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HW#4

Q1. $x = abd\bar{e} + c\bar{d}e + cd\bar{e} + ab\bar{d}e + abc$

$y = ab + c$ — 20 literals

(i) Substitute y into f_x by the algebraic division f_x / f_y

- $i=1$; $C_1^B = ab$

$D = \{abd\bar{e},\ ab\bar{d}e,\ abc\}$

$D_1 = \{d\bar{e},\ \bar{d}e,\ c\}$

$Q = D_1 = \{d\bar{e},\ \bar{d}e,\ c\}$

- $i=2$; $C_2^B = c$

$D = \{c\bar{d}e,\ cd\bar{e},\ abc\}$

$D_2 = \{\bar{d}e,\ d\bar{e},\ ab\}$

$Q = Q \cap D_2 = \{d\bar{e},\ \bar{d}e\}$

$R = \{abd\bar{e},\ c\bar{d}e,\ cd\bar{e},\ ab\bar{d}e,\ abc\}$
$\quad - \{d\bar{e},\ \bar{d}e\} \times \{ab,\ c\}$

$= \{abd\bar{e},\ c\bar{d}e,\ cd\bar{e},\ ab\bar{d}e,\ abc\}$
$\quad - \{abd\bar{e},\ ab\bar{d}e,\ cd\bar{e},\ c\bar{d}e\}$

$= \{abc\}$

$\Rightarrow$ $x = y \cdot (d\bar{e} + \bar{d}e) + abc$
$y = ab + c$

$\Rightarrow$ $x = yd\bar{e} + y\bar{d}e + abc$
$y = ab + c$ — 12 literals

# of literals saved in SOP = 8

$= n * l - n + \sum |b_i|$

$= 2 * 3 - 2 + 2 + 2 = 8$ literals

(Note that we are not using $-l$ in the formula since y is already implemented.).

(ii) SIS produces the same result.

Q2. $x = ach + bch + dgh + egh + fgh + i$

(i) Recursive Kernel Computation

We assume that the variables are ordered in lexicographic order : $\{a, b, c, d, e, f, g, h, i\}$

$i = 1 : \{a\}$

Cubes $(x, a) = \{ach\} < 2 \Rightarrow$ no kernels

$i = 2 : \{b\}$

Cubes $(x, b) = \{bch\} < 2 \Rightarrow$ no kernels

$i = 3 : \{c\}$

Cubes $(x, c) = \{ach, bch\} \geq 2$

$C = ch$

The kernel $a+b$ will be returned.

$i = 4 : \{d\}$

Cubes $(x, d) = \{dgh\} < 2 \Rightarrow$ no kernels

$i = 5 : \{e\}$

Cubes $(x, e) = \{egh\} < 2 \Rightarrow$ no kernels

$i = 6 : \{f\}$

Cubes $(x, f) = \{fgh\} < 2 \Rightarrow$ no kernels

$i=7$: $\{g\}$

Cubes $(x, g) = \{dgh, egh, fgh\} \geq 2$

$C = gh$

The Kernel $d+e+f$ will be returned.

$i=8$: $\{h\}$

Cubes $(x, h) = \{ach, bch, dgh, egh, fgh\} \geq 2$

$C = h$

The Kernel procedure will be called on the function $ac+bc+dg+eg+fg$ with index $= 9$ i.e. variable i.

So, the Kernel $ac+bc+dg+eg+fg$ will be returned.

$i=9$: $\{i\}$ $\Rightarrow$ no Kernels.

Cubes $(x, i) = \{i\} < 2$

Since x is cube-free, it is also a Kernel.

So, the Kernels and co-Kernels of x are:

Kernel	co-Kernel
$a+b$	ch
$d+e+f$	gh
$ac+bc+dg+eg+fg$	h
$ach+bch+dgh+egh+fgh+i$	$\emptyset$

(ii)

	var.	a	b	c	d	e	f	g	h	i
cube	R/C	1	2	3	4	5	6	7	8	9
ach	1	1	0	1	0	0	0	0	1	0
bch	2	0	1	1	0	0	0	0	1	0
dgh	3	0	0	0	1	0	0	1	1	0
egh	4	0	0	0	0	1	0	1	1	0
fgh	5	0	0	0	0	0	1	1	1	0
i	6	0	0	0	0	0	0	0	0	1

Prime Rectangle	cube	Kernel
({1,2}, {3,8})	ch	a+b
({3,4,5}, {7,8})	gh	d+e+f
({1,2,3,4,5}, {8})	h	ac+bc+dg+eg+fg

So, we got the same Kernels as in part (i).

Q3. $x = ac + bc + be + dc + de$ $\quad$ 10 literals

(i) Quick factor using first level-0 Kernel.

$L(a) = 1 \Rightarrow$ no Kernels

$L(b) = 2 > 1$

$C = b$

Then, we call the procedure one-level-0-Kernel on $\frac{x}{b} = c + e$

So, the first level-0 Kernel returned is $c+e$.

