Dec. 12, 2011

COMPUTER ENGINEERING DEPARTMENT

COE 561

Digital System Design and Synthesis

MAJOR EXAM II

(Open Book Exam)

First Semester (111)

Time: 8:00-10:30 PM

Student Name : _KEY____

Student ID. : ____

Question	Max Points	Score
Q1	15	
Q2	20	
Q3	15	
Q4	14	
Q5	18	
Q6	18	
Total	100	

[15 Points]

(Q1) Consider the function F(A, B, C, D) with **ON-SET=Σm(0, 5, 7, 8, 12)** and **OFF-SET=Σm(2, 10, 11, 13, 14, 15)**. Note that you do not need to use the positional-cube notation in your solution.

- **(i)** **Expand** the minterm **A'B'C'D'** using ESPRESSO heuristics.
- **(ii)** A cover of the function is given by F = A'B + C'D'. **Reduce** the cube **A'B** using Theorem 7.4.1.
- **(iii)** Use Corollary 7.4.1 to check if the implicant **A'B** is an **essential** prime implicant.

(i) Expand the minterm $\bar{A}\bar{B}\bar{C}\bar{D}$

Free-set = {2, 4, 6, 8}
Column 6 can't be raised as it has distance 1 from the off-set
The overexpanded cube = $\bar{C}$
Thus, we need to check the cubes $\bar{A}B\bar{C}D$, $AB\bar{C}\bar{D}$ and $A\bar{B}\bar{C}\bar{D}$ for being feasibly covered.

$SC(\bar{A}\bar{B}\bar{C}\bar{D}, \bar{A}B\bar{C}D) = \bar{A}\bar{C}$ feasible
$SC(\bar{A}\bar{B}\bar{C}\bar{D}, AB\bar{C}\bar{D}) = \bar{C}\bar{D}$ feasible
$SC(\bar{A}\bar{B}\bar{C}\bar{D}, A\bar{B}\bar{C}\bar{D}) = \bar{B}\bar{C}\bar{D}$ feasible

The minterm is expanded to $\bar{C}\bar{D}$ as it covers more cubes.
Free-set = {8}. Column 8 can't be raised as it has distance 1 from the off-set.
Thus, the expanded cube is $\bar{C}\bar{D}$.

(ii) Reduce the cube $\bar{A}B$

$$Q = \bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D$$

$$Q_{\bar{A}B} = \bar{C}\bar{D} + C\bar{D} = \bar{D}$$

$$\overline{Q_{\bar{A}B}} = D$$

$$\Rightarrow \bar{A}B \cap SC(\overline{Q_{\bar{A}B}}) = \bar{A}BD$$

Thus, the cube $\bar{A}B$ is reduced to $\bar{A}BD$

(iii) Check if $\bar{A}B$ is an essential prime implicant

$$\alpha = \bar{A}B$$

$$G = \{\bar{C}\bar{D}, \bar{A}\bar{B}\bar{C}D, \bar{A}\bar{B}CD, \bar{A}B\bar{C}\bar{D}, \bar{A}BC\bar{D}, A\bar{B}\bar{C}D\}$$

$$G\#\alpha = \{A\bar{C}\bar{D}, \bar{B}\bar{C}\bar{D}, \bar{A}\bar{B}\bar{C}D, \bar{A}\bar{B}CD, A\bar{B}\bar{C}D\}$$

$$H = \text{Consensus}(G\#\alpha, \alpha)$$
$$= \{B\bar{C}\bar{D}, \bar{A}\bar{C}\bar{D}, \bar{A}\bar{C}D, \bar{A}CD\}$$

$$H \cup DC = \{B\bar{C}\bar{D}, \bar{A}\bar{C}\bar{D}, \bar{A}\bar{C}D, \bar{A}CD, \bar{A}BC\bar{D}, A\bar{B}\bar{C}D\}$$

$$\{H \cup DC\}_\alpha = \{\bar{C}\bar{D}, \bar{C}\bar{D}, \bar{C}D, CD, C\bar{D}\}$$
$$\Rightarrow \text{Tautology}$$

Thus, the prime implicant $\bar{A}B$ is <u>not</u> an essential prime implicant.

[20 Points]

(Q2) Consider the following cover of a function F*(A,B,C,D)*

$$F = \overline{A}\overline{C} + \overline{A}B + B\overline{C} + BD + ACD$$

With $F^{DC} = \sum m(6, 12)$

(i) Determine the relatively essential set of cubes, E^r.

(ii) Determine the totally redundant, R^t, and partially redundant, R^p, sets of cubes.

(iii) Find a subset of R^p that, together with E^r, covers the function by solving a covering problem.

