HW #3

Q1. $X = ABEF + AB\bar{G}\bar{H} + CDEF + CD\bar{G}\bar{H} + \bar{A}\bar{B}\bar{C}\bar{D}$

$Y = AB + CD$ 24 literals

(i) Substitute y into x by algebraic division X/Y

- $i=1$, $C_1^B = AB$

$D = \{ABEF, AB\bar{G}\bar{H}\}$

$D_1 = \{EF, \bar{G}\bar{H}\}$

$Q = D_1 = \{EF, \bar{G}\bar{H}\}$

- $i=2$, $C_2^B = CD$

$D = \{CDEF, CD\bar{G}\bar{H}\}$

$D_2 = \{EF, \bar{G}\bar{H}\}$

$Q = Q \cap D_2 = \{EF, \bar{G}\bar{H}\}$

$R = \{ABEF, AB\bar{G}\bar{H}, CDEF, CD\bar{G}\bar{H}, \bar{A}\bar{B}\bar{C}\bar{D}\}$
$- \{AB, CD\} \times \{EF, \bar{G}\bar{H}\}$
$= \{\bar{A}\bar{B}\bar{C}\bar{D}\}$

$\Rightarrow X = Y \cdot (EF + \bar{G}\bar{H}) + \bar{A}\bar{B}\bar{C}\bar{D}$

$= YEF + Y\bar{G}\bar{H} + \bar{A}\bar{B}\bar{C}\bar{D}$

$Y = AB + CD$ 14 literals

No. of literals saved $= 24 - 14 =$ 10 literals

$= n * l - n + \sum |b_i|$

$= 2 * 4 - 2 + 2 + 2 =$ 10 literals

(ii) Sis produces the same result.

Q2. $X = ACE + A\bar{D}\bar{E} + B\bar{D}\bar{E} + BCE + DE + \bar{C}\bar{E}$

(i) Recursive Kernel computation

We assume that the variables are ordered in lexicographic order: $\{A, B, C, \bar{C}, D, \bar{D}, E, \bar{E}\}$

$i=1$: $\{A\}$

$cubes(X, A) = \{ACE, A\bar{D}\bar{E}\} \geq 2$

$C = A$

The Kernel $CE + \bar{D}\bar{E}$ will be returned

$i=2$: $\{B\}$

$cubes(X, B) = \{B\bar{D}\bar{E}, BCE\} \geq 2$

$C = B$

The Kernel $CE + \bar{D}\bar{E}$ will be returned

$i=3$: $\{C\}$

$cubes(X, C) = \{ACE, BCE\} \geq 2$

$C = CE$

The Kernel $A + B$ will be returned

$i=4$: $\{\bar{C}\}$

$cubes(X, \bar{C}) = \{\bar{C}\bar{E}\} < 2 \Rightarrow$ no Kernels

$i=5$: $\{D\}$

$cubes(X, D) = \{DE\} < 2 \Rightarrow$ no Kernels

$i=6$: $\{\bar{D}\}$

$cubes(X, \bar{D}) = \{A\bar{D}\bar{E}, B\bar{D}\bar{E}\}$

$C = \bar{D}\bar{E}$

The Kenel $A + B$ will be returned

$i=7$: $\{E\}$

cubes $(X, E) = \{ACE, BCE, DE\} \geq 2$

$C = E$

The Kernel $AC + BC + D$ will be returned

$i' = 8$: $\{\bar{E}\}$

Cubes $(X, \bar{E}) = \{A\bar{D}\bar{E}, B\bar{D}\bar{E}, \bar{C}\bar{E}\}$

$C = \bar{E}$

The Kernel $A\bar{D} + B\bar{D} + \bar{C}$ will be returned

Since X is cube-free, it is also a Kernel.

Thus, the Kernels and co-Kernels of X are:

Kernel	co-Kernel
$CE + \bar{D}\bar{E}$	A, B
$A + B$	$CE, \bar{D}\bar{E}$
$AC + BC + D$	E
$A\bar{D} + B\bar{D} + \bar{C}$	$\bar{E}$
$ACE + A\bar{D}\bar{E} + B\bar{D}\bar{E} + BCE + DE + \bar{C}\bar{E}$	$\emptyset$

(ii)

cube	vari. R/C	A 1	B 2	C 3	$\bar{C}$ 4	D 5	$\bar{D}$ 6	E 7	$\bar{E}$ 8
ACE	1	1	0	1	0	0	0	1	0
$A\bar{D}\bar{E}$	2	1	0	0	0	0	1	0	1
$B\bar{D}\bar{E}$	3	0	1	0	0	0	1	0	1
BCE	4	0	1	1	0	0	0	1	0
DE	5	0	0	0	0	0	0	1	0
$\bar{C}\bar{E}$	6	0	0	0	1	0	0	0	1

Prime Rectangle	cube	Kernel
$(\{1,2\},\{1\})$	A	$CE + \bar{D}\bar{E}$
$(\{3,4\},\{2\})$	B	$\bar{D}\bar{E} + CE$
$(\{1,4\},\{3,7\})$	CE	$A + B$
$(\{2,3\},\{6,8\})$	$\bar{D}\bar{E}$	$A + B$
$(\{1,4,5\},\{7\})$	E	$AC + BC + D$
$(\{2,3,6\},\{8\})$	$\bar{E}$	$A\bar{D} + B\bar{D} + \bar{C}$

we got the same set of kernels as was obtained in (i).

