COE 561, Term 111

Digital System Design and Synthesis

HW# 2 Solution

Q.1. Consider the function *F(A,B,C,D)* with the following ON-set and DC-set:

$F^{ON}= \sum m(0, 2, 3, 4, 5, 7, 8, 10, 12, 13, 15)$

$F^{DC}= \sum m(1, 11)$

Apply the EXPAND procedure on the given cover using Espresso heuristics and show the obtained expanded cover. Compare your solution with the result obtained by ESPRESSO tool.

Q.2. Consider the function F(A, B, C, D) with **ON-SET=Σm(0, 4, 5, 7, 8, 12, 13, 15)** and **DC-SET=Σm(1, 3, 9, 14)**.

(i) A cover of the function is given by F = C' + BD. **Reduce** the cube **C'** using Theorem 7.4.1.

(ii) Use Corollary 7.4.1 to check if the implicant **BD** is an **essential** prime implicant.

Q.3. Consider the following cover of a function F*(A,B,C,D)*

$$F = \overline{A}\,\overline{B} + \overline{A}D + \overline{B}\,\overline{D} + BCD + ABC + AC\overline{D}$$

(i) Determine the relatively essential set of cubes, E^r.

(ii) Determine the totally redundant, R^t, and partially redundant, R^p, sets of cubes.

(iii) Find a subset of R^p that, together with E^r, covers the function by solving a covering problem.

(iv) Compare your solution with the result obtained by ESPRESSO tool.

HW#2 Solution

Q1. $F^{ON} = \sum m(0, 2, 3, 4, 5, 7, 8, 10, 12, 13, 15)$

$F^{DC} = \sum m(1, 11)$

$F^{off} = a\bar{b}\bar{c}d + abc\bar{d} + \bar{a}bc\bar{d}$

We first compute the weights of the ON-set:

	a	b	c	d	weight
$\bar{a}\bar{b}\bar{c}\bar{d}$	10	10	10	10	23
$\bar{a}\bar{b}cd$	10	10	01	01	21
$\bar{a}\bar{b}c\bar{d}$	10	10	01	10	22
$\bar{a}b\bar{c}\bar{d}$	10	01	10	10	24
$\bar{a}b\bar{c}d$	10	01	10	01	23
$\bar{a}bcd$	10	01	01	01	22
$ab\bar{c}\bar{d}$	01	01	10	10	23
$ab\bar{c}d$	01	01	10	01	22
$abcd$	01	01	01	01	21
$a\bar{b}\bar{c}\bar{d}$	01	10	10	10	22
$a\bar{b}c\bar{d}$	01	10	01	10	21
	6 5	5 6	6 5	6 5	

We have 3 cubes with the same low weight (21), so we can select any one of them. Let us select the minterm $a\bar{b}c\bar{d}$ similar to what is selected by Espresso.

- <u>Expand $a\bar{b}c\bar{d}$</u>:

  Free = $\{1, 4, 5, 8\}$

  Since there is no column with all 0's in F^{off}, no column can always be raised.

  Intersection with the off-set implies that column 4 can't be raised $\Rightarrow$ Free= $\{1, 5, 8\}$ and over expanded cube = $\bar{b}$.

  We only need to check $\bar{a}\bar{b}\bar{c}\bar{d}$, $\bar{a}\bar{b}cd$, $\bar{a}\bar{b}c\bar{d}$ and $a\bar{b}\bar{c}\bar{d}$ for being feasibly covered.

  Supercube $(a\bar{b}c\bar{d}, \bar{a}\bar{b}\bar{c}\bar{d}) = \bar{b}\bar{d}$ (feasible)
  Supercube $(a\bar{b}c\bar{d}, \bar{a}\bar{b}cd) = \bar{b}c$ (feasible)
  Supercube $(a\bar{b}c\bar{d}, \bar{a}\bar{b}c\bar{d}) = \bar{b}c\bar{d}$ (feasible)
  Supercube $(a\bar{b}c\bar{d}, a\bar{b}\bar{c}\bar{d}) = a\bar{b}\bar{d}$ (feasible)

  We select $\bar{b}\bar{d}$ as it covers more cubes

  Free = $\{8\}$ which can't be reaised.

