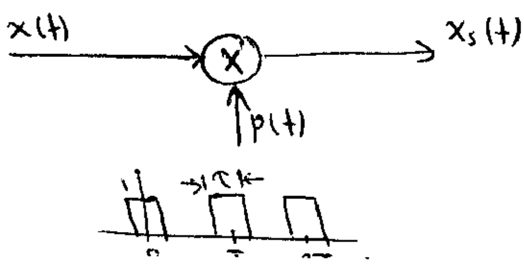


For the sampling process to be useful, we must be able to show that it is possible to ~~sample~~ reconstruct the signal $x(t)$ from its samples $x_s(t)$. This can be ~~done~~ ~~in~~ shown in the frequency domain as follows:

$$x_s(t) = x(t)\,p(t)$$

where $p(t)$ model the switch open-close operation

The sampling function $p(t)$ is assumed to be periodic pulse train of period T



Since $p(t)$ is a periodical pulses, then we can represen ~~as~~ by Fourier serier,

$$p(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_s t} \qquad f_s = \frac{1}{T} \quad \text{sampling } f$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-j2\pi n f_s t}\, dt$$

We then can write ~~the sampling~~ $x_s(t)$ as,

$$x_s(t) = \sum_{n=-\infty}^{\infty} C_n\, x(t)\, e^{j2\pi n f_s t}$$

<u>Note</u>: $x_s(t)$ is not a periodic function.

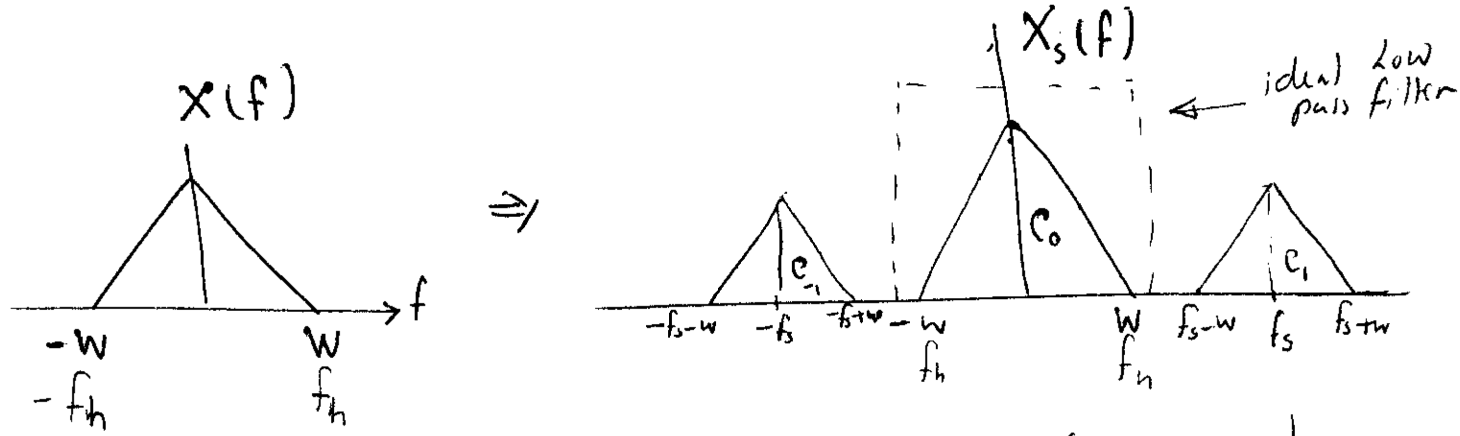
The F.T of $x_s(t)$,

$$X_s(f) = \int_{-\infty}^{\infty} x_s(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_n\, x(t)\, e^{j2\pi n f_s t}\, e^{-j2\pi f t}\, dt$$

$$= \cancel{\sum_{n=-\infty}^{\infty} C_n \int_{-\infty}^{\infty} x(t)\, e^{j2\pi(f-f_s)}}$$

