HW #2 Solution

Q1. $F^{ON} = \sum m(5, 7, 8, 10, 12, 14, 15)$

$F^{DC} = \sum m(0, 1, 4)$

(i) $F^{ON} \cup F^{DC} = \bar{a}b\bar{c}d + \bar{a}bcd + a\bar{b}\bar{c}\bar{d} + a\bar{b}c\bar{d} + ab\bar{c}\bar{d} + abc\bar{d} + abcd + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + \bar{a}b\bar{c}\bar{d}$

$= \bar{a}[b\bar{c}d + bcd + \bar{b}\bar{c}\bar{d} + \bar{b}\bar{c}d + b\bar{c}\bar{d}] + a[\bar{b}\bar{c}\bar{d} + \bar{b}c\bar{d} + b\bar{c}\bar{d} + bc\bar{d} + bcd]$

$= \bar{a}\,[\bar{b}\,[\bar{c}\bar{d} + \bar{c}d] + b\,[\bar{c}d + cd + \bar{c}\bar{d}]] + a\,[\bar{b}\,[\bar{c}\bar{d} + c\bar{d}] + b\,[\bar{c}\bar{d} + c\bar{d} + cd]]$

$= \bar{a}\,[\bar{b}\,[\bar{c}\,[\bar{d} + d] + c\,[0]] + b\,[\bar{c}\,[d + \bar{d}] + c\,[d]]] + a\,[\bar{b}\,[\bar{c}\,[\bar{d}] + c\,[\bar{d}]] + b\,[\bar{c}\,[\bar{d}] + c\,[\bar{d} + d]]]$

$\Rightarrow F^{off} = \bar{a}\,[\bar{b}\,[\bar{c}\,[0] + c\,[1]] + b\,[\bar{c}\,[0] + c\,[\bar{d}]]] + a\,[\bar{b}\,[\bar{c}\,[d] + c\,[d]] + b\,[\bar{c}\,[d] + c\,[0]]]$

$= \bar{a}\bar{b}c + \bar{a}bc\bar{d} + a\bar{b}\bar{c}d + a\bar{b}cd + ab\bar{c}d$

(ii) Expand Procedure:

F^{off}:

	a	b	c	d
$\bar{a}\bar{b}c$	10	10	01	11
$\bar{a}bc\bar{d}$	10	01	01	10
$a\bar{b}\bar{c}d$	01	10	10	01
$a\bar{b}cd$	01	10	01	01
$ab\bar{c}d$	01	01	10	01

We next compute the weights of the on-set;

	a	b	c	d	weights
$\bar{a}b\bar{c}d$	10	01	10	01	13
$\bar{a}bcd$	10	01	01	01	14
$a\bar{b}\bar{c}\bar{d}$	01	10	10	10	14
$a\bar{b}c\bar{d}$	01	10	01	10	15
$ab\bar{c}\bar{d}$	01	01	10	10	17
$abc\bar{d}$	01	01	01	10	18
$abcd$	01	01	01	01	17
	25	25	34	43	

We need to expand cubes in ascending order of weight

Thus, we expand $\bar{a}b\bar{c}d$ first.

* Expand $\bar{a}b\bar{c}d$:

Free $= \{2, 3, 6, 7\}$

Since there is no column with all 0's in F^{off}, no column can always be raised.

Intersecting with the off-set, column 2 can't be raised.

Free $= \{3, 6, 7\}$, overexpanded cube $= \bar{a}$.

we only need to check $\bar{a}bcd$ for being feasibly covered with $\bar{a}b\bar{c}d$.

Supercube $(\bar{a}b\bar{c}d, \bar{a}bcd) = \bar{a}bd$ (feasible)

The resulting expanded cube is $\bar{a}bd$ and free $= \{3, 7\}$.

Intersecting with the off-set, it can be concluded that none of them can be raised.

The covered minterm $\bar{a}bcd$ is removed.

* Expand $a\bar{b}\bar{c}\bar{d}$:

Free $= \{1, 4, 6, 8\}$

Intersecting with the off-set implies that column 8 cannot be raised. Thus, Free $= \{1, 4, 6\}$ and overexpanded cube is $\bar{d}$.

we need to check the cubes $ab\bar{c}\bar{d}$, $abc\bar{d}$ and $\bar{a}bc\bar{d}$ for being feasibly covered.