  Thus, we get the expanded cube $\bar{b}\bar{d}$ and the cubes $\bar{a}\bar{b}\bar{c}\bar{d}$, $\bar{a}\bar{b}c\bar{d}$ and $a\bar{b}\bar{c}\bar{d}$ are removed from the cover.

- <u>Expand $\bar{a}\bar{b}cd$</u>:

  Free = $\{2, 4, 5, 7\}$

  Intersection with the off-set implies that none of the columns can't be raised.

  Thus, overexpanded cube = 1 and we need to check all remaining cubes for being feasibly covered.

supercube ($\bar{a}\bar{b}cd$, $\bar{a}b\bar{c}\bar{d}$) = $\bar{a}$ (not feasible)
supercube ($\bar{a}\bar{b}cd$, $\bar{a}b\bar{c}d$) = $\bar{a}d$ (feasible)
supercube ($\bar{a}\bar{b}cd$, $\bar{a}b\,cd$) = $\bar{a}cd$ (feasible)
supercube ($\bar{a}\bar{b}cd$, $ab\bar{c}\bar{d}$) = 1 (not feasible)
supercube ($\bar{a}\bar{b}cd$, $ab\bar{c}d$) = d (not feasible)
supercube ($\bar{a}\bar{b}cd$, $abcd$) = cd (feasible)

Either $\bar{a}d$ or cd can be selected as they cover the same cubes. Let us select cd.
Free = {5, 7} which both can't be raised.
Thus, the expanded cube is cd, and the cubes $\bar{a}bcd$ and $abcd$ are removed.

The next cube selected with lowest weight is $ab\bar{c}\bar{d}$.

- Expand $ab\bar{c}\bar{d}$:

Free = {1, 3, 6, 7}
Intersection with the off-set implies that column 3 can't be raised ⇒ Free = {1, 6, 7}
and the overexpanded cube = b.
We consider the cubes $\bar{a}b\bar{c}\bar{d}$, $\bar{a}b\bar{c}d$ and $ab\bar{c}d$ for being feasibly covered.

Supercube ($ab\bar{c}\bar{d}$, $\bar{a}b\bar{c}\bar{d}$) = $b\bar{c}$ (feasible)
Supercube ($ab\bar{c}\bar{d}$, $\bar{a}b\bar{c}d$) = $b\bar{c}d$ (feasible)
Supercube ($ab\bar{c}\bar{d}$, $ab\bar{c}d$) = $ab\bar{c}$ (feasible)

$b\bar{c}$ is selected as it covers more cubes.
Free = {6} which can't be raised.
Thus, the cube is expanded to $b\bar{c}$.

The expanded cover = $\bar{b}\bar{d} + cd + b\bar{c}$
which is the same cover obtained by Espresso

```
# espresso -d -t -Dexpand hw2q1.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb y
# READ          Time was 0.00 sec, cost is c=11(11) in=44 out=11 tot=55
# COMPL         Time was 0.00 sec, cost is c=2(2) in=7 out=2 tot=9
# PLA is hw2q1.pla with 4 inputs and 1 outputs
# ON-set cost is  c=11(11) in=44 out=11 tot=55
# OFF-set cost is c=2(2) in=7 out=2 tot=9
# DC-set cost is  c=2(2) in=8 out=2 tot=10
EXPAND: 1010 1 (covered 3)
EXPAND: 0011 1 (covered 2)
EXPAND: 1101 1 (covered 3)
# EXPAND                Time was 0.00 sec, cost is c=3(0) in=6 out=3 tot=9
# READ           1 call(s) for 0.00 sec ( 0.0%)
# COMPL          1 call(s) for 0.00 sec ( 0.0%)
# EXPAND                 1 call(s) for 0.00 sec ( 0.0%)
# expand        Time was 0.00 sec, cost is c=3(0) in=6 out=3 tot=9
.i 4
.o 1
.ilb a b c d
.p 3
-0-0 1
--11 1
-10- 1
.e
# WRITE         Time was 0.00 sec, cost is c=3(0) in=6 out=3 tot=9
```

