COE 561, Term 091

Digital System Design and Synthesis

HW# 2 Solution

Due date: Sunday, Dec. 6

Q.1. Consider the function *F(A,B,C,D)* with the following ON-set and DC-set:

$$F^{ON} = \sum m(0, 1, 2, 3, 5, 7, 8, 10, 12, 13)$$

$$F^{DC} = \sum m(4, 15)$$

- **(i)** Compute the off-set using the recursive complementation procedure outlined in section 7.3.4
- **(ii)** Apply the EXPAND procedure on the given cover using Espresso heuristics and show the obtained expanded cover. Compare your solution with the result obtained by ESPRESSO tool. Note that if there are minterms of the same weight, expand the minterm with the least number first (i.e. expand minterm 8 before 10). Similarly if raising all literals has the same benefit, expand the literals according to their order (i.e. literal A before B).
- **(iii)** Apply the IRREDUNDANT procedure on the expanded cover using Espresso heuristics and show the obtained irredundant cover. Compare your solution with the result obtained by ESPRESSO tool.
- **(iv)** Determine if any of the obtained prime implicants is an essential prime implicant or not using the method outlined in section 7.4.4. If it is essential, remove it from the cover and make the on-sets covered by it don't cares.
- **(v)** Apply the REDUCE procedure on the irredundant cover using Espresso heuristics and show the obtained reduced cover. Compare your solution with the result obtained by ESPRESSO tool.
- **(vi)** Apply the EXPAND procedure again on the obtained reduced cover using Espresso heuristics and show the obtained expanded cover. Compare your solution with the result obtained by ESPRESSO tool.

HW #2 Solution

Q1. $F^{ON} = \sum m(0,1,2,3,5,7,8,10,12,13)$

$F^{DC} = \sum m(4, 15)$

(i) $F^{ON} \cup F^{DC} = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}c\bar{d} + \bar{a}\bar{b}cd + \bar{a}b\bar{c}d + \bar{a}bcd + a\bar{b}\bar{c}\bar{d} + a\bar{b}c\bar{d} + ab\bar{c}\bar{d} + ab\bar{c}d + \bar{a}b\bar{c}\bar{d} + abcd$

$= \bar{a}[\bar{b}\bar{c}\bar{d} + \bar{b}\bar{c}d + \bar{b}c\bar{d} + \bar{b}cd + b\bar{c}d + bcd + b\bar{c}\bar{d}] + a[\bar{b}\bar{c}\bar{d} + \bar{b}c\bar{d} + b\bar{c}\bar{d} + b\bar{c}d + bcd]$

$= \bar{a}[\bar{b}[\bar{c}\bar{d} + \bar{c}d + c\bar{d} + cd] + b[\bar{c}d + cd + \bar{c}\bar{d}]] + a[\bar{b}[\bar{c}\bar{d} + c\bar{d}] + b[\bar{c}\bar{d} + \bar{c}d + cd]]$

$= \bar{a}[\bar{b}[\bar{c}[\bar{d}+d] + c[\bar{d}+d]] + b[\bar{c}[d+\bar{d}] + c[d]]] + a[\bar{b}[\bar{c}[\bar{d}] + c[\bar{d}]] + b[\bar{c}[\bar{d}+d] + c[d]]]$

$\Rightarrow F^{off} = \bar{a}[\bar{b}[\bar{c}[0] + c[0]] + b[\bar{c}[\ ] + c[\bar{d}]]] + a[\bar{b}[\bar{c}[d] + c[d]] + b[\bar{c}[0] + c[\bar{d}]]]$

Thus, $F^{off} = \bar{a}bc\bar{d} + a\bar{b}\bar{c}d + a\bar{b}cd + abc\bar{d}$

(ii) Expand Procedure:

F^{off}:

	a	b	c	d
$\bar{a}bc\bar{d}$	10	01	01	10
$a\bar{b}\bar{c}d$	01	10	10	01
$a\bar{b}cd$	01	10	01	01
$abc\bar{d}$	01	01	01	10

We next compute the weights of the on-set:

	a	b	c	d	weights
$\bar{a}\bar{b}\bar{c}\bar{d}$	10	10	10	10	23
$\bar{a}\bar{b}\bar{c}d$	10	10	10	01	23
$\bar{a}\bar{b}c\bar{d}$	10	10	01	10	21
$\bar{a}\bar{b}cd$	10	10	01	01	21
$\bar{a}b\bar{c}d$	10	01	10	01	21
$\bar{a}bcd$	10	01	01	01	19
$a\bar{b}\bar{c}\bar{d}$	01	10	10	10	21
$a\bar{b}c\bar{d}$	01	10	01	10	19
$ab\bar{c}\bar{d}$	01	01	10	10	19
$ab\bar{c}d$	01	01	10	01	19
	64	64	64	55	

We have four cubes of the same weight. We select the minterm with the least number i.e. $\bar{a}bcd$.

