COE 561, Term 101

Digital System Design and Synthesis

HW# 2 Solution

Due date: Tuesday, Nov. 9

Q.1. Consider the function *F(A,B,C,D)* with the following ON-set and DC-set:

$F^{ON} = \sum m(0, 3, 5, 7, 9, 11, 12, 14)$

$F^{DC} = \sum m(4, 15)$

(i) Compute the off-set using the recursive complementation procedure outlined in section 7.3.4

(ii) Apply the EXPAND procedure on the given cover using Espresso heuristics and show the obtained expanded cover. Compare your solution with the result obtained by ESPRESSO tool. Note that if there are minterms of the same weight, expand the minterm with the least number first (i.e. expand minterm 8 before 10). Similarly if raising all literals has the same benefit, expand the literals according to their order (i.e. literal A before B).

(iii) Apply the IRREDUNDANT procedure on the expanded cover using Espresso heuristics and show the obtained irredundant cover. Compare your solution with the result obtained by ESPRESSO tool.

(iv) Determine if any of the obtained prime implicants is an essential prime implicant or not using the method outlined in section 7.4.4. If it is essential, remove it from the cover and make the on-sets covered by it don't cares.

(v) Apply the REDUCE procedure on the irredundant cover using Espresso heuristics and show the obtained reduced cover. Compare your solution with the result obtained by ESPRESSO tool.

Q.2. Consider the following cover of a function F*(A,B,C,D)*

$$F = \overline{B}\,\overline{C} + \overline{C}D + BD + \overline{A}\,\overline{D} + \overline{A}B + C\overline{D}$$

With $F^{DC} = \sum m(2, 10, 14)$

(i) Determine the relatively essential set of cubes, E^r.

(ii) Determine the totally redundant, R^t, and partially redundant, R^p, sets of cubes.

(iii) Find a subset of R^p that, together with E^r, covers the function by solving a covering problem.

(iv) Compare your solution with the result obtained by ESPRESSO tool.

COE 561 Term: 101

HW #2 Solution

Q1. $F^{ON} = \sum m\,(0, 3, 5, 7, 9, 11, 12, 14)$

$F^{DC} = \sum m\,(4, 15)$

(i) $$F^{ON} \cup F^{DC} = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}cd + \bar{a}b\bar{c}d + \bar{a}bcd$$
$$+ a\bar{b}\bar{c}d + a\bar{b}cd + ab\bar{c}\bar{d} + abc\bar{d}$$
$$+ \bar{a}b\bar{c}\bar{d} + abcd$$

$$= \bar{a}\,[\bar{b}\bar{c}\bar{d} + \bar{b}cd + b\bar{c}d + bcd + b\bar{c}\bar{d}]$$
$$+ a\,[\bar{b}\bar{c}d + \bar{b}cd + b\bar{c}\bar{d} + bc\bar{d} + bcd]$$

$$= \bar{a}\,[\bar{b}\,[\bar{c}\bar{d} + cd] + b\,[\bar{c}d + cd + \bar{c}\bar{d}]]$$
$$+ a\,[\bar{b}\,[\bar{c}d + cd] + b\,[\bar{c}\bar{d} + c\bar{d} + cd]]$$

$$= \bar{a}\,[\bar{b}\,[\bar{c}\,[\bar{d}] + c\,[d]] + b\,[\bar{c}\,[d + \bar{d}]$$
$$+ c\,[d]]]$$
$$+ a\,[\bar{b}\,[\bar{c}\,[d] + c\,[d]] + b\,[\bar{c}\,[\bar{d}] +$$
$$c\,[\bar{d} + d]]]$$

$$\Rightarrow F^{off} = \bar{a}\,[\bar{b}\,[\bar{c}\,[d] + c\,[\bar{d}]] + b\,[\bar{c}\,[0] + c\,[\bar{d}]]]$$
$$+ a\,[\bar{b}\,[\bar{c}\,[\bar{d}] + c\,[\bar{d}]] + b\,[\bar{c}\,[d] + c\,[0]]]$$
$$= \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}c\bar{d} + \bar{a}bc\bar{d} + a\bar{b}\bar{c}\bar{d} + a\bar{b}c\bar{d}$$
$$+ ab\bar{c}d$$

