King Fahd University of Petroleum and Minerals
College of Computer Science and Engineering
Computer Engineering Department

COE 202: Digital Logic Design (3-0-3)
Term 132 (Spring 2013-2014)
Major Exam 1
Saturday March 1, 2014

Time: 90 minutes, Total Pages: 11

Name:_____ ID:_____ Section: _____

Notes:

- Do not open the exam book until instructed
- **No Calculators are allowed** (*basic, advanced, cell phones, etc.*)
- Answer all questions
- All steps must be shown
- Any assumptions made must be clearly stated

Question	Maximum Points	Your Points
1	13	
2	12	
3	15	
4	15	
5	10	
Total	65	

Question 1. **(13 points)**

Perform the following number base conversion with fraction precision of 3-digit where needed. Show your work in the "Work/ Scratch Area"

Required Conversion	**Work / Scratch Area**
a. (i) $(0.339)_{10}$ = ( )$_2$. (ii) Convert the above obtained binary result back to decimal. = ( )$_{10}$ (iii) What is the conversion loss in accuracy?	0.399 X 2=0.798 0.798 X 2=1.596 0.596 X 2=1.182 Therefore $(0.399)_{10} = (0.011)_2$ $(0.011)_2=2^{-2}+2^{-3}=0.25+0.125=0.375$ 0.399-0.375=0.024 due to base conversion
b. $(80.125)_{10}$=( )$_2$ Fraction part 0.125 X 2=0.25 0.25 X 2=0.5 0.5 X 2=1.0 =$(1010000.001)_2$	Integer part 80 | . 40 | 0 20 | 0 10 | 0 5 | 0 2 | 1 1 | 0 0 | 1

c. $(10)_{13}=(\quad)_{10}$	1x13=10
d. $(F319)_{16} = (\quad)_2$.	$=(1111\ 0011\ 0001\ 1001)_2$
e. $(F319)_{16} = (\quad)_8$.	$=(1\ 111\ 001\ 100\ 011\ 001)_2$ $=(171431)_8$
f. $(9403)_{10} = (\quad)$ BCD 8421 code.	$=(1001\_0100\_0000\_0011)$ BCD 8421 code.

Question 2. **(12 points)**

I. Compute the following arithmetic operations in the indicated bases **(9 Points)**

a. $(A69C - 3F)_{16}$ =A65D	**b.** $(255 + 127)_8$ =404
c. $(1101\ 1000 - 1001\ 1111)_2$ 1101_1000 1001_1111 0011 1001	**d.** $(1101\ 1000 * 101)_2$ 1101 1000 101 1101 1000 0000 00000 1101 100000 10000111000

II. What is the radix r of the number system for which $(24 + 17 = 40)_r$. **(3 Points)**

(2b+4)+ (b+7) =(4b+0)

b=11

Question 3. **(15 points)**

Use Boolean algebra to solve the following questions. Show clearly all your steps.

a. Give the simplest form of $F = Y(X + Y) + \overline{(X + Y)}Z + YZ$ (4 points)

$$F = (XY + Y) + (\bar{X}\,\bar{Y}Z + YZ)$$
$$F = Y(X + 1) + Z(\bar{X}\,\bar{Y} + Y)$$
$$F = Y.1 + Z\big((\bar{X} + Y)(\bar{Y} + Y)\big)$$
$$F = Y + Z\big((\bar{X} + Y).1\big)$$
$$F = Y + Z(\bar{X} + Y)$$
$$F = Y + YZ + \bar{X}\,Z$$
$$F = Y(1 + Z) + \bar{X}\,Z$$
$$F = Y + \bar{X}\,Z$$

b. Given that $C = A\,\overline{B} + \overline{A}\,B$ show that $A\,\overline{C} + \overline{A}\,C = B$ (5 Points)

$$\bar{C} = (\bar{A} + B)(A + \bar{B}) = \overline{A}\,\overline{B} + A\,B$$
$$LHS = A\,\overline{C} + \overline{A}\,C$$
$$LHS = A\left(\overline{A}\,\overline{B} + A\,B\right) + \overline{A}\,(A\,\overline{B} + \overline{A}\,B)$$
$$LHS = AB + \overline{A}\,B$$
$$LHS = B\left(A + \overline{A}\right) = B = RHS$$

c. Find the values of the 4 Boolean variables A, B, C, and D by solving the following set of simultaneous Boolean equations: (3 Points)

i. $\bar{A} + B = 0$
ii. $A\,B = A\,C$
iii. $A\,B + A\,\bar{C} + C\,D = \bar{C}\,D$