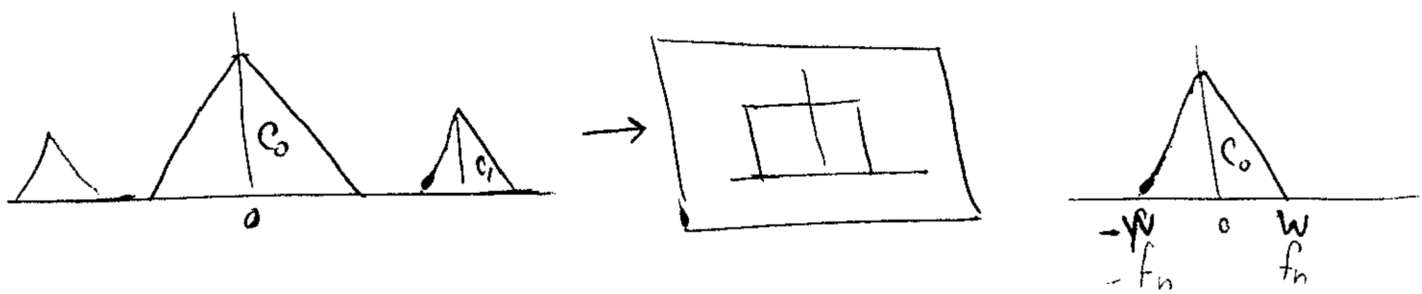
$$= \sum_{n=-\infty}^{\infty} C_n \underbrace{\int_{-\infty}^{\infty} x(t)\, e^{-j2\pi(f-nf_s)t}\, dt}_{X(f-nf_s)} = \sum_{n=-\infty}^{\infty} C_n X(f-$$

~~Therefore~~ Therefore the spectrum of the sampled signal ~~$x_s(t)$~~ is $X_s(f)$ is a shifted version of the ~~original sig~~ spectrum of the original signal $X(f)$ weighted by the coefficients C_n.



Q: how can we obtain $X(f)$ (i.e $x(t)$) from $X_s(f)$ (i.e $x_s(t)$)

A: If we pass $X_s(f)$ through ideal Low pass filter ~~centered~~ we obtain the following



we can normalize the output by dividing by C_0 to get the exact input.