(ii) Expand Procedure;

F^{off}:

	a	b	c	d
$\bar{a}\bar{b}\bar{c}d$	10	10	10	01
$\bar{a}\bar{b}c\bar{d}$	10	10	01	10
$\bar{a}bc\bar{d}$	10	01	01	10
$a\bar{b}\bar{c}\bar{d}$	01	10	10	10
$a\bar{b}c\bar{d}$	01	10	01	10
$ab\bar{c}d$	01	01	10	01

We next compute the weights of the ON-set:

	a	b	c	d	weight
$\bar{a}\bar{b}\bar{c}\bar{d}$	10	10	10	10	15
$\bar{a}\bar{b}cd$	10	10	01	01	17
$\bar{a}b\bar{c}d$	10	01	10	01	17
$\bar{a}bcd$	10	01	01	01	17
$a\bar{b}\bar{c}d$	01	10	10	01	17
$a\bar{b}cd$	01	10	01	01	17
$ab\bar{c}\bar{d}$	01	01	10	10	15
$abc\bar{d}$	01	01	01	10	15
	44	44	44	35	

We have 3 cubes of the same weight, we select the minterm with the least number i.e. $\bar{a}\bar{b}\bar{c}\bar{d}$.

* Expand $\bar{a}\bar{b}\bar{c}\bar{d}$:

Free = {2, 4, 6, 8}.
Since there is no column with all 0's in F^{off}, no column can always be raised.

Intersection with the off-set implies that columns 2, 6, and 8 can't be raised $\Rightarrow$ Free = {4} and overexpanded cube = $\bar{a}\bar{c}\bar{d}$.

Since we have only one free column, it is expanded and we get the cube $\underline{\bar{a}\bar{c}\bar{d}}$.

* <u>Expand $ab\bar{c}\bar{d}$</u>

Free = {1, 3, 6, 8}

Instersection with off-set implies that columns 3 and 8 can't be raised $\Rightarrow$ Free = {1, 6} and overexpanded cube = $b\bar{d}$

We only need to check $abc\bar{d}$ for being feasibly covered with $ab\bar{c}\bar{d}$

Super cube ($ab\bar{c}\bar{d}$, $abc\bar{d}$) = $ab\bar{d}$ (feasible)

Free set = {1} which can't be raised.

$\Rightarrow$ expand to $\underline{ab\bar{d}}$

The cube $abc\bar{d}$ is removed.

* <u>Expand $\bar{a}\bar{b}cd$</u>

Free = {2, 4, 5, 7}.

Intersection with the off-set implies that columns 5 and 7 can't be raised $\Rightarrow$ Free = {2, 4} and over expanded cube = cd

We need to check cubes $\bar{a}bcd$ and $a\bar{b}cd$ for being feasibly covered.

supercube $(\bar{a}\bar{b}cd, \bar{a}bcd) = \bar{a}cd$ (feasible)

supercube $(\bar{a}\bar{b}cd, a\bar{b}cd) = \bar{b}cd$ (feasible)

Any can be selected. Select $\bar{a}cd$ $\Rightarrow$ free $= \{2\}$ can be raised $\Rightarrow$ expanded cube is $\underline{cd}$.

Remove cubes $\bar{a}bcd$ and $a\bar{b}cd$.

* Expand $\bar{a}b\bar{c}d$

Free $= \{2, 3, 6, 7\}$

Intersection with off-set implies that columns 2 and 3 can't be raised $\Rightarrow$ Free $= \{6, 7\}$ and overexpanded cube $= \bar{a}b$.

No remaining cubes covered with overexpanded cube $\Rightarrow$ Find the largest prime implicant covering the cube. Either $\bar{a}b\bar{c}$ or $\bar{a}bd$ can be selected.
Let us select $\underline{\bar{a}bd}$.