* Expand $\bar{a}bcd$:

Free = {2, 3, 5, 7}
Since there is no column with all 0's in F^{off}, no column can always be raised.
Intersection with the off-set, column 7 can't be raised. Free = {2, 3, 5}, overexpanded cube = d.

We only need to check cubes $\bar{a}\bar{b}\bar{c}d$, $\bar{a}\bar{b}cd$, $\bar{a}b\bar{c}d$ and $ab\bar{c}d$ for being feasibly covered with $\bar{a}bcd$.

Supercube $(\bar{a}bcd, \bar{a}\bar{b}\bar{c}d) = \bar{a}d$ (feasible)
Supercube $(\bar{a}bcd, \bar{a}\bar{b}cd) = \bar{a}cd$ (feasible)
Supercube $(\bar{a}bcd, \bar{a}b\bar{c}d) = \bar{a}bd$ (feasible)
Supercube $(\bar{a}bcd, ab\bar{c}d) = bd$ (feasible)

The expanded cube $\bar{a}d$ is selected and Free $= \{2\}$ which cannot be raised. $\bar{a}d$ is selected as it covers more cubes.
The covered cubes $\bar{a}\bar{b}\bar{c}d$, $\bar{a}\bar{b}cd$ and $\bar{a}b\bar{c}d$ are removed.

* Expand $a\bar{b}c\bar{d}$

Free $= \{1, 4, 5, 8\}$

Intersecting with the off-set implies that columns 4 and 8 cannot be raised. Thus, Free $= \{1, 5\}$ and overexpanded cube $= \bar{b}\bar{d}$.

We need to check the cubes $\bar{a}\bar{b}\bar{c}\bar{d}$, $\bar{a}\bar{b}c\bar{d}$ and $a\bar{b}\bar{c}\bar{d}$ for being feasibly covered. Since the supercube does not cover off-set cubes, this implies that all of them will be feasibly covered.

Supercube $(a\bar{b}c\bar{d}, \bar{a}\bar{b}\bar{c}\bar{d}) = \bar{b}\bar{d}$ (feasible)
Supercube $(a\bar{b}c\bar{d}, \bar{a}\bar{b}c\bar{d}) = \bar{b}c\bar{d}$ (feasible)
Supercube $(a\bar{b}c\bar{d}, a\bar{b}\bar{c}\bar{d}) = a\bar{b}\bar{d}$ (feasible)

The expanded cube $\bar{b}\bar{d}$ is selected and Free $= \{\}$.

The covered cubes $\bar{a}\bar{b}\bar{c}\bar{d}$, $\bar{a}\bar{b}c\bar{d}$ and $a\bar{b}\bar{c}\bar{d}$ are removed.

* Expand $ab\bar{c}\bar{d}$:

Free = {1, 3, 6, 8}.

Intersecting with the off-set, column 6 can't be raised. Free = {1, 3, 8}, overexpanded cube = $\bar{c}$.

We only need to check cube $ab\bar{c}d$ for being feasibly covered.

Supercube ($ab\bar{c}\bar{d}$, $ab\bar{c}d$) = $ab\bar{c}$ (feasible)

The resulting expanded cube is $ab\bar{c}$ and Free = {1, 3}.

Intersecting with the off-set column 3 can't be raised and Free = {1}.

Finally, column 1 is raised an the expanded cube is $b\bar{c}$.

Thus, the expanded cover is $\{\bar{a}d, \bar{b}\bar{d}, b\bar{c}\}$.

In comparison with Espresso tool, the same cover is obtained.

hw2q1ii.pla

```
.i 4
.o 1
.ilb a b c d
.olb y
.p 10
0000 1
0001 1
0010 1
0011 1
0101 1
0111 1
1000 1
1010 1
1100 1
1101 1
0100 -
1111 -
.e
```