(iii) Quick factor of X based on first level-0 kernel found

$L(A) = 2 > 1$

$C = A$

Then, we call the procedure one-level-0-Kernel

on $\frac{X}{A} = CE + \bar{D}\bar{E}$

So, the first level-0 kernel found is $CE + \bar{D}\bar{E}$

$(h, r) = \text{Divide}(X, K = CE + \bar{D}\bar{E})$

$h = A + B$

$r = DE + \bar{C}\bar{E}$

Calling Quick factor on h and r will return them as is.

Thus, X is factored as follows:

$X = (A+B)(\bar{D}\bar{E} + CE) + DE + \bar{C}\bar{E}$ 10 literals

The same result is produced by sis using the command factor -q X.

Q3. $X = ABCD + AB\bar{C}\bar{D} + ABE\bar{F} + AB\bar{E}F + ABG$
$+ ABH + ABK + ABL + \bar{C}DG + C\bar{D}G$
$+ K\bar{G}\bar{H} + L\bar{G}\bar{H}$ $\quad\underline{\underline{40}}$ literals

(i) Double-cube divisors

Double-cube divisor	Base	Weight
$CD + \bar{C}\bar{D}$	AB	$2\times4-2-4+3=5$
$CD + E\bar{F}$	AB	$1\times4-1-4+2=1$
$CD + \bar{E}F$	AB	$1\times4-1-4+2=1$
$CD + G$	AB	$1\times3-1-3+2=1$
$CD + H$	AB	$1\times3-1-3+2=1$
$CD + K$	AB	$1\times3-1-3+2=1$
$CD + L$	AB	$1\times3-1-3+2=1$
$ABC + \bar{C}G$	D	$1\times5-1-5+1=0$
$ABD + \bar{D}G$	C	$1\times5-1-5+1=0$
$\bar{C}\bar{D} + E\bar{F}$	AB	$1\times4-1-4+2=1$
$\bar{C}\bar{D} + \bar{E}F$	AB	$1\times4-1-4+2=1$
$\bar{C}\bar{D} + G$	AB	$1\times3-1-3+2=1$
$\bar{C}\bar{D} + H$	AB	$1\times3-1-3+2=1$
$\bar{C}\bar{D} + K$	AB	$1\times3-1-3+2=1$
$\bar{C}\bar{D} + L$	AB	$1\times3-1-3+2=1$
$AB\bar{D} + DG$	$\bar{C}$	$1\times5-1-5+1=0$
$AB\bar{C} + CG$	$\bar{D}$	$1\times5-1-5+1=0$

Double-cube divisor	Base	Weight
$E\bar{F}+\bar{E}F$	AB	$1\times4-1-4+2=1$
$E\bar{F}+G$	AB	$1\times3-1-3+2=1$
$E\bar{F}+H$	AB	$1\times3-1-3+2=1$
$E\bar{F}+K$	AB	$1\times3-1-3+2=1$
$E\bar{F}+L$	AB	$1\times3-1-3+2=1$
$\bar{E}F+G$	AB	$1\times3-1-3+2=1$
$\bar{E}F+H$	AB	$1\times3-1-3+2=1$
$\bar{E}F+K$	AB	$1\times3-1-3+2=1$
$\bar{E}F+L$	AB	$1\times3-1-3+2=1$
$G+H$	AB	$1\times2-1-2+2+2=3$
$G+K$	AB	$1\times2-1-2+2=1$
$G+L$	AB	$1\times2-1-2+2=1$
$AB+\bar{C}D$	G	$1\times4-1-4+1=0$
$AB+C\bar{D}$	G	$1\times4-1-4+1=0$
$H+K$	AB	$1\times2-1-2+2=1$
$H+L$	AB	$1\times2-1-2+2=1$
$K+L$	$AB, \bar{G}\bar{H}$	$2\times2-2-2+4=4$
$AB+\bar{G}\bar{H}$	K, L	$2\times4-2-4+2=4$
$\bar{C}D+C\bar{D}$	G	$2\times4-2-4+3=5$