(i) Relatively Essential Set E^r:

– Check $\overline{A}\overline{C}$

$\{\overline{A}B, B\overline{C}, BD, ACD, \overline{A}BC\overline{D}, AB\overline{C}\overline{D}\}_{\overline{A}\overline{C}}$

$= \{B, B, BD, 0, 0, 0\}$ Not tautology ⇒ Rel. Ess.

– Check $\overline{A}B$

$\{\overline{A}\overline{C}, B\overline{C}, BD, ACD, \overline{A}BC\overline{D}, AB\overline{C}\overline{D}\}_{\overline{A}B}$

$= \{\overline{C}, \overline{C}, D, 0, C\overline{D}, 0\}$ Tautology ⇒ Not Rel. Es.

– Check $B\overline{C}$

$\{\overline{A}\overline{C}, \overline{A}B, BD, ACD, \overline{A}BC\overline{D}, AB\overline{C}\overline{D}\}_{B\overline{C}}$

$= \{\overline{A}, \overline{A}, D, 0, 0, A\overline{D}\}$ Tautology ⇒ Not Rel. Es.

– Check BD

$\{\overline{A}\overline{C}, \overline{A}B, B\overline{C}, ACD, \overline{A}BC\overline{D}, AB\overline{C}\overline{D}\}_{BD}$

$= \{\overline{A}\overline{C}, \overline{A}, \overline{C}, AC, 0, 0\}$ Tautology ⇒ Not Rel. Es.

– Check ACD

$\{\overline{A}\overline{C}, \overline{A}B, B\overline{C}, BD, \overline{A}BC\overline{D}, AB\overline{C}\overline{D}\}_{ACD}$

$= \{0, 0, 0, B, 0, 0\}$ Not Tautology ⇒ Rel Es.

Thus, $E^r = \{\overline{A}\overline{C}, ACD\}$

(ii) Totally redundant set R^t:

- Check $\bar{A}B$

$\{\bar{A}\bar{C}, ACD, \bar{A}BC\bar{D}, AB\bar{C}\bar{D}\}_{\bar{A}B}$
$= \{\bar{C}, 0, C\bar{D}, 0\}$ Not tautology $\Rightarrow$ Part. Red.

- Check $B\bar{C}$

$\{\bar{A}\bar{C}, ACD, \bar{A}BC\bar{D}, AB\bar{C}\bar{D}\}_{B\bar{C}}$
$= \{\bar{A}, 0, 0, A\bar{D}\}$ Not tautology $\Rightarrow$ Part. Red.

- Check BD

$\{\bar{A}\bar{C}, ACD, \bar{A}BC\bar{D}, AB\bar{C}\bar{D}\}_{BD}$
$= \{\bar{A}\bar{C}, AC, 0, 0\}$ Not tautology $\Rightarrow$ Part. Red.

Thus, $R^t = \{\}$ and $R^p = \{\bar{A}B, B\bar{C}, BD\}$

(iii) First, we find coverage relations:

- $\bar{A}B$:

$\{\bar{A}\bar{C}, ACD, \bar{A}BC\bar{D}, AB\bar{C}\bar{D}, B\bar{C}, BD\}_{\bar{A}B}$
$= \{\bar{C}, 0, C\bar{D}, 0, \bar{C}, D\}$

- Expand on D:
  - $D=1$: $\{\bar{C}, 0, 0, 0, \bar{C}, 1\} \Rightarrow$ add row (1, 0, 1)
  - $D=0$: $\{\bar{C}, 0, C, 0, \bar{C}, 0\} \Rightarrow$ No rows added

- $B\bar{C}$:

$\{\bar{A}\bar{C}, ACD, \bar{A}BC\bar{D}, AB\bar{C}\bar{D}, \bar{A}B, BD\}_{B\bar{C}}$
$= \{\bar{A}, 0, 0, A\bar{D}, \bar{A}, D\}$

- Expand on D:
  - $D=1$: $\{\bar{A}, 0, 0, 0, \bar{A}, 1\} \Rightarrow$ Add row (0, 1, 1)
  - $D=0$: $\{\bar{A}, 0, 0, A, \bar{A}, 0\} \Rightarrow$ No rows added

- $\underline{BD}$:

$\{\bar{A}\bar{C}, ACD, \bar{A}BC\bar{D}, AB\bar{C}\bar{D}, \bar{A}B, B\bar{C}\}_{BD}$

$= \{\bar{A}\bar{C}, AC, 0, 0, \bar{A}, \bar{C}\}$

- Expand on C:
  - $C = 1$: $\{0, A, 0, 0, \bar{A}, 0\} \Rightarrow$ Added row (1,0,1)
  - $C = 0$: $\{\bar{A}, 0, 0, 0, \bar{A}, 1\} \Rightarrow$ Added row (0,1,1)

Covering Matrix:

	$\bar{A}B$	$B\bar{C}$	BD
$\bar{A}B$	1	0	1
$B\bar{C}$	0	1	1
BD	1	0	1
	0	1	1

Thus, BD is selected and the minimal cover is $\{\bar{A}\bar{C}, ACD, BD\}$.