* Expand $a\bar{b}\bar{c}d$

Free $= \{1, 4, 6, 7\}$

Intersection with off-set implies that columns 1, 4 and 6 can't be raised $\Rightarrow$ column 7 is raised and the cube is expanded to $\underline{a\bar{b}d}$.

Thus, the expanded cover is:

$\{\bar{a}\bar{c}\bar{d}, ab\bar{d}, cd, \bar{a}bd, a\bar{b}d\}$.

In comparison with Espresso tool, the same expanded cover is obtained.

```
hw2q1ii.pla
.i 4
.o 1
.ilb a b c d
.olb y
.p 10
0000 1
0011 1
0101 1
0111 1
1001 1
1011 1
1100 1
1110 1
0100 -
1111 -
.e
# espresso -d -t -Dexpand hw2q1ii.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb y
# READ          Time was 0.00 sec, cost is c=8(8) in=32 out=8 tot=40
# COMPL         Time was 0.00 sec, cost is c=4(4) in=14 out=4 tot=18
# PLA is hw2q1ii.pla with 4 inputs and 1 outputs
# ON-set cost is  c=8(8) in=32 out=8 tot=40
# OFF-set cost is c=4(4) in=14 out=4 tot=18
# DC-set cost is  c=2(2) in=8 out=2 tot=10
EXPAND: 0000 1 (covered 0)
EXPAND: 1100 1 (covered 1)
EXPAND: 1001 1 (covered 1)
EXPAND: 0101 1 (covered 1)
EXPAND: 0011 1 (covered 0)
# EXPAND        Time was 0.00 sec, cost is c=5(0) in=14 out=5 tot=19
# READ           1 call(s) for 0.00 sec ( 0.0%)
# COMPL          1 call(s) for 0.00 sec ( 0.0%)
# EXPAND         1 call(s) for 0.00 sec ( 0.0%)
# expand        Time was 0.00 sec, cost is c=5(0) in=14 out=5 tot=19
.i 4
.o 1
.ilb a b c d
.p 5
0-00 1
11-0 1
10-1 1
01-1 1
--11 1
.e
# WRITE         Time was 0.00 sec, cost is c=5(0) in=14 out=5 tot=19
```

(iii) Irredundant Procedure:

The expanded cover is $\{\bar{a}\bar{c}\bar{d}, ab\bar{d}, cd, \bar{a}bd, a\bar{b}d\}$. First, we need to check whether each of these cubes is relatively essential.

- Check if $\{ab\bar{d}, cd, \bar{a}bd, a\bar{b}d, \bar{a}b\bar{c}\bar{d}, abcd\}$ covers $\bar{a}\bar{c}\bar{d}$

  $\{ab\bar{d}, cd, \bar{a}bd, a\bar{b}d, \bar{a}b\bar{c}\bar{d}, abcd\}_{\bar{a}\bar{c}\bar{d}}$

  $= \{0, 0, 0, 0, b, 0\}$

  Since the cofactor is not tautology, this implies that $\bar{a}\bar{c}\bar{d}$ is relatively essential.

- Check for $ab\bar{d}$:

  $\{\bar{a}\bar{c}\bar{d}, cd, \bar{a}bd, a\bar{b}d, \bar{a}b\bar{c}\bar{d}, abcd\}_{ab\bar{d}}$

  $= \{0, 0, 0, 0, 0, 0\} \Rightarrow$ relatively essential

- check for cd:

  $\{\bar{a}\bar{c}\bar{d}, ab\bar{d}, \bar{a}bd, a\bar{b}d, \bar{a}b\bar{c}\bar{d}, abcd\}_{cd}$

  $= \{0, 0, \bar{a}b, a\bar{b}, 0, ab\} \Rightarrow$ Not tautology $\Rightarrow$ relatively essential

- Check for $\bar{a}bd$:

  $\{\bar{a}\bar{c}\bar{d}, ab\bar{d}, cd, a\bar{b}d, \bar{a}b\bar{c}\bar{d}, abcd\}_{\bar{a}bd}$

  $= \{0, 0, c, 0, 0, 0\} \Rightarrow$ relatively essential

- check for $a\bar{b}d$:

  $\{\bar{a}\bar{c}\bar{d}, ab\bar{d}, cd, \bar{a}bd, \bar{a}b\bar{c}\bar{d}, abcd\}_{a\bar{b}d}$

  $= \{0, 0, c, 0, 0, 0\} \Rightarrow$ relatively essential.

