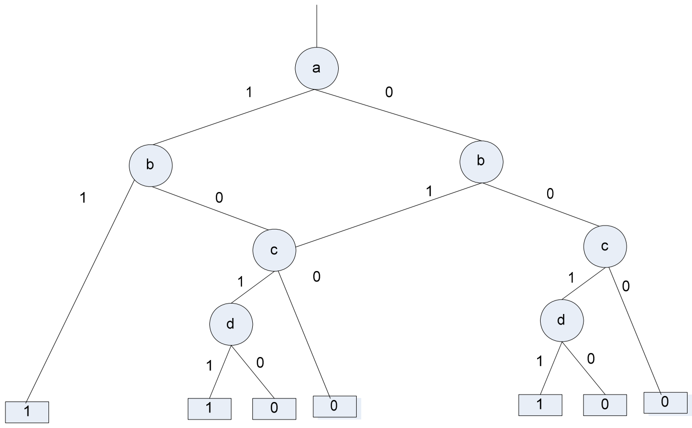
COE 561, Term 091

Digital System Design and Synthesis

HW# 1 Solution

Due date: Sunday, Nov. 1

Q.1. Consider the following OBDD with the variable ordering {a, b, c, d}. Reduce it based on **Reduce** function to obtain the ROBDD. Show the details of your work.



Q.2. Consider the function f=(a+bc)(d+b'c'):

(i) Draw the **ROBDD** for the function using the variable order {a, b, c, d}.

(ii) Draw the **ROBDD** for the function using the variable order {a, d, b, c}.

Q.3. Consider the two functions f=(a+bc)(d+b'c') and g=(a+d)(b+c):

(i) Compute the function $f \oplus g$ based on orthonormal basis expansion.

(ii) Draw the **ITE DAG** for the function f.g. Show the details of the ITE algorithm step by step. Use the variable order {a, b, c, d}

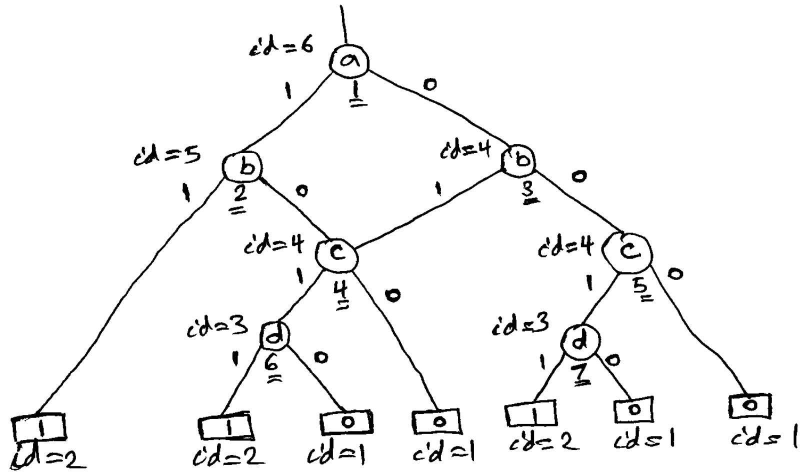
Q.4. Consider the following given matrix representing a covering problem:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Find a **minimum cover** using **EXACT_COVER** procedure. Show all the details of the algorithm. Assume the following order in branching selection when needed: C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8.

HW#1

Q1.



First, we set id(v) = 1 for all leaf vertices with value 0 and id(v) = 2 for all leaf vertices with value 1.

We initialize ROBDD with two leaf vertices for 0 and 1.

Then, we process vertices at level 4, i.e. nodes with index = d.

V = {6, 7}.

None of the vertices is removed since id(low(v)) ≠ id(high(v)).

We assign keys to all vertices ∈ V.

Key(6) = (1, 2), Key(7) = (1, 2).

oldKey = (0, 0).

We next sort the vertices in V according to their keys. Thus, V = {6, 7}.

v = {6} : since Key(6) ≠ oldKey, nextid = 3,

id(6) = 3, oldKey = (1, 2).

We add v = {6} to the ROBDD.

$v = \{7\}$: since $Key(7) = oldKey$, $id(7) = 3$

Next, we process vertices at level 3 with index = c.

$V = \{4, 5\}$.

None of the vertices is removed since $id(low(v)) \neq id(high(v))$.