D:\Courses\coe561\091>espresso -d -t -Dexpand hw2q1ii.pla > hw2q1ii_expand.pla

```
# espresso -d -t -Dexpand hw2q1ii.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb y
# READ            Time was 0.00 sec, cost is c=10(10) in=40 out=10 tot=50
# COMPL           Time was 0.00 sec, cost is c=2(2) in=6 out=2 tot=8
# PLA is hw2q1i.pla with 4 inputs and 1 outputs
# ON-set cost is  c=10(10) in=40 out=10 tot=50
# OFF-set cost is c=2(2) in=6 out=2 tot=8
# DC-set cost is  c=2(2) in=8 out=2 tot=10
EXPAND: 1100 1 (covered 2)
EXPAND: 1010 1 (covered 3)
EXPAND: 0111 1 (covered 2)
# EXPAND          Time was 0.00 sec, cost is c=3(0) in=6 out=3 tot=9
# READ             1 call(s) for 0.00 sec ( 0.0%)
# COMPL            1 call(s) for 0.00 sec ( 0.0%)
# EXPAND           1 call(s) for 0.00 sec ( 0.0%)
# expand    Time was 0.00 sec, cost is c=3(0) in=6 out=3 tot=9
.i 4
.o 1
.ilb a b c d
.p 3
-10- 1
-0-0 1
0--1 1
.e
# WRITE           Time was 0.00 sec, cost is c=3(0) in=6 out=3 tot=9
```

(iii) Irredundant Procedure:

The expanded cover is $\{\bar{a}d, \bar{b}\bar{d}, b\bar{c}\}$.

First, we need to check whether each of these cubes is relatively essential.

- check if $\{\bar{b}\bar{d}, b\bar{c}, \bar{a}b\bar{c}\bar{d}, abcd\}$ covers $\bar{a}d$

  $\{\bar{b}\bar{d}, b\bar{c}, \bar{a}b\bar{c}\bar{d}, abcd\}_{\bar{a}d} = \{0, b\bar{c}, 0, 0\}$

  Since the cofactor is not tautology, this implies that $\bar{a}d$ is relatively essential.

- check if $\{\bar{a}d, b\bar{c}, \bar{a}b\bar{c}\bar{d}, abcd\}$ covers $\bar{b}\bar{d}$

  $\{\bar{a}d, b\bar{c}, \bar{a}b\bar{c}\bar{d}, abcd\}_{\bar{b}\bar{d}} = \{0, 0, 0, 0\}$

  Not tautology $\Rightarrow$ $\bar{b}\bar{d}$ is relatively essential.

- check if $\{\bar{a}d, \bar{b}\bar{d}, \bar{a}b\bar{c}\bar{d}, abcd\}$ covers $b\bar{c}$.

  $\{\bar{a}d, \bar{b}\bar{d}, \bar{a}b\bar{c}\bar{d}, abcd\}_{b\bar{c}} = \{\bar{a}d, 0, \bar{a}\bar{d}, 0\}$

  Not tautology $\Rightarrow$ $b\bar{c}$ is relatively essential.

Since all implicants are relatively essential, then the cover is irredundant.

This is consistent with what is produced by Espresso tool.

D:\Courses\coe561\091>espresso -Dirred -t -d hw2q1ii_expand.pla > hw2q1iii_irred.pla

```
# espresso -Dirred -t -d hw2q1ii_expand.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb y
# READ            Time was 0.01 sec, cost is c=3(3) in=6 out=3 tot=9
# COMPL           Time was 0.00 sec, cost is c=0(0) in=0 out=0 tot=0
# PLA is hw2q1ii_expand.pla with 4 inputs and 1 outputs
# ON-set cost is  c=3(3) in=6 out=3 tot=9
# OFF-set cost is c=0(0) in=0 out=0 tot=0
# DC-set cost is  c=0(0) in=0 out=0 tot=0
# IRRED: F=3 E=3 R=0 Rt=0 Rp=0 Rc=0 Final=3 Bound=0
# IRRED           Time was 0.00 sec, cost is c=3(3) in=6 out=3 tot=9
# READ             1 call(s) for 0.01 sec (93.7%)
# COMPL            1 call(s) for 0.00 sec ( 0.0%)
# IRRED            1 call(s) for 0.00 sec ( 0.0%)
# irred     Time was 0.03 sec, cost is c=3(3) in=6 out=3 tot=9
.i 4
.o 1
.ilb a b c d
.p 3
-10- 1
-0-0 1
0--1 1
.e
# WRITE           Time was 0.00 sec, cost is c=3(3) in=6 out=3 tot=9
```

(iv) Essential Prime Implicants:

$F = \bar{a}d + \bar{b}\bar{d} + b\bar{c}$

$F^{DC} = \bar{a}b\bar{c}\bar{d} + abcd$

- Checking $\bar{a}d$:

$G = \{\bar{b}\bar{d},\ b\bar{c},\ \bar{a}b\bar{c}\bar{d},\ abcd\}$

$G \# \bar{a}d = \{\bar{b}\bar{d},\ b\bar{c}\bar{d},\ ab\bar{c},\ \bar{a}b\bar{c}\bar{d},\ abcd\}$

