King Fahd University of Petroleum and Minerals
College of Computer Science and Engineering
Computer Engineering Department

COE 202: Digital Logic Design (3-0-3)
Term 142 (Spring 2014-2015)
Major Exam 1
Saturday February 28, 2015

Time: 90 minutes, Total Pages: 7

Name:_KEY_____ ID:_____ Section: _____

Notes:

- Do not open the exam book until instructed
- **No Calculators are allowed** (*basic, advanced, cell phones, etc*.)
- Answer all questions
- All steps must be shown
- Any assumptions made must be clearly stated

Question	Maximum Points	Your Points
1	25	
2	20	
3	10	
Total	55	

Question 1. **(25 points)**

(I) Convert the following numbers from the given base to the other uncrossed bases listed in the table (if needed, express fractions up to 3 digits only). **(12 points)**

Decimal	Binary	Octal	HEX	EXCESS-3 BCD
109.39	1101101.011	155.307		0100 0011 1100. 0110 1100
173.625	10101101.101		AD.A	
231.281		347.22	E7.48	

(II) Perform the following arithmetic operations in the specified number system. **(8 points)**

Octal Subtraction	Hexadecimal Addition	Binary Subtraction	Binary Addition
4 5 1 2 - 2 5 3 7 1 7 5 3	F E A 3 + A F 9 D 1 A E 4 0	1 1 1 0 0 0 1 0 - 1 0 1 1 1 1 1 1 0 0 1 0 0 0 1 1	1 1 0 1 1 0 1 1 + 0 1 1 1 0 1 1 1 1 0 1 0 1 0 0 1 0

(III) Two number system with radixes r_1 *and* r_2, have the following two relations:

a. $(69)_{r_2} = (100)_{r_1}$, *and*

b. $(17)_{r_2} = (21)_{r_1}$

What are the values of r_1 *and* r_2 ? **(5 points)**

Solution

a. $6r_2 + 9 = r_1^2$

b. $r_2 + 7 = 2r_1 + 1 \rightarrow r_2 = 2r_1 - 6$ substituting in (a)

$r_1^2 - 12r_1 + 27 = 0 \rightarrow (r_1 - 3)(r_1 - 9) = 0 \rightarrow$ **2 Solutions** $(r_1 = 3)$ *and* $(r_1 = 9)$

1) $r_1 = 3 \rightarrow r_2 = 0 \rightarrow$ Infeasible solution

2) $r_1 = 9 \rightarrow r_2 = 12 \rightarrow$ Correct solution

Question 2. **(20 points)**

Use Boolean algebra to solve the following questions. Show clearly all your steps.

(I) Simplify each of the following Boolean functions to the specified number of literals in sum-of-products (SOP) representation:

a. $F1 = x + \overline{x}\,y$ **(2 literals)** **(1 point)**

$= (x + \overline{x})(x + y) = 1\,(x + y) = x + y$

b. $F2 = x\,y + \overline{x}\,z + y\,\overline{z}$ **(3 literals)** **(4 points)**

$= x\,y + \overline{x}\,z + y\,\overline{z} + y\,z \quad (by\ consensus\ of\ x\,y\ and\ \overline{x}\,z)$

$= x\,y + \overline{x}\,z + y\,(\overline{z} + z) = x\,y + \overline{x}\,z + y \quad (by\ distributive\ law)$

$= \overline{x}\,z + y \quad (by\ absorption\ xy\ is\ absrobed\ by\ y)$

c. $F3 = x\,\overline{w}\,\overline{z} + x\,\overline{w}\,\overline{y} + x\,w + x\,y\,z$ **(1 literal)** **(4 points)**

$= x\,(\overline{w}\,\overline{z} + \overline{w}\,\overline{y} + w + y\,z) \quad (by\ distributive\ law)$

$= x\,(\overline{z} + \overline{w}\,\overline{y} + w + y\,z) \quad (by\ simplification\ as\ w + \overline{w}\,\overline{z} = w + \overline{z})$

$= x\,(\overline{z} + \overline{y} + w + y\,z) \quad (by\ simplification\ as\ w + \overline{w}\,\overline{y} = w + \overline{y})$

$= x\,(\overline{z} + \overline{y} + w + z) \quad (by\ simplification\ as\ \overline{y} + y\,z = \overline{y} + z)$

$= x\,(1) = x \quad (since\ \overline{z} + z = 1)$

d. $F4 = \overline{(x + \overline{y})}\,\overline{(x\,y + \overline{x}\,z)}$ **(3 literals)** **(4 points)**

$= \overline{x}\,y\,(\overline{x} + \overline{y})(x + \overline{z}) \quad (by\ Demorgan's\ law)$

$= \overline{x}\,y\,(x + \overline{z}) \quad (by\ disibutive\ law)$

$= \overline{x}\,y\,\overline{z} \quad (by\ disibutive\ law)$

(II) Given the Boolean function $F(X,Y,Z) = (Y + \bar{Z})(\bar{X} + Y)$: **(5 points)**

a. Express F as a **product-of-maxterms**, $F = \prod M$.

