# KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
# COLLEGE OF COMPUTER SCIENCES & ENGINEERING
## COMPUTER ENGINEERING DEPARTMENT

**COE 540 – Computer Networks**
**Assignment 1 – Due Date Sept 13st, 2015**

**Problem (70 points):** On the Fourier series/transform and channel capacity.

Consider the periodic signal specified by

$$g(t) = \begin{cases} A & 0 \le t < \frac{2T}{8} \cup \frac{6T}{8} \le t < T \\ 0 & otherwise \end{cases}$$

where $A$ and $T$ are the amplitude in volts and period of the signal in seconds, respectively.

a) (7 points) Compute the DC component and the total power for the signal.

b) (8 points) Compute the FSE for the signal $g(t)$ and specify the expressions for the coefficients $a_n$, $b_n$, and $c$. Write the FSE for the signal $g(t)$.

c) (10 points) Plot the signal $g(t)$ for $t \in [-T, T]$ and plot also the corresponding power spectral density $PSD_{g(t)}(f)$.

d) (7 points) If this signal is passed through an ideal low-pass filter (LPF) that suppresses frequency components greater or equal to $8/T$ Hz. Let the output of the filter be specified by $g_{LPF}(t)$. Write the expression for $g_{LPF}(t)$ and compute its power.

e) (8 points) Plot the function $g_{LPF}(t)$ $t \in [-T, T]$ and plot also the corresponding power spectral density $PSD_{g_{LPF}(t)}(f)$.

f) (7 points) If this signal is passed through an ideal band-pass filter (BPF) that passes frequency components belonging to $f \in [6/T, 12/T]$ Hz. Let the output of the filter be specified by $g_{BPF}(t)$. Write the expression for $g_{BPF}(t)$ and compute its power.

g) (8 points) Plot the function $g_{BPF}(t)$ $t \in [-T, T]$ and plot also the corresponding power spectral density $PSD_{g_{BPF}(t)}(f)$.

h) (15 points) Assume the noise power spectral density is equal to $10^{-6}$ Watts/Hz and considering the system in part (f). What would be maximum theoretical capacity of the system in bits per second. What is the minimum number of bits per symbol needed to achieve this capacity?

For all your calculations, assume $T = 2$ msec and $A = 2$ volts.

Solution:

a) DC is given by $\frac{1}{T}\int_0^T g(t)\,dt = A/2 = 1$ volts.

The power is given by by $\frac{1}{T}\int_0^T |g(t)|^2\,dt = \frac{A^2}{2} = 2$ Watts.

b) Computing FSE: To consider one full period – we take $t$ ranging from $-T/4$ to $3T/4$, this way the integral is non zero for only ONE subinterval which is $t \in [-T/4, T/4]$. Then confidents are given by

$$c = \frac{2}{T}\int_{-T/4}^{T/4} g(t)\,dt = \frac{2}{T}\int_{-T/4}^{T/4} A\,dt = A$$

$$a_n = \frac{2}{T}\int_{-T/4}^{T/4} g(t)\sin\left(\frac{2\pi n t}{T}\right)dt = \cdots = 0$$

$$b_n = \frac{2}{T}\int_{-T/4}^{T/4} g(t)\cos\left(\frac{2\pi n t}{T}\right)dt = \cdots = \frac{2A}{\pi n}\sin\left(\frac{n\pi}{2}\right)$$

Then, $g(t)$ can be written as:

$$\begin{aligned} g(t) &= \frac{A}{2} + \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{2}\right)}{n}\cos\left(\frac{2\pi n t}{T}\right) \\ &= \frac{A}{2} + \frac{2A}{\pi\times 1}\cos\left(\frac{2\pi\times 1\times t}{T}\right) + \frac{-2A}{\pi\times 3}\cos\left(\frac{2\pi\times 3\times t}{T}\right) + \frac{2A}{\pi\times 5}\cos\left(\frac{2\pi\times 5\times t}{T}\right) + \cdots \end{aligned}$$

It is worth noting that:

- The coefficient bn is ZERO for EVEN n. Therefore, the signal g(t) has frequency components ONLY at $0\times f_0, 1\times f_0, 3\times f_0, 5\times f_0, \cdots$
- The DC component $\frac{A}{2}$ is the same as that computed in part (a)
- The above summation contains infinite number of terms

d) The low-pass filtered signal contains all frequency components less that 8/T or $8\times f_0$ – Therefore, low-pass filtered signal is specified by

$$\begin{aligned} g_{LPF}(t) &= \frac{A}{2} + \frac{2A}{\pi}\sum_{n=1}^{7}\frac{\sin\left(\frac{n\pi}{2}\right)}{n}\cos\left(\frac{2\pi n t}{T}\right) \\ &= \frac{A}{2} + \frac{2A}{\pi\times 1}\cos\left(\frac{2\pi\times 1\times t}{T}\right) + \frac{-2A}{\pi\times 3}\cos\left(\frac{2\pi\times 3\times t}{T}\right) + \frac{2A}{\pi\times 5}\cos\left(\frac{2\pi\times 5\times t}{T}\right) \\ &\quad + \frac{2A}{\pi\times 7}\cos\left(\frac{2\pi\times 7\times t}{T}\right) \end{aligned}$$