Since all implicants are relatively essential, then the cover is irredundant. This is consistent with what is produced by Espresso tool.

Input:

```
.i 4
.o 1
.ilb a b c d
.p 7
0-00 1
11-0 1
10-1 1
01-1 1
--11 1
0100 -
1111 -
.e
# espresso -Dirred -t -d hw2q1ii_irred_input.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
# READ          Time was 0.00 sec, cost is c=5(5) in=14 out=5 tot=19
# COMPL         Time was 0.00 sec, cost is c=0(0) in=0 out=0 tot=0
# PLA is hw2q1ii_irred_input.pla with 4 inputs and 1 outputs
# ON-set cost is  c=5(5) in=14 out=5 tot=19
# OFF-set cost is c=0(0) in=0 out=0 tot=0
# DC-set cost is  c=2(2) in=8 out=2 tot=10
# IRRED: F=5 E=5 R=0 Rt=0 Rp=0 Rc=0 Final=5 Bound=0
# IRRED         Time was 0.00 sec, cost is c=5(5) in=14 out=5 tot=19
# READ           1 call(s) for 0.00 sec ( 0.0%)
# COMPL          1 call(s) for 0.00 sec ( 0.0%)
# IRRED          1 call(s) for 0.00 sec ( 0.0%)
# irred  Time was 0.00 sec, cost is c=5(5) in=14 out=5 tot=19
.i 4
.o 1
.ilb a b c d
.p 5
0-00 1
11-0 1
10-1 1
01-1 1
--11 1
.e
# WRITE         Time was 0.00 sec, cost is c=5(5) in=14 out=5 tot=19
```

(iv) Essential Prime Implicants:

$F = \bar{a}\bar{c}\bar{d} + ab\bar{d} + cd + \bar{a}bd + a\bar{b}d$

$F^{DC} = \bar{a}b\bar{c}\bar{d} + abcd$

- Checking $\bar{a}\bar{c}\bar{d}$:

$G = \{ab\bar{d}, cd, \bar{a}bd, a\bar{b}d, \bar{a}b\bar{c}\bar{d}, abcd\}$

$G \# \bar{a}\bar{c}\bar{d} = \{ab\bar{d}, cd, \bar{a}bd, a\bar{b}d, abcd\}$

$H = \text{Consensus}(G \# \bar{a}\bar{c}\bar{d}, \bar{a}\bar{c}\bar{d}) = \{b\bar{c}\bar{d}, \bar{a}b\bar{c}\}$

Then, we check if $H \cup F^{DC}$ covers $\bar{a}\bar{c}\bar{d}$

$\Rightarrow \{b\bar{c}\bar{d}, \bar{a}b\bar{c}, \bar{a}b\bar{c}\bar{d}, abcd\}_{\bar{a}\bar{c}\bar{d}}$

$= \{b, b, b, 0\}$

Since it is not tautology, this means that $\bar{a}\bar{c}\bar{d}$ is an essential prime implicant.

- Checking $ab\bar{d}$:

$G = \{\bar{a}\bar{c}\bar{d}, cd, \bar{a}bd, a\bar{b}d, \bar{a}b\bar{c}\bar{d}, abcd\}$

$G \# ab\bar{d} = \{\bar{a}\bar{c}\bar{d}, cd, \bar{a}bd, a\bar{b}d, \bar{a}b\bar{c}\bar{d}, abcd\}$

$H = \text{Consensus}(G \# ab\bar{d}, ab\bar{d}) = \{b\bar{c}\bar{d}, abc, b\bar{c}\bar{d}, abc\}$

Then, check if $H \cup F^{DC}$ covers $ab\bar{d}$

$\Rightarrow \{b\bar{c}\bar{d}, abc, \bar{a}b\bar{c}\bar{d}, abcd\}_{ab\bar{d}}$

$= \{\bar{c}, c, 0, 0\}$ = Tautology

$\Rightarrow ab\bar{d}$ is not an essential prime implicant

- Checking cd:

$G = \{ \bar{a}\bar{c}\bar{d},\ ab\bar{d},\ \bar{a}bd,\ a\bar{b}d,\ \bar{a}b\bar{c}\bar{d},\ abcd \}$

$G \# cd = \{ \bar{a}\bar{c}\bar{d},\ ab\bar{d},\ \bar{a}b\bar{c}d,\ a\bar{b}\bar{c}d,\ \bar{a}b\bar{c}\bar{d} \}$

$H = \text{Consensus}(G \# cd, cd) = \{ abc,\ \bar{a}bd,\ a\bar{b}d \}$

Then, check if $H \cup F^{DC}$ covers CD

$\Rightarrow \{ abc,\ \bar{a}bd,\ a\bar{b}d,\ \bar{a}b\bar{c}\bar{d},\ abcd \}_{cd}$

$= \{ ab,\ \bar{a}b,\ a\bar{b},\ 0,\ ab \} \Rightarrow$ Not Tautology

$\Rightarrow$ cd is an essential prime implicant.

- Checking $\bar{a}bd$:

$G = \{ \bar{a}\bar{c}\bar{d},\ ab\bar{d},\ cd,\ a\bar{b}d,\ \bar{a}b\bar{c}\bar{d},\ abcd \}$

$G \# \bar{a}bd = \{ \bar{a}\bar{c}\bar{d},\ ab\bar{d},\ acd,\ \bar{b}cd,\ a\bar{b}d,\ \bar{a}b\bar{c}\bar{d},\ abcd \}$

$H = \text{Consensus}(G \# \bar{a}bd, \bar{a}bd) = \{ \bar{a}b\bar{c},\ bcd,\ \bar{a}cd,\ \bar{a}b\bar{c},\ bcd \}$.

Then, check if $H \cup F^{DC}$ covers $\bar{a}bd$

$\Rightarrow \{ \bar{a}b\bar{c},\ bcd,\ \bar{a}cd,\ \bar{a}b\bar{c},\ bcd,\ \bar{a}b\bar{c}\bar{d},\ abcd \}_{\bar{a}bd}$

$= \{ \bar{c},\ c,\ c,\ \bar{c},\ c,\ 0,\ 0 \} \Rightarrow$ Tautology

$\Rightarrow$ $\bar{a}bd$ is not an essential prime implicant.

- Checking $a\bar{b}d$:

$G = \{ \bar{a}\bar{c}\bar{d},\ ab\bar{d},\ cd,\ \bar{a}bd,\ \bar{a}b\bar{c}\bar{d},\ abcd \}$

$G \# a\bar{b}d = \{ \bar{a}\bar{c}\bar{d},\ ab\bar{d},\ \bar{a}cd,\ bcd,\ \bar{a}bd,\ \bar{a}b\bar{c}\bar{d},\ abcd \}$

$H = \text{Consensus}(G \# a\bar{b}d, a\bar{b}d) = \{ \bar{b}cd,\ acd,\ acd \}$

Then, check if $H \cup F^{DC}$ covers $a\bar{b}d$

$\Rightarrow \{ \bar{b}cd,\ acd,\ \bar{a}b\bar{c}\bar{d},\ abcd \}_{a\bar{b}d}$

$= \{ c,\ c,\ 0,\ 0 \} \Rightarrow$ Not Tautology

$\Rightarrow$ $a\bar{b}d$ is an essential prime implicant.

Thus, the essential prime implicants are $\{ \bar{a}\bar{c}\bar{d},\ cd,\ a\bar{b}d \}$.

Thus, after removing the essential prime implicants from the cover, the cover becomes $\{ab\bar{d}, \bar{a}bd\}$ and F^{DC} becomes $\{\bar{a}\bar{c}\bar{d}, cd, a\bar{b}d\}$.

(v) Reduce Procedure:

First, we compute the weight of each implicant in the cover.

	a	b	c	d	weight
$ab\bar{d}$	01	01	11	10	8
$\bar{a}bd$	10	01	11	01	8
	11	02	22	11	

Since both have the same weight, we can reduce any of them.