- Equation (i) implies that: **B = 0** AND **A = 1**
- Substituting into Equation (ii) → : $LHS = A\,B = 1.0 = 0 = AC = 1.C = C$ *which implies that:* **C = 0**
- Substituting into Equation (iii) → : $LHS = A\,B + A\,\bar{C} + C\,D = 1.0 + 1.1 + 0.D = 1$
- $Thus, RHS = 1 = \bar{C}\,D = 1.D = D$ *which implies that:* **D = 1**
- $Thus, ABCD = 1001$

d. **Without simplification**, write out the **complement** and **dual** forms of the following expression:

$(x + \overline{y}\,\overline{z})(w\,\overline{x}\,z + \overline{w}\,y\,\overline{z})$: (3 Points)

$$Let\ F = (x + \overline{y}\,\overline{z})(w\,\overline{x}\,z + \overline{w}\,y\,\overline{z}) = (x + (\overline{y}\,\overline{z}))\big((w\,\overline{x}\,z) + (\overline{w}\,y\,\overline{z})\big)$$

Using De-Morgan Theorem, we get the complement expression as follows:

$$\bar{F} = \bar{x}.(y + z) + (\overline{w} + x + \bar{z}).(w + \bar{y} + z)$$

The dual expression of F is given by:

$$Dual\ (F) = x.(\bar{y} + \bar{z}) + (w + \bar{x} + z).(\overline{w} + y + \bar{z})$$

Question 4. **(15 points)**

I. The truth table of a digital circuit which has two inputs (A, B) and two outputs (Y, Z) is shown: **(4 points)**

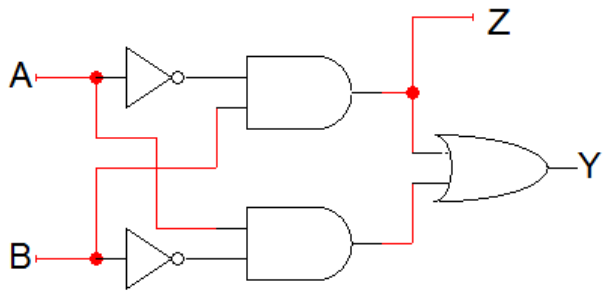
A	B	Y	Z
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

a. Write the Boolean expressions of the circuit outputs (Y , Z).

$$Y = \bar{A}B + A\bar{B}$$

$$Z = \bar{A}B$$

b. Draw the logic diagram of this circuit (i.e., its gate-level implementation).



II. Given the Boolean function $F(X,Y,Z) = (X+Y)(X+Z)(\bar{X}+\bar{Z})$: **(4 points)**

a. Express F as a **sum-of-minterms**, $F = \sum m$.

b. Find the ***algebraic*** **product-of-Maxterms** expression for *F*.

a. F' = X' Y' + X' Z' + X Z = $\sum m\,(0,1,2,5,7)$

$F = \sum m(3,4,6)$.

b. $F = \prod M(0,1,2,5,7) = (X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+Z)(\bar{X}+Y+\bar{Z})(\bar{X}+\bar{Y}+\bar{Z})$

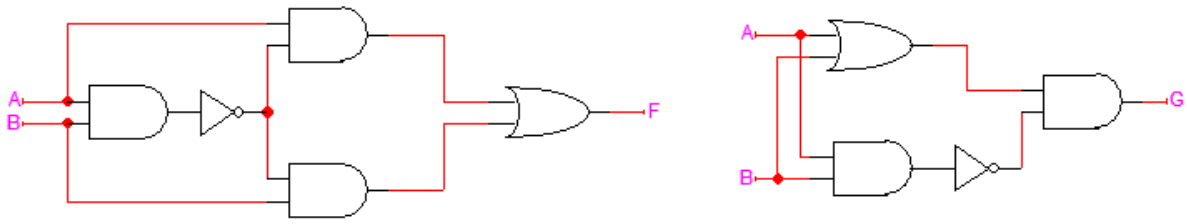
III. Given $F(A,B,C) = \sum m\,(0,3,5,7)$ and $G(A,B,C) = \prod M\,(1,2,4,7)$, express the function $F + \bar{G}$ as a sum-of-minterms. **(3 points)**

$\bar{G} = \sum m(1,2,4,7)$

$F + \bar{G} = \sum m(0,1,2,3,4,5,7)$

IV. Given the following two circuits representing the functions F and G. Determine whether the two functions F and G are equivalent or not. Justify your answer.

(4 points)



F = (A B)' A + (A B)' B = (A' + B') A + (A' + B') B = A B' + A' B = ∑m(1,2)

G = (A + B)(A B)' = (A + B)(A' + B') = A' B + A' B = ∑m(1,2)

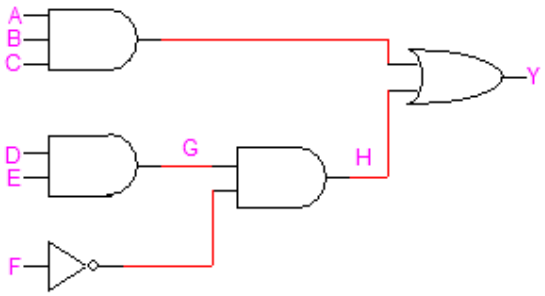
Thus, F and G implement equivalent functions.