Supercube $(a\bar{b}\bar{c}\bar{d}, ab\bar{c}\bar{d}) = a\bar{c}\bar{d}$ (feasible)
Supercube $(a\bar{b}\bar{c}\bar{d}, abc\bar{d}) = a\bar{d}$ (feasible)
Supercube $(a\bar{b}\bar{c}\bar{d}, a\bar{b}c\bar{d}) = a\bar{b}\bar{d}$ (feasible)

The expanded cube $a\bar{d}$ is selected and Free = {1} which cannot be raised.

The covered minterms $ab\bar{c}\bar{d}$, $abc\bar{d}$ and $a\bar{b}c\bar{d}$ are removed.

* Expand abcd:

Free = {1, 3, 5, 7}

Intersecting with the off-set, it is concluded that columns 3 and 5 cannot be raised. Thus, Free = {1, 7} and overexpanded cube is bc.

Since there are no feasibly covered cubes and the overexpanded cube does not intersect any remaining cube in the on-set, we need to find the largest prime implicant covering abcd.

There are two prime implicants covering abcd which are bcd and abc. We select bcd.

Thus, the expanded cover is { $\bar{a}bd$, $a\bar{d}$, bcd }.

In comparison with Espresso tool, the same cover is obtained.

(iii) Irredundant Procedure:

The expanded cover is $\{\bar{a}bd, a\bar{d}, bcd\}$.

First, we need to check whether each of these cubes is relatively essential.

- Check if $\{a\bar{d}, bcd, \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}\bar{b}\bar{c}d, \bar{a}b\bar{c}\bar{d}\}$ covers $\bar{a}bd$.

  $\{a\bar{d}, bcd, \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}\bar{b}\bar{c}d, \bar{a}b\bar{c}\bar{d}\}_{\bar{a}bd} = \{0, c, 0, 0, 0\}$

  Since the cofactor is not tautology, this implies that $\bar{a}bd$ is relatively essential.

- check if $\{\bar{a}bd, bcd, \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}\bar{b}\bar{c}d, \bar{a}b\bar{c}\bar{d}\}$ covers $a\bar{d}$.

  $\{\bar{a}bd, bcd, \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}\bar{b}\bar{c}d, \bar{a}b\bar{c}\bar{d}\}_{a\bar{d}} = \{0, 0, 0, 0, 0\}$.

  Since the cofactor is not tautology, this implies that $a\bar{d}$ is relatively essential.

- check if $\{\bar{a}bd, a\bar{d}, \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}\bar{b}\bar{c}d, \bar{a}b\bar{c}\bar{d}\}$ covers bcd.

  $\{\bar{a}bd, a\bar{d}, \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}\bar{b}\bar{c}d, \bar{a}b\bar{c}\bar{d}\}_{bcd} = \{\bar{a}, 0, 0, 0, 0\}$.

  Since the cofactor is not tautology, this implies that bcd is relatively essential

Since all implicants are relatively essential, then the cover is irredundant.

This is consistent with what is produced by Espresso tool.

(iv) Essential Prime Implicants:

$F = \bar{a}bd + a\bar{d} + bcd$

$F^{DC} = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + \bar{a}b\bar{c}\bar{d}$

* Checking $\bar{a}bd$:

$G = \{a\bar{d},\ bcd,\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}$

$G \# \bar{a}bd = \{a\bar{d},\ abcd,\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}$

$H = \text{Consensus}(G\#\bar{a}bd,\ \bar{a}bd)$

$= \{bcd,\ \bar{a}\bar{c}d,\ \bar{a}b\bar{c}\}$

Then, we check if $H \cup F^{DC}$ covers $\bar{a}bd$.

$\Rightarrow \{bcd,\ \bar{a}\bar{c}d,\ \bar{a}b\bar{c},\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}_{\bar{a}bd}$

$= \{c,\ \bar{c},\ \bar{c},\ 0,\ 0,\ 0\} \Rightarrow$ tautology

Since it is tautology, this means that $\bar{a}bd$ is not an essential prime implicant.