$(h, r) = \text{Divide}(x, k = c+e)$

$h = b+d$

$r = ac$

Calling Quick factor on h and r will return them as is.

Thus, x is factored as follows:

$x = (c+e)(b+d) + ac$ $\underline{\underline{6}}$ literals

The solution returned by SIS is

$x = e(d+b) + c(a+b+d)$ $\underline{\underline{7}}$ literals

This is clearly due to the order of the variables processed.

(ii) <u>Good Factoring</u>

We need to compute all the kernels with their value and select the one that has the highest value.

Note that the value of a kernel is

$= n * l - n - l \quad \sum |b_i|$

Kernel	Co-Kernel	Value
$c+e$	$\{b\}, \{d\}$	$2 \times 2 - 2 - 2 + 2 = 2$
$b+d$	$\{e\}$	$1 \times 2 - 1 - 2 + 1 = 0$
$a+b+d$	$\{c\}$	$1 \times 3 - 1 - 3 + 1 = 0$

Thus, the divisor selected will be $k = c + e$

So, we will get the same result

$x = (c+e)(b+d) + ac$ $\underline{\underline{6}}$ literals

This is the same result as produced by the SIS tool.

$\underline{\underline{Q4}}$. $x = acd + ae\bar{f} + a\bar{e}\bar{g} + \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{d} + \bar{a}\bar{b}ef + \bar{a}\bar{b}\bar{e}g + bcd$

$\underline{\underline{26}}$ literals

Double-cube Divisor	Base	weight
$cd + e\bar{f}$	a	$1\times4-1-4+1 = 0$
$cd + \bar{e}\bar{g}$	a	$1\times4-1-4+1 = 0$
$acd + \bar{a}\bar{b}\bar{c}$	$\emptyset$	$1\times6-1-6 = -1$
$acd + \bar{a}\bar{b}\bar{d}$	$\emptyset$	-1
$acd + \bar{a}\bar{b}ef$	$\emptyset$	-1
$acd + \bar{a}\bar{b}\bar{e}g$	$\emptyset$	-1
$a + b$	cd	$1\times2-1-2+2+4 = 5$
$e\bar{f} + \bar{e}\bar{g}$	a	$2\times4-2-4+1+2 = 5$
$ae\bar{f} + \bar{a}\bar{b}\bar{c}$	$\emptyset$	-1
$ae\bar{f} + \bar{a}\bar{b}\bar{d}$	$\emptyset$	-1
$a\bar{f} + \bar{a}\bar{b}f$	e	$1\times5-1-5+1 = 0$
$ae\bar{f} + \bar{a}\bar{b}\bar{e}g$	$\emptyset$	-1
$ae\bar{f} + bcd$	$\emptyset$	-1
$a\bar{e}\bar{g} + \bar{a}\bar{b}\bar{c}$	$\emptyset$	-1
$a\bar{e}\bar{g} + \bar{a}\bar{b}\bar{d}$	$\emptyset$	-1
$a\bar{e}\bar{g} + \bar{a}\bar{b}ef$	$\emptyset$	-1
$a\bar{g} + \bar{a}\bar{b}g$	$\bar{e}$	$1\times5-1-5+1 = 0$
$a\bar{e}\bar{g} + bcd$	$\emptyset$	-1

Double-cube Divisor	Base	Weight
$\bar{c} + \bar{d}$	$\bar{a}\bar{b}$	$1\times 2-1-2+2+2 = 3$
$\bar{c} + ef$	$\bar{a}\bar{b}$	$1\times 3-1-3+2 = 1$
$\bar{c} + \bar{e}g$	$\bar{a}\bar{b}$	$1\times 3-1-3+2 = 1$
$\bar{a}\bar{b}\bar{c} + bcd$	$\emptyset$	-1
$\bar{d} + ef$	$\bar{a}\bar{b}$	$1\times 3-1-3+2 = 1$
$\bar{d} + \bar{e}g$	$\bar{a}\bar{b}$	$1\times 3-1-3+2 = 1$
$\bar{a}\bar{b}\bar{d} + bcd$	$\emptyset$	-1
$ef + \bar{e}g$	$\bar{a}\bar{b}$	$2\times 4-2-4+2+1 = 5$
$\bar{a}\bar{b}ef + bcd$	$\emptyset$	-1
$\bar{a}\bar{b}\bar{e}g + bcd$	$\emptyset$	-1

As we can see, the double-cube divisors $(a+b)$, $(e\bar{f} + \bar{e}\bar{g})$, and $(ef + \bar{e}g)$ have the same weight and any one of them can be selected. $W_{dmax} = 5$.

$W_{smax} = 2$ since $W(\bar{a}\bar{b}) = 4-2 = 2$

$\Rightarrow W_{max} = 5$

Let us choose the double cube divisor

$d = a+b$

The resulting network after extracting the double-cube divisor is:

$$[1] = a+b$$
$$x = [1]cd + ae\bar{f} + a\bar{e}\bar{g} + \overline{[1]}\,\bar{c} + \overline{[1]}\,\bar{d} + [1]ef + \overline{[1]}\,\bar{e}g$$

<u>21</u> literals

The process is repeated and we select the next best divisor. We will list only double-cube divisors with non-empty base.