- Reduce $ab\bar{d}$

$\alpha = ab\bar{d}$
$Q = \{F \cup F^{DC}\} - \alpha = \{\bar{a}bd, \bar{a}\bar{c}\bar{d}, cd, a\bar{b}d\}$
$Q_\alpha = \{0, 0, 0, 0\} = \{0\}$
$\bar{Q}_\alpha = 1$, $\tilde{\alpha} = \alpha \cap \text{supercube}(\bar{Q}_\alpha) = ab\bar{d} \cap 1 = ab\bar{d}$

- Reduce $\bar{a}bd$

$\alpha = \bar{a}bd$
$Q = \{F \cup F^{DC}\} - \alpha = \{ab\bar{d}, \bar{a}\bar{c}\bar{d}, cd, a\bar{b}d\}$
$Q_\alpha = \{0, 0, c, 0\} = \{c\}$
$\bar{Q}_\alpha = \{\bar{c}\}$
$\tilde{\alpha} = \alpha \cap \text{Supercube}(\bar{Q}_\alpha) = \bar{a}bd \cap \bar{c} = \bar{a}b\bar{c}d$

Thus, the reduced cover is $\{ab\bar{d}, \bar{a}b\bar{c}d\}$.

This is consistent with the result produced by Espresso.

Input:
.i 4
.o 1
.ilb a b c d
.p 5
0-00 -
11-0 1
10-1 -
01-1 1
--11 -
.e

espresso -Dreduce -t -d hw2q1v.pla
UC Berkeley, Espresso Version #2.3, Release date 01/31/88
READ Time was 0.00 sec, cost is c=2(2) in=6 out=2 tot=8
COMPL Time was 0.00 sec, cost is c=0(0) in=0 out=0 tot=0
PLA is hw2q1v.pla with 4 inputs and 1 outputs
ON-set cost is c=2(2) in=6 out=2 tot=8
OFF-set cost is c=0(0) in=0 out=0 tot=0
DC-set cost is c=3(3) in=8 out=3 tot=11
REDUCE: 01-1 1 to 0101 1 0.00 sec
REDUCE Time was 0.00 sec, cost is c=2(1) in=7 out=2 tot=9
READ 1 call(s) for 0.00 sec (0.0%)
COMPL 1 call(s) for 0.00 sec (0.0%)
REDUCE 1 call(s) for 0.00 sec (0.0%)
reduce Time was 0.00 sec, cost is c=2(1) in=7 out=2 tot=9
.i 4
.o 1
.ilb a b c d
.p 2
11-0 1
0101 1
.e
WRITE Time was 0.00 sec, cost is c=2(1) in=7 out=2 tot=9

Q2. $F = \bar{B}\bar{C} + \bar{C}D + BD + \bar{A}\bar{D} + \bar{A}B + C\bar{D}$

$F^{DC} = \bar{a}\bar{b}c\bar{d} + a\bar{b}c\bar{d} + abc\bar{d}$

(1) <u>Relatively Essential set E^r</u>:

- <u>Check $\bar{B}\bar{C}$</u>

$\{\bar{C}D, BD, \bar{A}\bar{D}, \bar{A}B, C\bar{D}, \bar{A}\bar{B}C\bar{D}, A\bar{B}C\bar{D}, ABC\bar{D}\}_{\bar{B}\bar{C}}$

$= \{D, 0, \bar{A}\bar{D}, 0, 0, 0, 0, 0\}$ $\Rightarrow$ Not Tautology

$\Rightarrow$ $\bar{B}\bar{C}$ is relatively essential.

- <u>Check $\bar{C}D$</u>

$\{\bar{B}\bar{C}, BD, \bar{A}\bar{D}, \bar{A}B, C\bar{D}, \bar{A}\bar{B}C\bar{D}, A\bar{B}C\bar{D}, ABC\bar{D}\}_{\bar{C}D}$

$= \{\bar{B}, B, 0, \bar{A}B, 0, 0, 0, 0\}$ $\Rightarrow$ Tautology

$\Rightarrow$ $\bar{C}D$ is not relatively essential.