From the spectrum $X_s(f)$ we observe that ~~following~~

$X(f)$ is assumed zero for $|f| \geq W$ also ~~we as~~

$$f_s - \cancel{W} f_h \geq \cancel{W} f_h$$

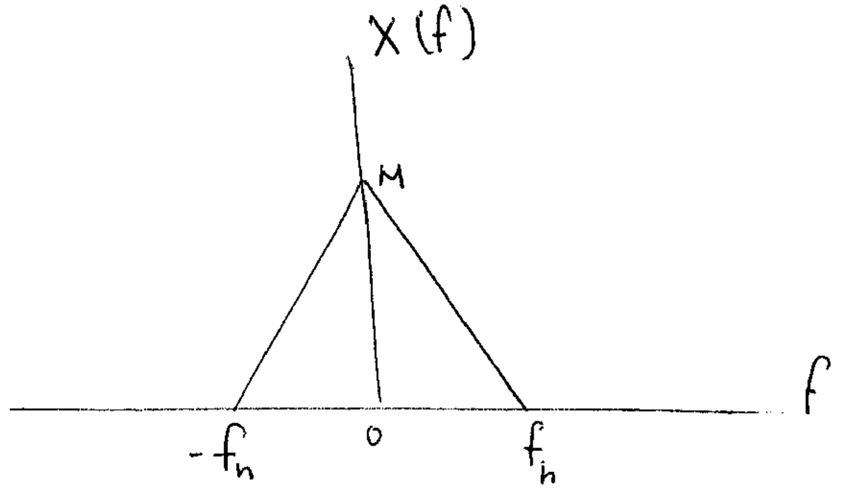
or

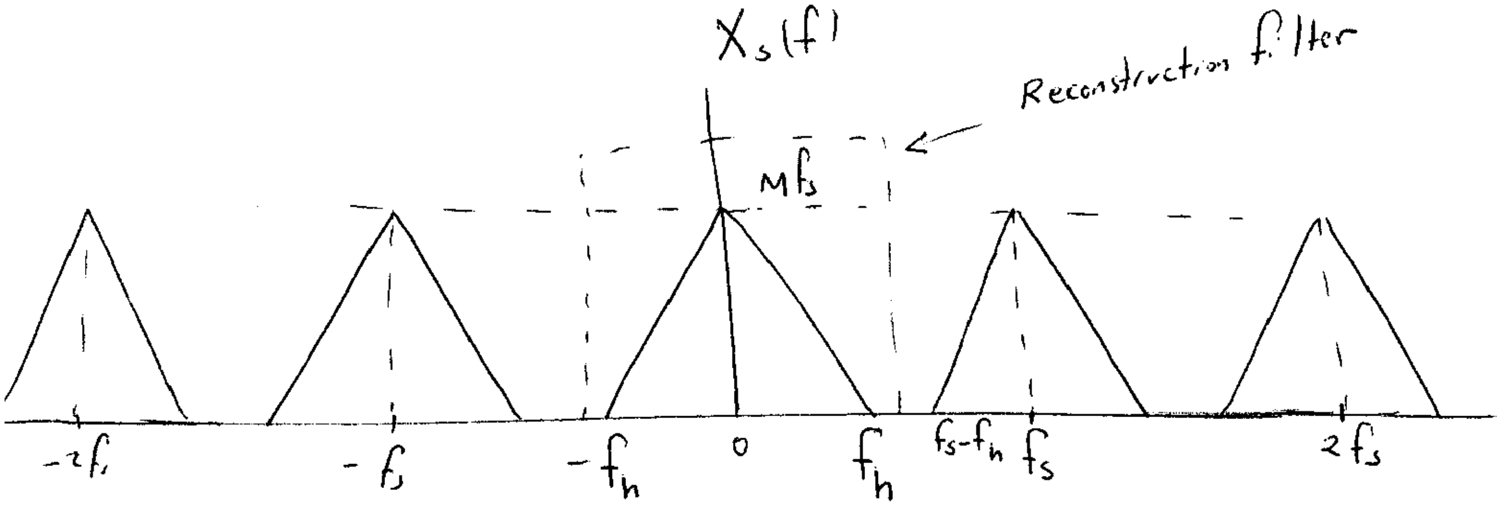
$$f_s \geq 2\cancel{W} f_h$$

Thus the minimum sampling frequency is $2\cancel{W} f_h$ where W is the maximum frequency of $X(f)$.

Sampling Thm A bandlimited signal $x(t)$, having no freq. Component above f_h hertz, is completely specified by samples that are taken at a uniform rate greater than $2f_h$. In other words the time between samples is no greater than $\frac{1}{2f_h}$ seconds.

The frequency $2f_h$ is known as the Nyquist rate.

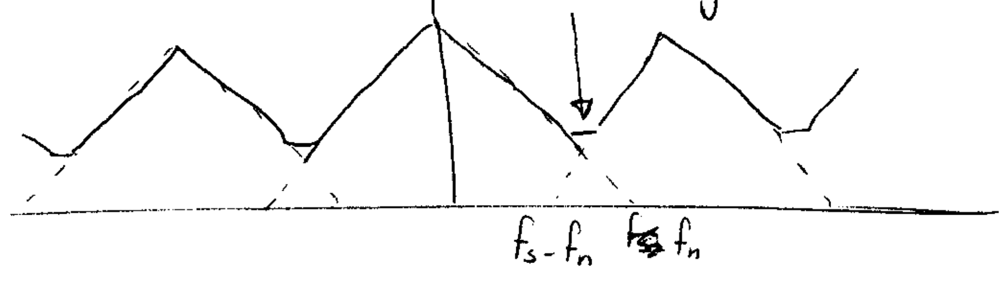




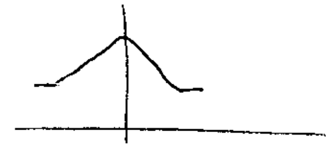
If the sampling frequency f_s is less than the Nyquist rate

$\Rightarrow \quad f_s - f_h < f_n$

we will have ~~ove~~ overlapping of adjecent spectrum "Aliasing effect"

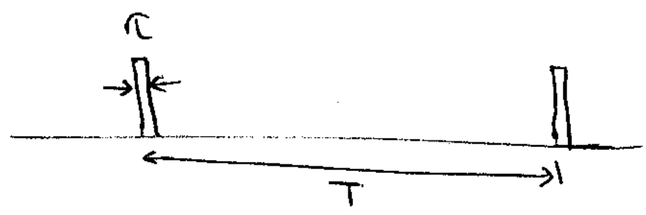


when we have aliasing, it will be impossible to reconstruct the original signal.