Double-cube Divisor	Base	Weight
$e\bar{f} + \bar{e}\bar{g}$	a	$2\times4-2-4+1+1=4$
$a\bar{f} + \overline{[1]}f$	e	$1\times4-1-4+1=0$
$a\bar{g} + \overline{[1]}g$	$\bar{e}$	$1\times4-1-4+1=0$
$\bar{c} + \bar{d}$	$\overline{[1]}$	$1\times2-1-2+1+1=1$
$\bar{c} + ef$	$\overline{[1]}$	$1\times3-1-3+1=0$
$\bar{c} + \bar{e}g$	$\overline{[1]}$	$1\times3-1-3+1=0$
$\bar{d} + ef$	$\overline{[1]}$	$1\times3-1-3+1=0$
$\bar{d} + \bar{e}g$	$\overline{[1]}$	$1\times3-1-3+1=0$
$ef + \bar{e}g$	$\overline{[1]}$	$2\times4-2-4+1+1=4$

Thus, either of the double-cube divisors $(e\bar{f} + \bar{e}\bar{g})$ or $(ef + \bar{e}g)$ can be extracted as they have the same weight.

Let us extract the double-cube divisor $ef + \bar{e}g$.

$\Rightarrow$ $[1] = a + b$

$[2] = ef + \bar{e}g$

$x = [1]cd + a\overline{[2]} + \overline{[1]}\bar{c} + \overline{[1]}\bar{d} + \overline{[1]}[2]$ $\quad$ $\underline{\underline{17}}$ literals

Next, we can see that the only double-cube divisor with a positive weight is $(\bar{c}+\bar{d})$ with a weight of 1. So, we extract it.

$$\Rightarrow \quad [1] = a+b$$
$$[2] = ef + \bar{e}g$$
$$[3] = \bar{c} + \bar{d}$$
$$x = [1]\overline{[3]} + a\overline{[2]} + \overline{[1]}[3] + \overline{[1]}[2]$$

<u>16</u> literals

The solution produced by SIS has the same cost of 16 literals.

<u>Q5</u>.

$$d = a+b$$
$$e = \bar{b}+c$$
$$f = cd + ae + b\bar{c}$$

(i) The cut is $\{a, b, c, d, e\}$ for the inputs of f.

$$\Rightarrow \quad CDC_{cut} = d \oplus (a+b) + e \oplus (\bar{b}+c)$$
$$= d\bar{a}\bar{b} + \bar{d}a + \bar{d}b + eb\bar{c} + \bar{e}\bar{b} + \bar{e}c$$

(ii) $SDC_d = d \oplus (a+b) = d\bar{a}\bar{b} + \bar{d}a + \bar{d}b$

$SDC_e = e \oplus (\bar{b}+c) = eb\bar{c} + \bar{e}\bar{b} + \bar{e}c$

(iii) Simplifying f using its CDC

$e=0$

$ab \backslash cd$	00	01	11	10
00	x	x	x	x
01	x	1	x	x
11	x	1	x	x
10	x	x	x	x

$e=1$

$ab \backslash cd$	00	01	11	10
00	0	x	x	0
01	x	x	1	x
11	x	x	1	x
10	x	1	1	x

$$\Rightarrow \quad f = d$$

$$d = a + b$$

Note that node will be eliminated since it is not used and it is not a primary output.

(iv) $$ODC_e = f_e \ \overline{\oplus}\ f_{\bar{e}}$$

$$= [cd + a + b\bar{c}] \ \overline{\oplus}\ [cd + b\bar{c}]$$

$$= \{\bar{c}[a+b] + c[a+d]\} \ \overline{\oplus}\ \{\bar{c}[b] + c[d]\}$$

$$= \bar{c}[b + \bar{a}\bar{b}] + c[d + \bar{a}\bar{d}]$$

$$= \bar{c}[b + \bar{a}] + c[d + \bar{a}]$$

$$= \bar{c}b + \bar{c}\bar{a} + cd + c\bar{a}$$

$$= \bar{a} + b\bar{c} + cd$$

$$= \bar{a} + b\bar{c} + ca + cb$$

$$= \bar{a} + b + c$$

Since the perturbation $\delta = \bar{b} + c \not\subseteq ODC_e$ then the perturbation is not feasible.

Thus, the fault e stuck-at-0 is testable.

The tests detecting the fault e stuck-at-0 are ;

$$e \cdot \overline{ODC_e} = (\bar{b}+c)\,\overline{(\bar{a}+b+c)}$$

$$= (\bar{b}+c)(a\bar{b}\bar{c})$$

$$= a\bar{b}\bar{c}$$

So, there is only one test.

(v) The sis command full_simplify produces the same result as the one obtained in (iii).