* Checking $a\bar{d}$:

$G = \{\bar{a}bd,\ bcd,\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}$

$G \# a\bar{d} = \{\bar{a}bd,\ bcd,\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}$

$H = \text{Consensus}(G \# a\bar{d},\ a\bar{d})$

$= \{abc,\ \bar{b}\bar{c}\bar{d},\ b\bar{c}\bar{d}\}$

Then, we check if $H \cup F^{DC}$ covers $a\bar{d}$.

$\Rightarrow \{abc,\ \bar{b}\bar{c}\bar{d},\ b\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}_{a\bar{d}} = \{bc,\ \bar{b}\bar{c},\ b\bar{c}\}$

Since it is not tautology, this means that $a\bar{d}$ is an essential prime implicant.

* Checking bcd:

$G = \{\bar{a}bd,\ a\bar{d},\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}$

$G \# bcd = \{\bar{a}b\bar{c}d,\ a\bar{d},\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}$

$H = \text{Consensus}(G \# bcd,\ bcd)$
$\quad = \{\bar{a}bd,\ abc\}$

Then, we check if $H \cup F^{DC}$ covers bcd.

$\Rightarrow \{\bar{a}bd,\ abc,\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}_{bcd}$

$= \{\bar{a},\ a,\ 0,\ 0,\ 0\} \Rightarrow$ tautology

Since it is tautology, this means that bcd is not an essential prime implicant.

Thus, only $a\bar{d}$ is an essential prime implicant.

Thus, the cover F becomes $\{\bar{a}bd,\ bcd\}$ and F^{DC} becomes $\{a\bar{d},\ \bar{a}\bar{b}\bar{c}\bar{d},\ \bar{a}\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d}\}$.

(v) Reduce Procedure:

First, we compute the weight of each implicant in the cover.

	a	b	c	d	weight
$\bar{a}bd$	10	01	11	01	9
bcd	11	01	01	01	9
	21	02	12	02	

Since both have the same weight, we will start reducing $\bar{a}bd$.

- Reduce $\bar{a}bd$:

$\alpha = \bar{a}bd$

$Q = \{F \cup F^{DC}\} - \alpha = \{ bcd, a\bar{d}, \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}\bar{b}\bar{c}d, \bar{a}b\bar{c}\bar{d} \}$

$Q_\alpha = \{ c, 0, 0, 0, 0 \} = c$

$\overline{Q_\alpha} = \bar{c}$

$\tilde{\alpha} = \alpha \cap \text{supercube}(\overline{Q_\alpha}) = \bar{a}bd \cap \bar{c} = \bar{a}b\bar{c}d$

- Reduce bcd:

$\alpha = bcd$

$Q = \{ \bar{a}b\bar{c}d, a\bar{d}, \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}\bar{b}\bar{c}d, \bar{a}b\bar{c}\bar{d} \}$

$Q_\alpha = \{ 0, 0, 0, 0, 0 \} = 0$

$\overline{Q_\alpha} = 1$

$\tilde{\alpha} = bcd \cap 1 = bcd$

Thus, the reduced cover is $\{ \bar{a}b\bar{c}d, bcd \}$.

(vi) Expand Procedure on reduced covers:

$F = \bar{a}b\bar{c}d + bcd$

we compute the weigh of each cube.

	a	b	c	d	weight
$\bar{a}b\bar{c}d$	10	01	10	01	7
bcd	11	01	01	01	8
	21	02	11	02	

Thus, we expand $\bar{a}b\bar{c}d$ first.

* Expand $\bar{a}b\bar{c}d$:

Free = $\{2, 3, 6, 7\}$

Column 2 cannot be raised and hence Free = $\{3, 6, 7\}$

and the overexpanded cube is $\bar{a}$.

None of the cubes is feasibly covered and none are covered by the overexpanded cube. Thus, the cube is expanded to find the largest prime implicant covering the cube. Thus, the cube is expanded to $\bar{a}\bar{c}$.

* Expand bcd:

  Free = $\{3, 5, 7\}$

  Since none of the columns can be raised, the cube cannot be expanded.

Thus, the expanded cover is $\{\bar{a}\bar{c}, bcd\}$.

This is consistent with the results produced by Espresso.