- <u>Check BD</u>

$\{\bar{B}\bar{C}, \bar{C}D, \bar{A}\bar{D}, \bar{A}B, C\bar{D}, \bar{A}\bar{B}C\bar{D}, A\bar{B}C\bar{D}, ABC\bar{D}\}_{BD}$

$= \{0, \bar{C}, 0, \bar{A}, 0, 0, 0, 0\}$ $\Rightarrow$ Not Tautology

$\Rightarrow$ BD is relatively essential.

- <u>Check $\bar{A}\bar{D}$</u>

$\{\bar{B}\bar{C}, \bar{C}D, BD, \bar{A}B, C\bar{D}, \bar{A}\bar{B}C\bar{D}, A\bar{B}C\bar{D}, ABC\bar{D}\}_{\bar{A}\bar{D}}$

$= \{\bar{B}\bar{C}, 0, 0, B, C, \bar{B}C, 0, 0\}$ $\Rightarrow$ Tautology

$\Rightarrow$ $\bar{A}\bar{D}$ is not relatively essential.

- <u>Check $\bar{A}B$</u>

$\{\bar{B}\bar{C}, \bar{C}D, BD, \bar{A}\bar{D}, C\bar{D}, \bar{A}\bar{B}C\bar{D}, A\bar{B}C\bar{D}, ABC\bar{D}\}_{\bar{A}B}$

$= \{0, \bar{C}D, D, \bar{D}, C\bar{D}, 0, 0, 0\}$ $\Rightarrow$ Tautology

$\Rightarrow$ $\bar{A}B$ is not relatively essential.

- Check $C\bar{D}$

$\{ \bar{B}\bar{C},\ \bar{C}D,\ BD,\ \bar{A}\bar{D},\ \bar{A}B,\ \bar{A}\bar{B}C\bar{D},\ A\bar{B}C\bar{D},\ ABC\bar{D} \}_{C\bar{D}}$

$= \{ 0,\ 0,\ 0,\ \bar{A},\ \bar{A}B,\ \bar{A}\bar{B},\ A\bar{B},\ AB \} \Rightarrow$ Tautology

$\Rightarrow$ $C\bar{D}$ is not relatively essential.

Thus, $E^r = \{ \bar{B}\bar{C},\ BD \}$

(ii) Totally redundant set R^t:

- Check $\bar{C}D$

$\{ \bar{B}\bar{C},\ BD,\ \bar{A}\bar{B}C\bar{D},\ A\bar{B}C\bar{D},\ ABC\bar{D} \}_{\bar{C}D}$

$= \{ \bar{B},\ B,\ 0,\ 0,\ 0 \} \Rightarrow$ Tautology $\Rightarrow$ redundant

- Check $\bar{A}\bar{D}$

$\{ \bar{B}\bar{C},\ BD,\ \bar{A}\bar{B}C\bar{D},\ A\bar{B}C\bar{D},\ ABC\bar{D} \}_{\bar{A}\bar{D}}$

$= \{ \bar{B}\bar{C},\ 0,\ \bar{B}C,\ 0,\ 0 \} \Rightarrow$ Not Tautology $\Rightarrow$ not redundant

- Check $\bar{A}B$

$\{ \bar{B}\bar{C},\ BD,\ \bar{A}\bar{B}C\bar{D},\ A\bar{B}C\bar{D},\ ABC\bar{D} \}_{\bar{A}B}$

$= \{ 0,\ D,\ 0,\ 0,\ 0 \} \Rightarrow$ Not Tautology $\Rightarrow$ Not redundant

- Check $C\bar{D}$

$\{ \bar{B}\bar{C},\ BD,\ \bar{A}\bar{B}C\bar{D},\ A\bar{B}C\bar{D},\ ABC\bar{D} \}_{C\bar{D}}$

$= \{ 0,\ 0,\ \bar{A}\bar{B},\ A\bar{B},\ AB \} \Rightarrow$ Not Tautology $\Rightarrow$ Not redund.