Q2. $F^{ON} = \sum m\,(0, 4, 5, 7, 8, 12, 13, 15)$

$F^{DC} = \sum m\,(1, 3, 9, 14)$

(i) $F = \overline{C} + BD$

<u>Reduce the cube $\overline{C}$:</u>

$$Q = BD + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}CD + ABC\overline{D} + A\overline{B}\,\overline{C}$$

$$Q_{\overline{C}} = BD + \overline{A}\,\overline{B}D + A\overline{B}D$$

$$= \overline{B}\,\overline{D}\,[0] + \overline{B}D\,[1] + B\overline{D}\,[0] + BD\,[1]$$

$$\overline{Q_{\overline{C}}} = \overline{B}\,\overline{D} + B\overline{D}$$

$$SC(\overline{Q_{\overline{C}}}) = \overline{D}$$

$$\Rightarrow \quad \overline{C} \cap SC(\overline{Q_{\overline{C}}}) = \overline{C}\,\overline{D}$$

Thus, the cube $\overline{C}$ is reduced to $\overline{C}\,\overline{D}$.

(ii) <u>BD check for Essential Prime Implicant:</u>

$\alpha = BD$

$G = \{\overline{C},\ \overline{A}\,\overline{B}\,\overline{C}D,\ \overline{A}\,\overline{B}CD,\ ABC\overline{D},\ A\overline{B}\,\overline{C}D\}$

$G \# \alpha = \{\overline{B}\,\overline{C},\ \overline{C}\,\overline{D},\ \overline{A}\,\overline{B}\,\overline{C}D,\ \overline{A}\,\overline{B}CD,\ ABC\overline{D},\ A\overline{B}\,\overline{C}D\}$

$H = \text{Consensus}(G \# \alpha, \alpha) = \{\overline{C}D,\ B\overline{C},\ \overline{A}\,\overline{C}D,\ \overline{A}CD,\ ABC,\ A\overline{C}D\}$

$H \cup DC = \{\bar{C}D,\ B\bar{C},\ \bar{A}\bar{C}D,\ \bar{A}CD,\ ABC,\ A\bar{C}D\}$

$\{H \cup DC\}_{\alpha} = \{\bar{C},\ \bar{C},\ \bar{A}\bar{C},\ \bar{A}C,\ AC,\ A\bar{C}\}$

$\Rightarrow$ Tautology

Thus, the prime implicant BD is <u>not</u> an essential prime implicant.

Q3. $F = \bar{A}\bar{B} + \bar{A}D + \bar{B}\bar{D} + BCD + ABC + AC\bar{D}$

(1) <u>Relatively essential set E^r</u>:

- <u>Check $\bar{A}\bar{B}$</u>:

  $\{\bar{A}D,\ \bar{B}\bar{D},\ BCD,\ ABC,\ AC\bar{D}\}_{\bar{A}\bar{B}}$
  $= \{D,\ \bar{D},\ 0,\ 0,\ 0\}$ = Tautology $\Rightarrow$ Not Rel. Ess.

- <u>Check $\bar{A}D$</u>:

  $\{\bar{A}\bar{B},\ \bar{B}\bar{D},\ BCD,\ ABC,\ AC\bar{D}\}_{\bar{A}D}$
  $= \{\bar{B},\ 0,\ BC,\ 0,\ 0\}$ = Not Taut. $\Rightarrow$ Rel. Ess.

- <u>Check $\bar{B}\bar{D}$</u>:

  $\{\bar{A}\bar{B},\ \bar{A}D,\ BCD,\ ABC,\ AC\bar{D}\}_{\bar{B}\bar{D}}$
  $= \{\bar{A},\ 0,\ 0,\ 0,\ AC\}$ = Not Taut. $\Rightarrow$ Rel. Ess.

- <u>Check BCD</u>:

  $\{\bar{A}\bar{B},\ \bar{A}D,\ \bar{B}\bar{D},\ ABC,\ AC\bar{D}\}_{BCD}$
  $= \{0,\ \bar{A},\ 0,\ A,\ 0\}$ = Taut. $\Rightarrow$ Not Rel. Ess.

- $\underline{\text{Check } ABC}$:

$\{\bar{A}\bar{B}, \bar{A}D, \bar{B}\bar{D}, BCD, AC\bar{D}\}_{ABC}$

$= \{0, 0, 0, D, \bar{D}\}$ = Taut. $\Rightarrow$ Not Rel. Ess.