Q2. $F = \bar{A}\bar{C} + AD + \bar{C}D + \bar{A}B\bar{D} + ABC + BC\bar{D}$

(i) Relatively essential set of cubes E^r:

- Checking $\bar{A}\bar{C}$

  $\alpha = \bar{A}\bar{C}$

  $F - \alpha = \{AD, \bar{C}D, \bar{A}B\bar{D}, ABC, BC\bar{D}\}$

  $\{F - \alpha\}_{\bar{A}\bar{C}} = \{0, D, B\bar{D}, 0, 0\}$

  Since it is not tautology, $\bar{A}\bar{C} \in E^r$.

- Checking AD

$\alpha = AD$

$F-\alpha = \{\bar{A}\bar{C}, \bar{C}D, \bar{A}B\bar{D}, ABC, BC\bar{D}\}$

$\{F-\alpha\}_{AD} = \{0, \bar{C}, 0, BC, 0\}$

Since it is not tautology, $AD \in E^r$.

- Checking $\bar{C}D$

$\alpha = \bar{C}D$

$F-\alpha = \{\bar{A}\bar{C}, AD, \bar{A}B\bar{D}, ABC, BC\bar{D}\}$

$\{F-\alpha\}_{\bar{C}D} = \{\bar{A}, A, 0, 0, 0\}$

Since it is tautology, $\bar{C}D \notin E^r$.

- Checking $\bar{A}B\bar{D}$

$\alpha = \bar{A}B\bar{D}$

$F-\alpha = \{\bar{A}\bar{C}, AD, \bar{C}D, ABC, BC\bar{D}\}$

$\{F-\alpha\}_{\bar{A}B\bar{D}} = \{\bar{C}, 0, 0, 0, C\}$

Since it is tautology, $\bar{A}B\bar{D} \notin E^r$.

- Checking ABC

$\alpha = ABC$

$F-\alpha = \{\bar{A}\bar{C}, AD, \bar{C}D, \bar{A}B\bar{D}, BC\bar{D}\}$

$\{F-\alpha\}_{ABC} = \{0, D, 0, 0, \bar{D}\}$

Since it is tautology, $ABC \notin E^r$.

- Checking $BC\bar{D}$

$\alpha = BC\bar{D}$

$F-\alpha = \{\bar{A}\bar{C}, AD, \bar{C}D, \bar{A}B\bar{D}, ABC\}$

$\{F-\alpha\}_{BC\bar{D}} = \{0, 0, 0, \bar{A}, A\}$

Since it is tautology, $BC\bar{D} \notin E^r$.

Thus, $E^r = \{AD, \bar{A}\bar{C}\}$.

(ii) Totally redundant, R^t, and partially redundant R^p:

$F - E^r = \{\bar{C}D, \bar{A}B\bar{D}, ABC, BC\bar{D}\}$, $E^r = \{\bar{A}\bar{C}, AD\}$

- Checking $\bar{C}D$

  $E^r_{\bar{C}D} = \{\bar{A}, A\}$

  Since it is tautology, $\bar{C}D \in R^t$

- Checking $\bar{A}B\bar{D}$

  $E^r_{\bar{A}B\bar{D}} = \{\bar{C}, 0\}$

  Since it is not tautology, $\bar{A}B\bar{D} \in R^p$

- Checking ABC

  $E^r_{ABC} = \{0, D\}$

  Since it is not tautology, $ABC \in R^p$

- Checking $BC\bar{D}$

  $E^r_{BC\bar{D}} = \{0, 0\}$

  Since it is not tautology, $BC\bar{D} \in R^p$

Thus, $R^t = \{\bar{C}D\}$ and $R^p = \{\bar{A}B\bar{D}, ABC, BC\bar{D}\}$.

(iii) We formulate a covering problem

	$\bar{A}B\bar{D}$	ABC	$BC\bar{D}$
$\bar{A}B\bar{D}$	1	0	1
ABC	0	1	1
$BC\bar{D}$	1	1	1

The cube $BC\bar{D}$ is selected and hence the irredundant cover is $F = \bar{A}\bar{C} + AD + BC\bar{D}$.

(iv) This is consistent with what is produced by Espresso.