We assign keys to all vertices $\in V$.

$Key(4) = (1, 3)$, $Key(5) = (1, 3)$.

$oldKey = (0, 0)$.

We sort the vertices according to their keys.

$V = \{4, 5\}$.

$v = \{4\}$: since $Key(4) \neq oldKey$, $nextid = 4$, $id(4) = 4$, $oldKey = (1, 3)$. We add $v = \{4\}$ to the ROBDD.

$v = \{5\}$: since $Key(5) = oldKey$, $id(5) = 4$.

Next, we process vertices at level 2 with index = b.

$V = \{2, 3\}$.

since $id(low(3)) = id(high(3)) = 4$, $id(3) = 4$ and vertex 3 is removed from V.

$Key(2) = (4, 2)$, $oldKey = (0, 0)$.

since $Key(2) \neq oldKey$, $nextid = 5$, $id(2) = 5$, $oldKey = (4, 2)$. We add $v = \{2\}$ to the ROBDD.

Finally, we process vertices at level 1 with index = a.

$V = \{1\}$.

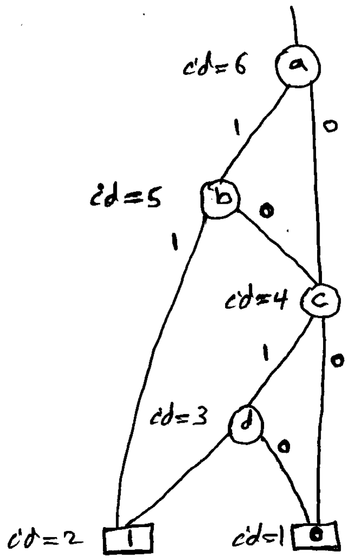
Since $id(low(1)) \neq id(high(1))$, the vertex is not removed.

$Key(1) = (4, 5)$, $oldKey\ (0, 0)$.

Since $Key(1) \neq oldKey$, $nextid = 6$, $id(1) = 6$.

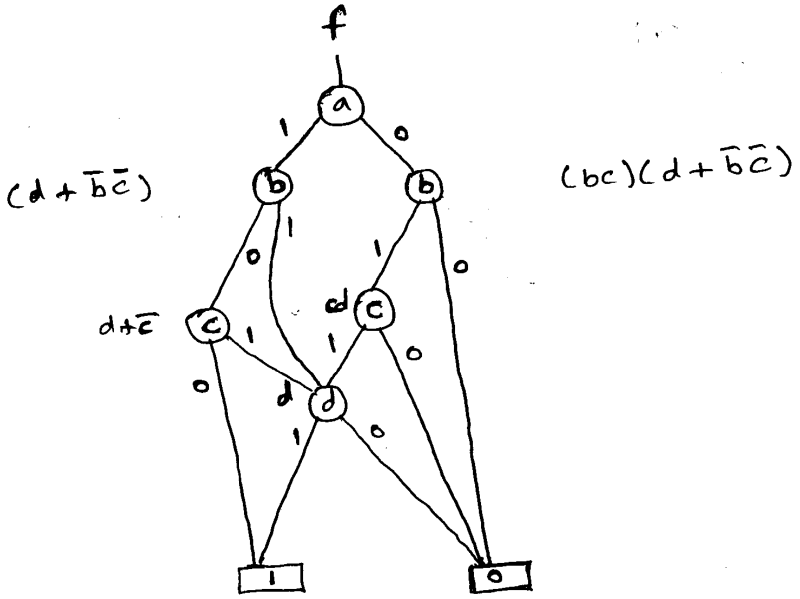
We add $v = \{1\}$ to the ROBDD.

Thus, the formed ROBDD is :