$H = \text{Consensus}(G \# \bar{a}d,\ \bar{a}d) = \{\bar{a}\bar{b},\ \bar{a}b\bar{c},\ b\bar{c}d,\ \bar{a}b\bar{c},\ bcd\}$

Then, we check if $H \cup F^{DC}$ covers $\bar{a}d$

$\Rightarrow \{\bar{a}\bar{b},\ \bar{a}b\bar{c},\ b\bar{c}d,\ \bar{a}b\bar{c},\ bcd,\ \bar{a}b\bar{c}\bar{d},\ abcd\}_{\bar{a}d}$

$= \{\bar{b},\ b\bar{c},\ b\bar{c},\ b\bar{c},\ bc,\ 0,\ 0\} \Rightarrow$ tautology

Since it is tautology, this means that $\bar{a}d$ is not an essential prime implicant.

- Checking $\bar{b}\bar{d}$:

$G = \{\bar{a}d,\ b\bar{c},\ \bar{a}b\bar{c}\bar{d},\ abcd\}$

$G \# \bar{b}\bar{d} = \{\bar{a}d,\ b\bar{c},\ \bar{a}b\bar{c}\bar{d},\ abcd\}$

$H = \text{consensus}(G \# \bar{b}\bar{d},\ \bar{b}\bar{d})$

$= \{\bar{a}\bar{b},\ \bar{c}\bar{d},\ \bar{a}\bar{c}\bar{d}\}$

Then, we check if $H \cup F^{DC}$ covers $\bar{b}\bar{d}$.

$\Rightarrow \{\bar{a}\bar{b},\ \bar{c}\bar{d},\ \bar{a}\bar{c}\bar{d},\ \bar{a}b\bar{c}\bar{d},\ abcd\}_{\bar{b}\bar{d}}$

$= \{\bar{a},\ \bar{c},\ \bar{a}\bar{c},\ 0,\ 0\} \Rightarrow$ Not tautology

This means that $\bar{b}\bar{d}$ is an essential prime implicant.

- <u>Checking $b\bar{c}$</u>:

$G = \{\bar{a}d, \bar{b}\bar{d}, \bar{a}b\bar{c}\bar{d}, abcd\}$

$G \# b\bar{c} = \{\bar{a}\bar{b}d, \bar{a}cd, \bar{b}\bar{d}, abcd\}$

$H = \text{consensus}(G \# b\bar{c}, b\bar{c})$

$= \{\bar{a}\bar{c}d, \bar{a}bd, \bar{c}\bar{d}, abd\}$

Then, we check if $H \cup F^{bc}$ covers $b\bar{c}$.

$\{\bar{a}\bar{c}d, \bar{a}bd, \bar{c}\bar{d}, abd, \bar{a}b\bar{c}\bar{d}, abcd\}_{b\bar{c}}$

$= \{\bar{a}d, \bar{a}d, \bar{d}, ad, \bar{a}\bar{d}, 0\} \Rightarrow$ Tautology

This means that $b\bar{c}$ is not an essential prime implicant.

Thus, only $\bar{b}\bar{d}$ is an essential prime implicant.

The cover becomes $\{\bar{a}d, b\bar{c}\}$ and F^{bc} becomes

$\{\bar{b}\bar{d}, \bar{a}b\bar{c}\bar{d}, abcd\}$.

(v) <u>Reduce Procedure</u>:

First, we compute the weight of each implicant in the cover.

	a	b	c	d	weight
$\bar{a}d$	10	11	11	01	10
$b\bar{c}$	11	01	10	11	10
	21	12	21	12	

Since both have the same weight, we can't start reducing any one of them.

- Reduce $\bar{a}d$:

$\alpha = \bar{a}d$

$Q = \{F \cup F^{DC}\} - \alpha = \{b\bar{c},\ \bar{b}\bar{d},\ \bar{a}b\bar{c}\bar{d},\ abcd\}$

$Q_\alpha = \{b\bar{c},\ 0,\ 0,\ 0\}$

$\overline{Q_\alpha} = \bar{b} + c$

$\tilde{\alpha} = \alpha \cap \text{supercube}(\overline{Q_\alpha}) = \bar{a}d \cap 1 = \bar{a}d$

- Reduce $b\bar{c}$:

$\alpha = b\bar{c}$

$Q = \{F \cup F^{DC}\} - \alpha = \{\bar{a}d,\ \bar{b}\bar{d},\ \bar{a}b\bar{c}\bar{d},\ abcd\}$

$Q_\alpha = \{\bar{a}d,\ 0,\ \bar{a}\bar{d},\ 0\} = \{\bar{a}\}$

$\overline{Q_\alpha} = a$

$\tilde{\alpha} = b\bar{c} \cap a = ab\bar{c}$

Thus, the reduced cover is $\{\bar{a}d,\ ab\bar{c}\}$.