Thus, $R^t = \{ \bar{C}D \}$ and $R^p = \{ \bar{A}\bar{D},\ \bar{A}B,\ C\bar{D} \}$.

(iii) First, we find coverage relations.

- $\bar{A}\bar{D}$:

$\{\bar{B}\bar{C}, BD, \bar{A}\bar{B}C\bar{D}, A\bar{B}C\bar{D}, ABC\bar{D}, \bar{A}B, C\bar{D}\}_{\bar{A}\bar{D}}$

$= \{\bar{B}\bar{C}, 0, \bar{B}C, 0, 0, B, C\}$

- Expand on B:
  - $B=1$: $\{0, 0, 0, 0, 0, 1, C\}$ ⇒ added row (1, 1, 0)
  - $B=0$: $\{\bar{C}, 0, C, 0, 0, 0, C\}$ ⇒ no row added

- $\bar{A}B$:

$\{\bar{B}\bar{C}, BD, \bar{A}\bar{B}C\bar{D}, A\bar{B}C\bar{D}, ABC\bar{D}, \bar{A}\bar{D}, C\bar{D}\}_{\bar{A}B}$

$= \{0, D, 0, 0, 0, \bar{D}, C\bar{D}\}$ ⇒ added row (1, 1, 0)

- $C\bar{D}$:

$\{\bar{B}\bar{C}, BD, \bar{A}\bar{B}C\bar{D}, A\bar{B}C\bar{D}, ABC\bar{D}, \bar{A}\bar{D}, \bar{A}B\}_{C\bar{D}}$

$= \{0, 0, \bar{A}\bar{B}, A\bar{B}, AB, \bar{A}, \bar{A}B\}$

- Expand on A:
  - $A=1$: $\{0, 0, 0, \bar{B}, B, 0, 0\}$ ⇒ no rows added
  - $A=0$: $\{0, 0, \bar{B}, 0, 0, 1, B\}$ ⇒ added row (1, 1, 1)

Coverage Matrix:

	$\bar{A}\bar{D}$	$\bar{A}B$	$C\bar{D}$
$\bar{A}\bar{D}$	1	1	0
$\bar{A}B$	1	1	0
$C\bar{D}$	1	1	1

Thus, either $\bar{A}\bar{D}$ or $\bar{A}B$ can be selected.

Minimal cover is $\{\bar{B}\bar{C}, BD, \bar{A}\bar{D}\}$ or $\{\bar{B}\bar{C}, BD, \bar{A}B\}$.

(iv) Espresso tool generated the second cover.

Q2 input

```
.i 4
.o 1
.ilb a b c d
.olb y
.p 9
-00- 1
--01 1
-1-1 1
0--0 1
01-- 1
--10 1
0010 -
1010 -
1110 -
.e
```

```
# espresso -Dirred -t -d hw2q2.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb y
# READ        Time was 0.00 sec, cost is c=6(6) in=12 out=6 tot=18
# COMPL       Time was 0.00 sec, cost is c=0(0) in=0 out=0 tot=0
# PLA is hw2q2.pla with 4 inputs and 1 outputs
# ON-set cost is  c=6(6) in=12 out=6 tot=18
# OFF-set cost is c=0(0) in=0 out=0 tot=0
# DC-set cost is  c=3(3) in=12 out=3 tot=15
# IRRED: F=6 E=2 R=4 Rt=1 Rp=3 Rc=1 Final=3 Bound=0
# IRRED       Time was 0.00 sec, cost is c=3(3) in=6 out=3 tot=9
# READ         1 call(s) for 0.00 sec ( 0.0%)
# COMPL        1 call(s) for 0.00 sec ( 0.0%)
# IRRED        1 call(s) for 0.00 sec ( 0.0%)
# irred Time was 0.00 sec, cost is c=3(3) in=6 out=3 tot=9
.i 4
.o 1
.ilb a b c d
.p 3
-00- 1
-1-1 1
01-- 1
.e
# WRITE       Time was 0.00 sec, cost is c=3(3) in=6 out=3 tot=9
```