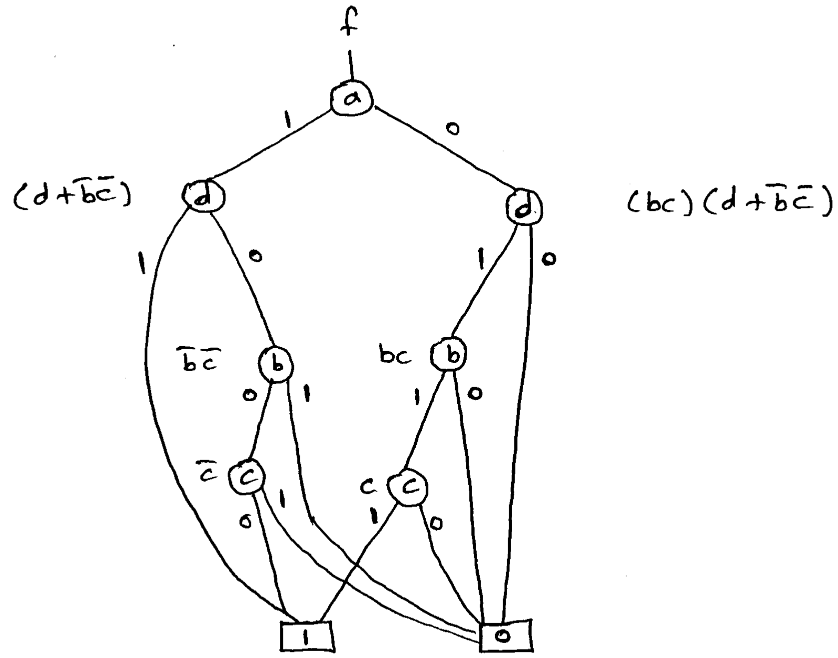


Q2. $f = (a + bc)(d + \bar{b}\bar{c})$

(i) ROBDD with variable order {a, b, c, d}



(ii) ROBDD with variable order $\{a, d, b, c\}$



Q3. $f = (a+bc)(d+\bar b\bar c)$ $\quad g = (a+d)(b+c)$

(i) $f \oplus g$ based on orthonormal basis expansion

$$f = \bar a\bar b(0) + \bar a b(cd) + a\bar b(d+\bar c) + ab(d)$$

$$g = \bar a\bar b(cd) + \bar a b(d) + a\bar b(c) + ab(1)$$

$$\begin{aligned} f\oplus g &= \bar a\bar b(cd) \\ &+ \bar a b(cd\cdot\bar d + (\bar c+\bar d)d) \\ &+ a\bar b((d+\bar c)\bar c + \bar d c\cdot c) \\ &+ ab(\bar d) \\ &= \bar a\bar b(cd) + \bar a b(\bar c d) + a\bar b(\bar c) + ab(\bar d) \end{aligned}$$

(ii) ITE diagram for the function $f \cdot g$

$$f \cdot g = ITE(f, g, 0)$$
$$= ITE((a+bc)(d+\overline{b}\overline{c}), (a+d)(b+c), 0)$$

- $x = a$

  $t = ITE(d+\overline{b}\overline{c}, b+c, 0)$

  - $x = b$

    $t = ITE(d, 1, 0) = d$ (trivial case)

    we assign id $= 3 \Rightarrow t = 3$

    $e = ITE(d+\overline{c}, c, 0)$

    - $x = c$

      $t = ITE(d, 1, 0) = d \Rightarrow t = 3$

      $e = ITE(1, 0, 0) = 0 \Rightarrow e = 1$

      since $t \neq e$, an entry will be added in the table for $(c, 3, 1)$ with id $= 4$

    $\Rightarrow e = 4$

    since $t \neq e$, an entry will be added in the table for $(b, 3, 4)$ with id $= 5$

  $\Rightarrow t = 5$

  $e = ITE(bc(d+\overline{b}\overline{c}), d(b+c), 0)$

  - $x = b$

    $t = ITE(cd, d, 0)$

    - $x = c$

      $t = ITE(d, d, 0) = d \Rightarrow t = 3$

      $e = ITE(0, d, 0) = 0 \Rightarrow e = 1$

      since $t \neq e$, and the entry $(c, 3, 1)$ is already in the table then $t = 4$.

$e = ITE(0, cd, 0) = 0$ (trivial case)

$\Rightarrow e = 1$

since $t \neq e$, an entry will be added in the table for $(b, 4, 1)$ with $id = 6$

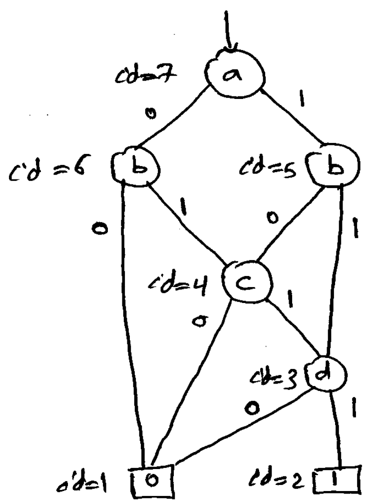
$\Rightarrow e = 6$

since $t \neq e$, an entry will be added in the table for $(a, 5, 6)$ with $id = 7$.