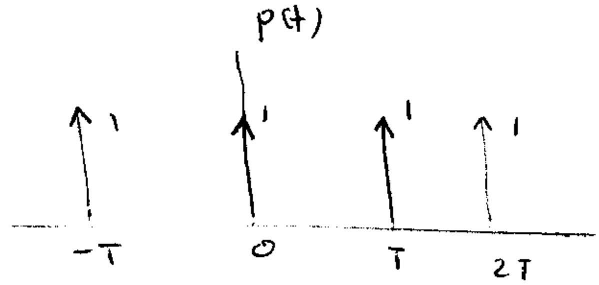


Impulse - Train Sampling Model

In Practice the width of the pulse in the pulse train is small compared to the period of the pulse train



Therefore we can model the pulse train as a train of impulses



$$\Rightarrow \quad p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

Note: when using this model the weight of the impuls carri the sample val

$\cancel{x_s(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)}$

The sampling using the infinite train of impulses is called **Ideal Sampling** or **Instantenous Sampling**.

The values of C_n's (The Fourier Coefficients)

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-j2\pi n f_s t}\, dt = \frac{1}{T} = f_s \quad \text{Constant}$$

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$

$$\Rightarrow \quad X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)$$

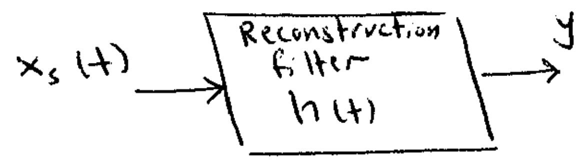
Data Reconstruction

We saw previously that $x(t)$ can be reconstructed from $x_s(t)$ by passing $x_s(t)$ through a low-pass filte[r]

We now investigate this process in more detail.

$$x_s(t) = x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT) = \sum_{k=-\infty}^{\infty} x(t)\,\delta(t-kT)$$

$$= \sum_{k=-\infty}^{\infty} x(kT)\,\delta(t-kT)$$



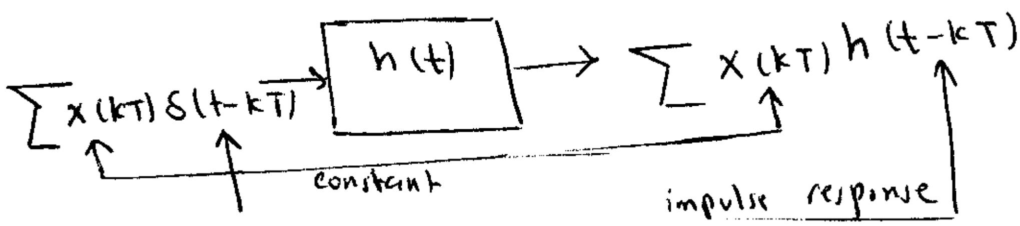
$$y(t) = x_s(t) * h(t) = x(t)$$

$$y(t) = x_s(t) * h(t) = \int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty} x(kT)\,\delta(\lambda-kT)\,h(t-\lambda)\,d\lambda$$

changing order of summation and integration.

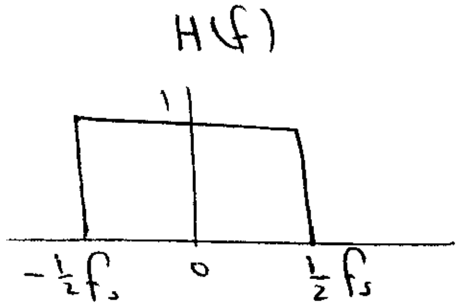
$$y(t) = \sum_{k=-\infty}^{\infty} x(kT)\int_{-\infty}^{\infty}\delta(\lambda-kT)\,h(t-\lambda)\,d\lambda$$

$$= \sum_{k=-\infty}^{\infty} x(kT)\,h(t-kT) \qquad \text{"sifting property"}$$



~~This 2 results~~

Now assume $h(t)$, the reconstruction filter is an ideal filter with B.W $= \frac{1}{2} f_s$