(ii) Fast Extraction

From part (i), we can see that $W_{dmax} = 5$
and either of the douple-cube divisors
$CD + \bar{C}\bar{D}$ or $\bar{C}D + C\bar{D}$ can be selected.
However, we can see that the single-cube
divisor AB has a weight $= 8 - 2 = 6$.
Thus, the single-cube divisor AB is extracted
first and the resulting network after extraction
is as follows:

$$[1] = AB$$
$$X = [1]CD + [1]\bar{C}\bar{D} + [1]E\bar{F} + [1]\bar{E}F + [1]G + [1]H + [1]K + [1]L + \bar{C}DG + C\bar{D}G + K\bar{G}\bar{H} + L\bar{G}\bar{H}$$

$\underline{\underline{34}}$ literals

The same set of double-cube divisors exists
with the base replaced from AB to [1]
and hence the weight is reduced by 1 for
each double-cube divisor that has a base AB.

Thus, $W_{dmax} = 4$ and either of the double-cube
divisors $CD + \bar{C}\bar{D}$ or $\bar{C}D + C\bar{D}$ is extracted.
We will extract $\bar{C}D + C\bar{D}$, and the resulting
network after extraction becomes:

$$[1] = AB$$
$$[2] = \bar{C}D + C\bar{D}$$
$$X = [1]\overline{[2]} + [1]E\bar{F} + [1]\bar{E}F + [1]G + [1]H + [1]K + [1]L + [2]G + K\bar{G}\bar{H} + L\bar{G}\bar{H}$$

$\underline{\underline{30}}$ literals

Next, the set of double-cube divisors is computed and the one with the highest weight is extracted. We will show only those with a positive weight.

Double-cube divisor	Base	Weight
$G+H$	[1]	$1\times 2-1-2+1+2=2$
$K+L$	$[1], \overline{G}\overline{H}$	$2\times 2-2-2+3=3$
$[1]+\overline{G}\overline{H}$	K, L	$2\times 3-2-3+2=3$

Either of the double-cube divisors $K+L$ or $[1]+\overline{G}\overline{H}$ can be extracted. We will extract $[1]+\overline{G}\overline{H}$ and the resulting network after extraction is as follows:

$$[1] = AB \qquad [2] = \overline{C}D + C\overline{D}$$

$$[3] = [1] + \overline{G}\overline{H}$$

$$X = [1]\overline{[2]} + [1]E\overline{F} + [1]\overline{E}F + [1]G + [1]H + K[3] + L[3] + [2]G$$

$\underline{\underline{27}}$ literals

The only remaining double-cube divisor with a positive weight is $G+H$ with weight $=1$. The resulting network after its extraction is:

$$[1] = AB \qquad [2] = \overline{C}D + C\overline{D}$$

$$[3] = [1] + \overline{[4]} \qquad [4] = G + H$$

$$X = [1]\overline{[2]} + [1]E\overline{F} + [1]\overline{E}F + [1][4] + K[3] + L[3] + [2]G$$

$\underline{\underline{26}}$ literals

Sis produced the same result.

Q4.

$$D = A + B$$

$$E = A\bar{C} + BC$$

$$F = DE + \bar{A}C\bar{D}$$

(i)

$$\begin{aligned} SDC_D &= D \oplus (A+B) \\ &= D\,(\overline{A+B}) + \bar{D}\,(A+B) \\ &= D\bar{A}\bar{B} + \bar{D}A + \bar{D}B \end{aligned}$$

$$\begin{aligned} SDC_E &= E \oplus (A\bar{C} + BC) \\ &= E\,(\overline{A\bar{C} + BC}) + \bar{E}\,(A\bar{C} + BC) \\ &= E\,(\overline{A\bar{C}} + \bar{B}C) + \bar{E}\,(A\bar{C} + BC) \\ &= E\bar{A}\bar{C} + E\bar{B}C + \bar{E}A\bar{C} + \bar{E}BC \end{aligned}$$

(ii) Cut including all inputs of F is $\{A, C, D, E\}$.

$$\begin{aligned} CDC_{cut} &= SDC_D + SDC_E \\ &= D\bar{A}\bar{B} + \bar{D}A + \bar{D}B + E\bar{A}\bar{C} + E\bar{B}C \\ &\quad + \bar{E}A\bar{C} + \bar{E}BC \end{aligned}$$

We then eliminate B since it is not in the cut.