This is consistent with the results produced by Espresso.

```
.i 4
.o 1
.ilb a b c d
.p 3
0--1 1
-10- 1
-0-0 -
0100 -
1111 -
.e

D:\Courses\coe561\091>espresso -Dreduce -t -d hw2q1v.pla > hw2q1v_red.pla

# espresso -Dreduce -t -d hw2q1v.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
# READ            Time was 0.00 sec, cost is c=2(2) in=4 out=2 tot=6
# COMPL           Time was 0.00 sec, cost is c=0(0) in=0 out=0 tot=0
# PLA is hw2q1v.pla with 4 inputs and 1 outputs
# ON-set cost is  c=2(2) in=4 out=2 tot=6
# OFF-set cost is c=0(0) in=0 out=0 tot=0
# DC-set cost is  c=3(3) in=10 out=3 tot=13
REDUCE: -10- 1 to 110- 1 0.00 sec
# REDUCE          Time was 0.00 sec, cost is c=2(1) in=5 out=2 tot=7
# READ             1 call(s) for 0.00 sec ( 0.0%)
# COMPL            1 call(s) for 0.00 sec ( 0.0%)
# REDUCE           1 call(s) for 0.00 sec ( 0.0%)
# reduce    Time was 0.00 sec, cost is c=2(1) in=5 out=2 tot=7
.i 4
.o 1
.ilb a b c d
.p 2
110- 1
0--1 1
.e
# WRITE           Time was 0.00 sec, cost is c=2(1) in=5 out=2 tot=7
```

(vi) Expand Procedure on Reduced Cover:

$F = \bar{a}d + ab\bar{c}$

$F^{DC} = \bar{b}\bar{d} + \bar{a}b\bar{c}\bar{d} + abcd$

We compute the weight of each cube:

	a	b	c	d	weight
$\bar{a}d$	10	11	11	01	9
$ab\bar{c}$	01	01	10	11	8
	11	12	21	12	

Thus, we expand $ab\bar{c}$ first

- Expand $ab\bar{c}$:

  Free = {1, 3, 6}

  Intersecting with the off-set, columns 3 and 6 can't be raised. Free = {1} and over expanded cube is $b\bar{c}$.

  Cube $\bar{a}d$ is not feasibly covered since it is not covered by the overexpanded cube $b\bar{c}$.

  Then, column 1 is raised and the cube is expanded to $b\bar{c}$.

- Expand $\bar{a}d$:

  Free = {2, 7}

  Intersecting with the off-set, none of the columns can be raised and Free = {}.

Thus, the expanded cover is $\{\bar{a}d, b\bar{c}\}$.
This is consistent with the results produced by Espresso.

```
.i 4
.o 1
.ilb a b c d
.p 2
0--1 1
110- 1
-0-0 -
0100 -
1111 -
.e
```

D:\Courses\coe561\091>espresso -Dexpand -t -d hw2q1vi.pla > hw2q1vi_expand.pla

```
# espresso -Dexpand -t -d hw2q1vi.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
# READ            Time was 0.00 sec, cost is c=2(2) in=5 out=2 tot=7
# COMPL           Time was 0.00 sec, cost is c=2(2) in=6 out=2 tot=8
# PLA is hw2q1vi.pla with 4 inputs and 1 outputs
# ON-set cost is  c=2(2) in=5 out=2 tot=7
# OFF-set cost is c=2(2) in=6 out=2 tot=8
# DC-set cost is  c=3(3) in=10 out=3 tot=13
EXPAND: 110- 1 (covered 0)
EXPAND: 0--1 1 (covered 0)
# EXPAND          Time was 0.00 sec, cost is c=2(0) in=4 out=2 tot=6
# READ             1 call(s) for 0.00 sec ( 0.0%)
# COMPL            1 call(s) for 0.00 sec ( 0.0%)
# EXPAND           1 call(s) for 0.00 sec ( 0.0%)
# expand    Time was 0.00 sec, cost is c=2(0) in=4 out=2 tot=6
.i 4
.o 1
.ilb a b c d
.p 2
-10- 1
0--1 1
.e
# WRITE           Time was 0.00 sec, cost is c=2(0) in=4 out=2 tot=6
```