## COE 405, Term 041

## COE 561 Digital System Design and Synthesis

## HW\# 3

## Due date: Tuesday, Nov. 23

Q.1. Consider the logic network defined by the following expressions:

$$
\begin{aligned}
& x=a d^{\prime}+a^{\prime} b^{\prime}+a^{\prime} d^{\prime}+b c+b d^{\prime}+a c \\
& y=a+b \\
& z=a^{\prime} c^{\prime}+a^{\prime} d^{\prime}+b^{\prime} c^{\prime}+b^{\prime} d^{\prime}+e \\
& u=a^{\prime} c+a^{\prime} d+b^{\prime} d+e^{\prime}
\end{aligned}
$$

(i) Substitute $y$ into $f_{x}$ by performing the algebraic division $f_{x} / f_{y}$. Show all steps. Determine the number of literals saved. Compare your solution with the result obtained by running the sis command resub -d (resubstitute without complement).
(ii) Compute all kernels and co-kernels of z and u . Extract a multiple-cube subexpression common to $f_{z}$ and $f_{u}$. Show all the steps. Determine the number of literals saved. Compare your solution with the result obtained by running the sis command $\boldsymbol{g k x}$.
Q.2. Consider the logic network defined by the following expressions:

$$
\begin{aligned}
& x=a b c f+e f c+d e \\
& y=a c d e f+b d e f \\
& z=b c d+a c f
\end{aligned}
$$

(i) Determine the cube-variable matrix and all prime rectangles.
(ii) Determine the minimum-literal network that can be derived by cube extraction. Determine the number of literals saved. Compare your solution with the result obtained by running the sis command $\boldsymbol{g c x}$.
Q.3. Consider the following function:

$$
x=b d+c d+b e+c e+a f d+a f e+a b g+a c g+a f g
$$

(i) Find a quick factor of $x$ by using the first level-0 kernel found. Assume that input variables are sorted in lexicographic order. Determine the number of literals obtained. Compare your solution with the result obtained by running the sis commands factor -q $\mathbf{x}$; print_factor; print_stats -f.
(ii) Find a good factor of $x$ based on using the best kernel. Determine the number of literals obtained. Compare your solution with the result obtained by running the sis commands factor -g x; print_factor; print_stats -f.
(iii) Decompose $x$ using quick decomposition based on using the first level-0 kernel found. Determine the number of literals obtained. Compare your solution with the result obtained by running the sis commands decomp -q. Compare your solution to the factored solution obtained in (i).
(iv) Decompose $x$ using good decomposition based on using based on using the best kernel. Determine the number of literals obtained. Compare your solution with the result obtained by running the sis commands decomp -g. Compare your solution to the factored solution obtained in (ii).
(v) Run the sis command $f x$ and compare the solution obtained to that obtained in (iii) and (iv).
Q.4. Consider the logic network defined by the following expressions::

$$
\begin{aligned}
& x=a^{\prime} b^{\prime} c d+a^{\prime} b^{\prime} c^{\prime} d^{\prime}+a c d^{\prime}+a c^{\prime} d+b c d^{\prime}+b c^{\prime} d \\
& y=a^{\prime} b^{\prime} c+a^{\prime} b^{\prime} d+a c^{\prime} d^{\prime}+b c^{\prime} d^{\prime}
\end{aligned}
$$

(i) Compute all double-cube divisors of $x$ and $y$ along with their bases.
(ii) Apply the fast extraction algorithm based on extracting double-cube divisors along with complements or single-cube divisors with two-literals. Show all steps of the algorithm. Determine the number of literals saved. Compare your solution with the result obtained by running the sis commands $f x$.
(iii) Run the sis commands gkx followed by gcx and compare the solution obtained to that obtained in (ii).
Q.5. Consider the logic network defined by the following expressions:

$$
\begin{aligned}
& d=b^{\prime} \\
& f=(a+d)^{\prime} \\
& e=(c a)^{\prime} \\
& x=f e \\
& y=d \oplus e
\end{aligned}
$$

Inputs are $\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ and output are $\{\mathrm{x}, \mathrm{y}\}$.
(i) Assume $\mathrm{CDC}_{\text {in }}=$ abc'. Compute $\mathrm{CDC}_{\text {out }}$.
(ii) Compute the ODC sets for all internal and input vertices, assuming that the outputs are fully observable.
Q.6. Consider the logic network defined by the following expressions:

$$
\begin{aligned}
& u=a b^{\prime}+b c \\
& x=a u+b \\
& y=a^{\prime} u^{\prime}+c^{\prime}
\end{aligned}
$$

Inputs are $\{a, b, c\}$ and outputs are $\{x, y\}$.
(i) Compute the SDC set and ODC set for node $u$.
(ii) Simplify $u$ using its ODC set.
(iii) Simplify $x$ and $y$ using the SDC set.
(iv) Apply the sis command full_simplify and compare the solution obtained to what you obtained as a result of applying steps (ii) and (iii).
Q.7. Consider the logic network defined by the following expressions:

$$
\begin{aligned}
& o=e g \\
& e=(a+b)^{\prime} \\
& g=d+c \\
& d=a b
\end{aligned}
$$

Inputs are $\{a, b, c\}$ and output is $\{o\}$.
(i) Consider the network perturbation replacing d by 0 , i.e. $\delta=\mathrm{a} \mathrm{b} \oplus 0=\mathrm{a}$ b. Determine if this perturbation is feasible or not.
(ii) Is the fault $\boldsymbol{d}$ stuck-at-0 testable. If it is testable find all tests detecting the fault. If the fault is untestable optimize the network by eliminating redundancy.
(iii) Is the fault $\boldsymbol{e}$ stuck-at- $\mathbf{1}$ testable. If it is testable find all tests detecting the fault. If the fault is untestable optimize the network by eliminating redundancy.