$$\begin{aligned} CDC_{cut,B} &= \bar{D}A + \bar{D} + E\bar{A}\bar{C} + \bar{E}A\bar{C} + \bar{E}C \\ &= \bar{D} + E\bar{A}\bar{C} + \bar{E}A\bar{C} + \bar{E}C \end{aligned}$$

$$CDC_{cut,\bar{B}} = D\bar{A} + \bar{D}A + E\bar{A}\bar{C} + EC + \bar{E}A\bar{C}$$

$$\begin{aligned} CDC_{cut,B} \cdot CDC_{cut,\bar{B}} &= \bar{D}A + \bar{D}E\bar{A}\bar{C} + \bar{D}EC + \bar{D}\bar{E}A\bar{C} \\ &\quad + ED\bar{A}\bar{C} + E\bar{A}\bar{C} + \bar{D}A\bar{E}\bar{C} + \bar{E}A\bar{C} \\ &\quad + \bar{E}CD\bar{A} + \bar{E}C\bar{D}A \end{aligned}$$

$$\Rightarrow \quad CDC_{cut} = \bar{D}A + \bar{D}\cancel{E}\bar{A}\bar{C} + \bar{D}EC + E\bar{A}\bar{C} + \bar{E}A\bar{C} + \bar{E}CD\bar{A}$$

(iii) Simplifying F using CDC

AC \ DE	00	01	11	10
00	0	x	x	0
01	1	x	1	x
11	x	x	1	0
10	x	x	1	x

$$\Rightarrow \quad F = E + C\bar{D}$$

(iv) $ODC_D = F_D \,\bar{\oplus}\, F_{\bar{D}} = E \,\bar{\oplus}\, (E+C)$

$$= E(E+C) + \bar{E}\,\bar{E}\bar{C}$$

$$= E + \bar{E}\bar{C} = E + \bar{C}$$

$$= A\bar{C} + BC + \bar{C} = B + \bar{C}$$

Simplifying D using ODC_D;

since D does not depend on variable C, we can eliminate by consensus

$$ODC_{D,\bar{C}} \cdot ODC_{D,C} = 1 \cdot B = B$$

A \ B	0	1
0	0	x
1	1	x

$$\Rightarrow \quad D = A$$

(v) $ODC_E = F_E \overline{\oplus} F_{\overline{E}} = 1 \overline{\oplus} C\overline{D} = C\overline{D}$

$= C\overline{A}\overline{B}$

Simplifying E using ODC_E ;

A \ BC	00	01	11	10
0	0	x	1	0
1	1	0	1	1

No simplification

$\Rightarrow E = A\overline{C} + BC$

sis produced the same result for F and E but it did not simplify D while it could be simplified as we did.

SIS Results:

Q1 (ii)

```
    {X} = A B E F + A B G' H' + A' B' C' D' + C D E F + C D G' H'
    {Y} = A B + C D
sis> print_stats
hw3q1.eqn  pi= 8  po= 2  node=  2  latch= 0  lits(sop)=  24  lits(ff)=  16
sis> resub -d
sis> print
    {X} = A' B' C' D' + E F {Y} + G' H' {Y}
    {Y} = A B + C D
sis> print_stats
hw3q1.eqn  pi= 8  po= 2  node=  2  latch= 0  lits(sop)=  14  lits(ff)=  13
```

Q2 (iii)

```
    {X} = A C E + A D' E' + B C E + B D' E' + C' E' + D E
sis> print_stats
hw3q2.eqn  pi= 6  po= 1  node=  1  latch= 0  lits(sop)=  16  lits(ff)=  10
sis> factor -q X
sis> print_factor
   {X} = (D' E' + C E) (B + A) + C' E' + D E
sis> print_stats -f
hw3q2.eqn  pi= 6  po= 1  node=  1  latch= 0  lits(sop)=  16  lits(ff)=  10
```

Q3 (ii)

```
        {X} = A B C D + A B C' D' + A B E F' + A B E' F + A B G + A B H + A B K + A
 B L + C D' G + C' D G + G' H' K + G' H' L
sis> print_stats
hw3q3.eqn  pi=10  po= 1  node=  1  latch= 0  lits(sop)=  40  lits(ff)=  23
sis> fx
sis> print
    {X} = E F' [1] + E' F [1] + G [2] + K [3] + L [3] + [1] [2]' + [1] [4]
    [1] = A B
    [2] = C D' + C' D
    [3] = [1] + [4]'
    [4] = G + H
sis> print_stats
hw3q3.eqn  pi=10  po= 1  node=  5  latch= 0  lits(sop)=  26  lits(ff)=  22
```

Q4 (vi)

```
    {F} = A' C D' + D E
    D = A + B
    E = A C' + B C
sis> print_stats
hw3q4.eqn  pi= 3  po= 1  node=  3  latch= 0  lits(sop)=  11  lits(ff)=  11
sis> full_simplify
sis> print
    {F} = C D' + E
    D = A + B
    E = A C' + B C
sis> print_stats
hw3q4.eqn  pi= 3  po= 1  node=  3  latch= 0  lits(sop)=   9  lits(ff)=   9
```