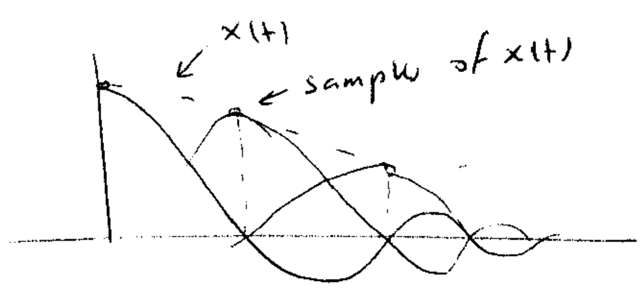
$$\Longleftrightarrow \quad h(t) = \mathrm{sinc}(f_s t) = \frac{\sin \pi f_s t}{\pi f_s t}$$

$$y(t) = x(t) = \sum_{k=-\infty}^{\infty} x(kT)\ \mathrm{sinc}\ f_s(t - kT)$$

$$= \sum_{k=-\infty}^{\infty} x(kT)\ \mathrm{sinc}\ f_s T\left(\frac{t}{T} - k\right)$$

$$= \sum_{k=-\infty}^{\infty} x(kT)\ \mathrm{sinc}\left(\frac{t}{T} - k\right)$$

The equation shows that the original data signal x can be reconstructed by weighting each sample ~~cto~~ by a sinc function.



The sinc function becomes like an interpolating function.

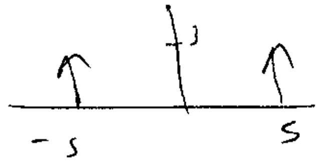
Ex 8-1

$x(t) = 6\cos 2\pi(5)t$

let $f_s = 7$ and 14 Hz

f_h (highest freq in $x(t)$) $= 5$

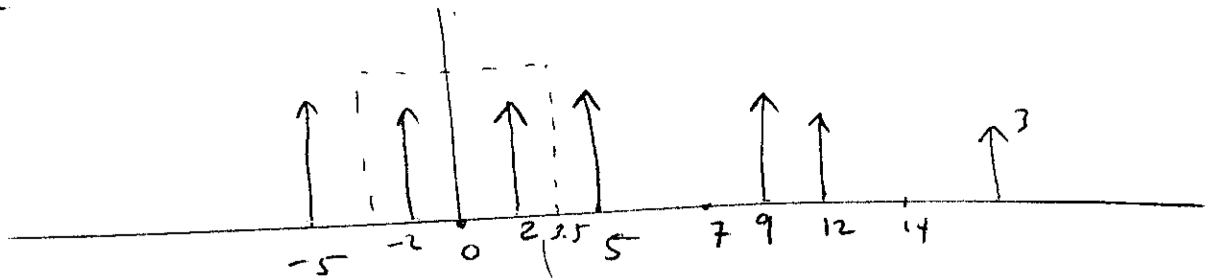
$X(f) = 3\,\delta(f-5) + 3\,\delta(f+5)$



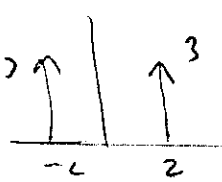
$X_s(f) = f_s \sum x(f - nf_s)$

$= 3f_s \sum [\delta(f-5-nf_s) + \delta(f+5-nf_s)]$

$\underline{f_s = 7}$



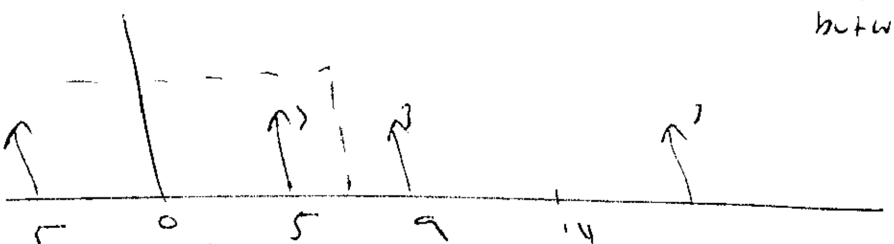
Hs B.w $(H_s(f)) = \frac{f_s}{2} = 3.5$

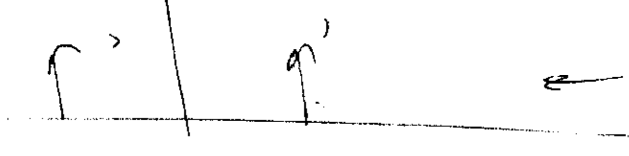


Aliasing
proper ampl
but wrong f

$\underline{f_s = 14}$

B.w $= \frac{14}{2} = 7$





Correct signal.