- $\underline{\text{Check } AC\bar{D}}$:

$\{\bar{A}\bar{B}, \bar{A}D, \bar{B}\bar{D}, BCD, ABC\}_{AC\bar{D}}$

$= \{0, 0, \bar{B}, 0, B\}$ = Taut. $\Rightarrow$ Not Rel. Ess.

Thus, $E^r = \{\bar{A}D, \bar{B}\bar{D}\}$.

(ii) Totally redundant set R^t :

- $\underline{\text{check } \bar{A}\bar{B}}$:

$\{\bar{A}D, \bar{B}\bar{D}\}_{\bar{A}\bar{B}} = \{D, \bar{D}\}$ = Taut. $\Rightarrow$ Totally Red.

- $\underline{\text{check } BCD}$:

$\{\bar{A}D, \bar{B}\bar{D}\}_{BCD} = \{\bar{A}, 0\}$ = Not Taut. $\Rightarrow$ Part. Red.

- $\underline{\text{Check } ABC}$:

$\{\bar{A}D, \bar{B}\bar{D}\}_{ABC} = \{0, 0\}$ = Not Taut. $\Rightarrow$ Part. Red.

- $\underline{\text{Check } AC\bar{D}}$:

$\{\bar{A}D, \bar{B}\bar{D}\}_{AC\bar{D}} = \{0, \bar{B}\}$ = Not Taut. $\Rightarrow$ Part. Red.

Thus, $R^t = \{\bar{A}\bar{B}\}$, $R^p = \{BCD, ABC, AC\bar{D}\}$.

(iii) First, we find coverage relations:

- $\underline{BCD}$:

$\{ \bar{A}D, \bar{B}\bar{D}, ABC, AC\bar{D} \}_{BCD}$

$= \{ \bar{A}, 0, A, 0 \}$

$\Rightarrow$ added row $(1, 1, 0)$

- $\underline{ABC}$:

$\{ \bar{A}D, \bar{B}\bar{D}, BCD, AC\bar{D} \}_{ABC}$

$= \{ 0, 0, D, \bar{D} \}$

$\Rightarrow$ added row $(1, 1, 0)$

$\Rightarrow$ added row $(0, 1, 1)$

- $\underline{AC\bar{D}}$:

$\{ \bar{A}D, \bar{B}\bar{D}, BCD, ABC \}_{AC\bar{D}}$

$= \{ 0, \bar{B}, 0, B \}$

$\Rightarrow$ added row $(0, 1, 1)$

<u>Covering Matrix</u>:

$$\begin{array}{c|ccc} & BCD & ABC & AC\bar{D} \\ \hline BCD & 1 & 1 & 0 \\ \multirow{2}{*}{ABC} & 1 & 1 & 0 \\ & 0 & 1 & 1 \\ AC\bar{D} & 0 & 1 & 1 \end{array}$$

Thus, ABC is selected and the minimal cover is $\{ \bar{A}D, \bar{B}\bar{D}, ABC \}$.
This is the same result obtained by Espresso.

```
# espresso -Dirred -t -d hw2q3.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb y
# READ          Time was 0.00 sec, cost is c=6(6) in=15 out=6 tot=21
# COMPL         Time was 0.00 sec, cost is c=0(0) in=0 out=0 tot=0
# PLA is hw2q3.pla with 4 inputs and 1 outputs
# ON-set cost is  c=6(6) in=15 out=6 tot=21
# OFF-set cost is c=0(0) in=0 out=0 tot=0
# DC-set cost is  c=0(0) in=0 out=0 tot=0
# IRRED: F=6 E=2 R=4 Rt=1 Rp=3 Rc=1 Final=3 Bound=0
# IRRED         Time was 0.00 sec, cost is c=3(3) in=7 out=3 tot=10
# READ           1 call(s) for 0.00 sec ( 0.0%)
# COMPL          1 call(s) for 0.00 sec ( 0.0%)
# IRRED          1 call(s) for 0.00 sec ( 0.0%)
# irred Time was 0.00 sec, cost is c=3(3) in=7 out=3 tot=10
.i 4
.o 1
.ilb a b c d
.p 3
0--1 1
-0-0 1
111- 1
.e
# WRITE         Time was 0.00 sec, cost is c=3(3) in=7 out=3 tot=10
```