Thus, the unique table produced is:

id	var	right child	left child
3	d	2	1
4	c	3	1
5	b	3	4
6	b	4	1
7	a	5	6

The corresponding ITE DAG is:



Q4. The matrix to be covered:

	c1	c2	c3	c4	c5	c6	c7	c8
r1	0	0	1	0	1	0	1	1
r2	0	1	0	1	0	1	0	1
r3	1	1	1	0	0	0	0	0
r4	0	0	0	1	1	0	0	0
r5	1	0	0	0	0	1	1	0
r6	1	0	0	0	0	0	0	1

The matrix can't be reduced as there are no essential columns, no row dominance and no column dominance.

Thus, we select c1 and call exact_cover with $x = (1,0,0,0,0,0,0,0)$ and $b = (1,1,1,1,1,1,1,1)$ and the matrix:

	c2	c3	c4	c5	c6	c7	c8
r1	0	1	0	1	0	1	1
r2	1	0	1	0	1	0	1
r4	0	0	1	1	0	0	0

There are no essential columns and no row dominance. However, c8 dominates c2, c3, c6 and c7. Thus, c2, c3, c6 and c7 are removed.

The reduced matrix is

	c4	c5	c8
r1	0	1	1
r2	1	0	1
r4	1	1	0

Since the matrix can't be reduced further, c_4 is selected for branching, $x_4 = 1$, rows r_2 and r_4 are removed.

Exact-Cover is called with $x = (1, 0, 0, 1, 0, 0, 0, 0)$, $b = (1, 1, 1, 1, 1, 1, 1, 1)$ and the matrix:

$$\begin{array}{ccc} & c_5 & c_8 \\ r_1 & 1 & 1 \end{array}$$

Since c_5 dominates c_8, c_8 is removed and c_5 is selected as it becomes essential and the solution returned is $x = (1, 0, 0, 1, 1, 0, 0, 0)$.

Since $|x| < |b|$, $b = (1, 0, 0, 1, 1, 0, 0, 0)$.

Next, exact-cover is called with x_4 not selected i.e. $x_4 = 0$. Thus, $x = (1, 0, 0, 0, 0, 0, 0, 0)$, $b = (1, 0, 0, 1, 1, 0, 0, 0)$ and the matrix is:

$$\begin{array}{ccc} & c_5 & c_8 \\ r_1 & 1 & 1 \\ r_2 & 0 & 1 \\ r_3 & 1 & 0 \end{array}$$

Both c_5 and c_8 are essential and selected, So, $x = (1, 0, 0, 0, 1, 0, 0, 1)$. Since the current estimate $= 3 = |b|$, the solution returned will be $b = (1, 0, 0, 1, 1, 0, 0, 0)$.

Next, the exact-cover algorithm is called with c_1 not selected i.e. $x = (0, 0, 0, 0, 0, 0, 0, 0)$, $b = (1, 0, 0, 1, 1, 0, 0, 0)$.

The matrix passed with the call is:

	c2	c3	c4	c5	c6	c7	c8
r1	0	1	0	1	0	1	1
r2	1	0	1	0	1	0	1
r3	1	1	0	0	0	0	0
r4	0	0	1	1	0	0	0
r5	0	0	0	0	1	1	0
r6	0	0	0	0	0	0	1

Since c8 is essential, it is selected and rows r1, r2 and r6 are removed and we get the following reduced matrix:

	c2	c3	c4	c5	c6	c7
r3	1	1	0	0	0	0
r4	0	0	1	1	0	0
r5	0	0	0	0	1	1

Since c2 dominates c3, c4 dominates c5 and c6 dominates c7, c3, c5 and c7 are removed and we get the following reduced matrix:

	c2	c4	c6
r3	1	0	0
r4	0	1	0
r5	0	0	1

Now the three columns are essential and get selected with $x = (0,1,0,1,0,1,0,1)$

Since current estimate $= 4 > |b|$, the solution returned is $(1,0,0,1,1,0,0,0)$.