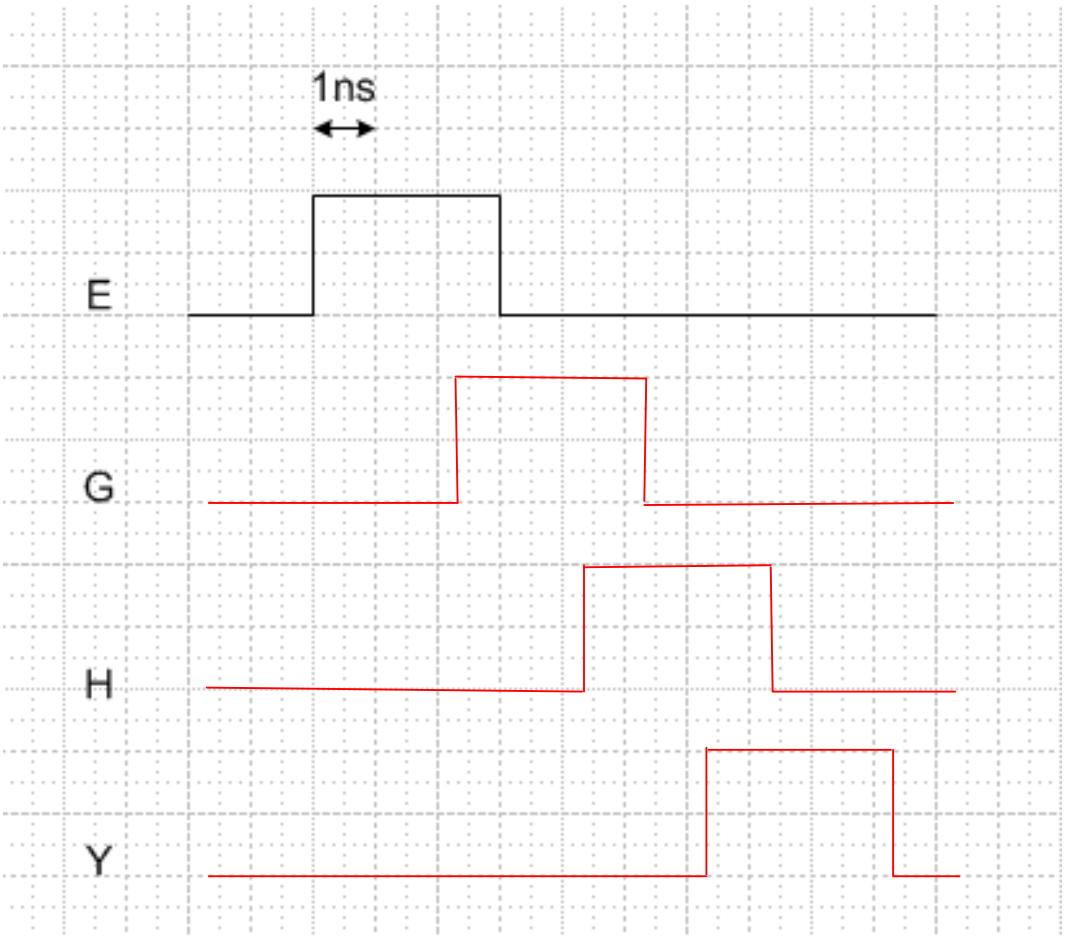
Question 5. **(10 points)**

I. Assume that the propagation delay of a gate depends only on its number of inputs. Thus, the propagation delay of an Inverter is 1 ns, of a 2-input gate (AND or OR) is 2 ns, and of a 3-input gate is 3 ns. For the circuit shown below;

a. What is the longest propagation delay from an input to the output? **(2 point)**

b. If **A=0**, **B=1**, **C=1**, **D=1**, and **F=0**, draw the signal waveforms at points **G, H**, and **Y** due to the shown applied signal at **E** by completing the timing diagram given below. **(3 points)**





II. Given an inverter with the following parameters V_{OH}=5v V_{OL}=0v, V_{IH}=2.8v, V_{IL}=1.6, the noise margins NM_H= V_{OH} - V_{IH}=5-2.8=2.2v and NM_L= V_{IL} - V_{OL}=1.6v. **(2 points)**

III. The Boolean function implemented by the circuit given below expressed as a sum-of-products is

$F = C (A + B) + C' A B = A C + B C + A B C' = A C + B C + A B$. **(3 points)**