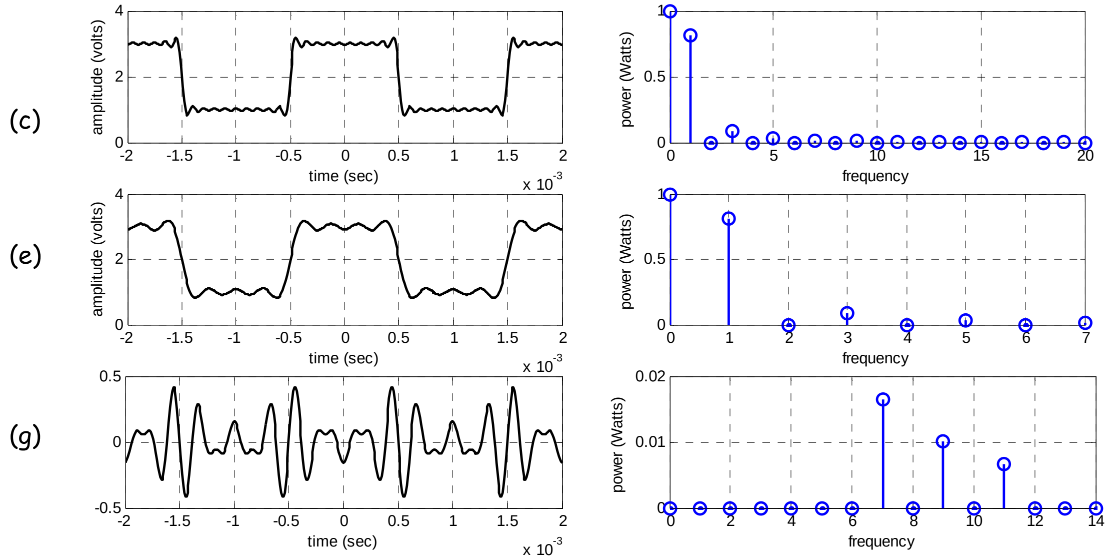
Note that we have only the DC component and the first 4 harmonic components.

f) The band-pass filtered signal contains all frequency components from 6/T (or $6\times f_0$) to 12/T (or $12\times f_0$) – since the original harmonics exist only at odd multiples of f0, then we will have only the following THREE components: $7\times f_0, 9\times f_0,$ and $11\times f_0$. Therefore, the band-pass filtered signal can be specified as

$$g_{BPF}(t) = \frac{2A}{\pi\times 7}\cos\left(\frac{2\pi\times 7\times t}{T}\right) + \frac{-2A}{\pi\times 9}\cos\left(\frac{2\pi\times 9\times t}{T}\right) + \frac{2A}{\pi\times 11}\cos\left(\frac{2\pi\times 11\times t}{T}\right)$$

Note that the above signal DOES NOT have DC component.

The plots for parts (c), (e), and (g) are as shown below.




h) C = B log2( 1 + SNR)

SNR = avg signal power / avg noise power

Avg signal power is the power of the signal in part (f) – given by

$$P_{g_{BPF}} = \frac{\left[\left(\frac{2A}{\pi \times 7}\right)^2 + \left(\frac{2A}{\pi \times 9}\right)^2 + \left(\frac{2A}{\pi \times 11}\right)^2\right]}{2} = 0.0332 \text{ Watts}$$

The bandwidth of the system, BW is from 6f0 till 12f0 which is 6f0 → BW = 6 x (500) = 3000 Hz.

Remember that f0 = 1/T = 1/0.002 = 500 Hz.

Therefore Avg noise power = BW x N0 = (3000) x (1e-6) = 0.003 Watts

Hence SNR = 0.0332 / 0.003 = 11.083 or 10.4 dB

Capacity C = Bxlog2(1 + SNR) = (3000) log2(1 + 11.083) = 10784.6 bit/sec

To achieve this capacity we need at least ceil(C/(2B)) = ceil(log2(V)) = ceil(10784.6/(6000)) = 2 bits per symbol.