$F = \prod M\,(1,4,5)$

b. Find the ***algebraic*** **sum-of-minterms** expression for *F*.

$F = \sum m(0,2,3,6,7)$

$= \overline{X}\,\overline{Y}\,\overline{Z} + \overline{X}\,Y\,\overline{Z} + \overline{X}\,Y\,Z + X\,Y\,\overline{Z} + X\,Y\,Z$

(III) Given the following Boolean function expressed using sum-of-products representation. $F(X,Y,Z) = X\,Y + \bar{X}\,Z$, express F as a product-of-sums (NOT as product-of-maxterms) representation. **(2 points)**

$\bar{F} = (\bar{X} + \bar{Y})(X + \bar{Z})$ (by Demonrgan\s law)

$= \bar{X}\,\bar{Z} + X\,\bar{Y} + \bar{Y}\,\bar{Z}$ (by distributive law)

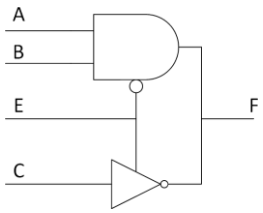
$= \bar{X}\,\bar{Z} + X\,\bar{Y}$ (by consensus)

$F = (X + Z)(\bar{X} + Y)$ $(By\ taking\ the\ complement\ of\ \bar{F}\ using\ Demongan's law)$

Question 3. **(10 points)**

I. Without simplification, write the Boolean algebra equation that represents F: **(2 points)**

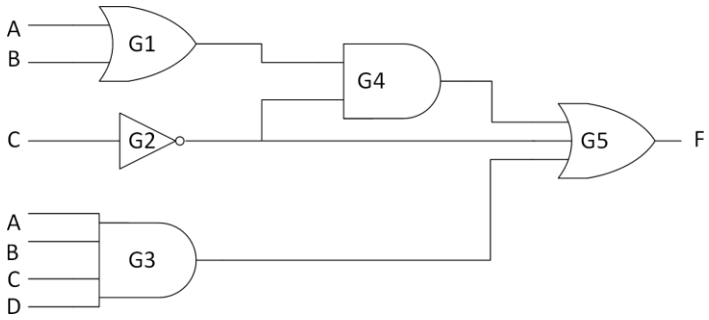
F = EC'+ E'AB



II.

a. Fill the table based on the Logic diagram **(3 points)**

Gate	Delay (*ns*)	Fan_{in}	Driving Load
G1	2		1
G2	1		2
G3	3	4	
G4	2	2	1
G5	2	3	



b. What is the worst-case delay? **(1 point)**

2+2+2=6ns

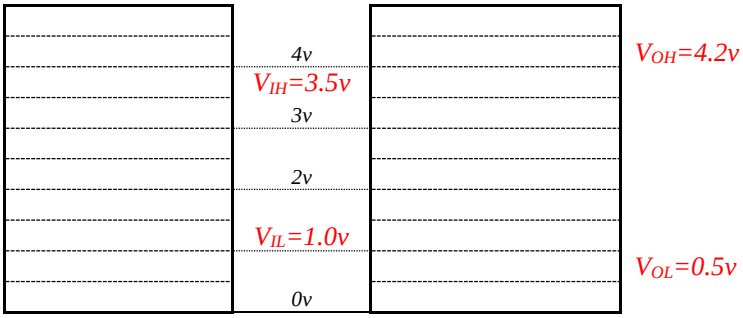
c. What is the worst-case delay path? **(1 point)**

G1, G4, G5

III.

a. You are required to mark the V_{IL}, V_{IH}, V_{OL}, V_{OH} parameters on the following diagram given that the values of these parameters are *selected* from the set **{0.5*v*, 1.0*v*, 3.5*v*, 4.2*v*}***. **(2 points)**

*(*Voltage values are given in ascending order, i.e. not necessarily in the same order of the V_{IL}, V_{IH}, V_{OL}, V_{OH} parameters*)



b. Calculate the Noise Margin for logic 1 (NM_1)? **(1 point)**

$NM_1 = V_{OH} - V_{IH} = 4.2 - 3.5 = 0.7$ v