[15 Points]

(Q3) Consider the logic network defined by the following expression:

$$x = a\,c\,e + a\,c'\,e' + a\,d + b\,c\,e + b\,c'\,e' + b\,d$$

Using the recursive procedure **KERNELS**, compute all the kernels and co-kernels of x. Show all the steps of the algorithm. Assume the following lexicographic order: {a, b, c, c', d, e, e'}.

- $i = 1$ (a):

  Cubes$(x, a) = \{ace, a\bar{c}\bar{e}, ad\} \geq 2$, $C = a$

  The Kernel $ce + \bar{c}\bar{e} + d$ will be found

  Recursive call with $i = 2$:

  Since the number of cubes containing each variable is < 2, no Kernels will be found.

- $i = 2$ (b):

  Cubes$(x, b) = \{bce, b\bar{c}\bar{e}, bd\} \geq 2$, $C = b$

  The Kernel $ce + \bar{c}\bar{e} + d$ will be found

  Recursive call with $i = 3$:

  Since the number of cubes containing each variable is < 2, no Kernels will be found.

- $i = 3$ (c):

  Cubes$(x, c) = \{ace, bce\} \geq 2$, $C = ce$

  The Kernel $a + b$ will be found

- $i = 4$ ($\bar{c}$):

  Cubes$(x, \bar{c}) = \{a\bar{c}\bar{e}, b\bar{c}\bar{e}\} \geq 2$, $C = \bar{c}\bar{e}$

  The Kernel $a + b$ will be found

- $i = 5$ (d):

  $Cubes(x, d) = \{ad, bd\} \geq 2$, $C = d$

  The Kernel $a+b$ will be found

- $i = 6$ (e):

  $Cubes(x, e) = \{ace, bce\} \geq 2$, $C = ce$

  Since the cube contains literal $c < 6$, no Kernels will be found.

- $i = 7$ ($\bar{e}$):

  $Cubes(x, \bar{e}) = \{a\bar{c}\bar{e}, b\bar{c}\bar{e}\} \geq 2$, $C = \bar{c}\bar{e}$

  Since the cube contains literal $\bar{c} < 7$, no Kernels will be found.

Thus, the list of Kernels and co-Kernels of x are:

Kernel	Co-Kernel
$ce + \bar{c}\bar{e} + d$	b, a
$a + b$	ce, $\bar{c}\bar{e}$, d

[14 Points]

(Q4) Consider the logic network defined by the following expression:

$$x = a\,b\,c + a\,b\,d + a\,b'\,c'\,d' + a'\,b\,c'\,d' + a'\,b'\,c + a'\,b'\,d + c\,e + c\,f + d\,e + d\,f$$

Compute the weight of the double cube divisors $d_1 = a\,b + a'\,b'$ and $d_2 = c + d$. Extract the double cube divisor with the highest weight and show the resulting network after extraction and the number of literals saved.

Double Cube Divisor	Base
$d_1 = ab + \bar{a}\bar{b}$	c, d
$\bar{d_1} = a\bar{b} + \bar{a}b$	$\bar{c}\bar{d}$
$d_2 = c + d$	$ab, e, f, \bar{a}\bar{b}$

$\text{weight}(d_1) = 3 * 4 - 3 - 4 + 1 + 1 + 2 = 9$

$\text{weight}(d_2) = 4 * 2 - 4 - 2 + 2 + 1 + 1 + 2 + 2 = 10$

Since d_2 has higher weight, it will be extracted.

The resulting network after extraction of d_2 is:

$[1] = c + d$

$x = ab[1] + a\bar{b}\,\overline{[1]} + \bar{a}b\,\overline{[1]} + \bar{a}\bar{b}[1] + [1]e + [1]f$

18 literals

Original number of literals = 28 literals

Number of literals saved = 10 literals

[18 Points]

(Q5) Consider the logic network defined by the following expressions with inputs *{a, b, c}* and output {*z*}:

$$x = a\ b' + a'\ b$$

$$y = a'\ c'\ x' + b\ x$$

$$z = y + a\ b'$$

(i) Compute the SDC set for nodes *x* and *y*.

(ii) Use the SDC computed in (i) to simplify *z*.

(iii) Compute the CDC and ODC of Y based on the simplified network in (ii) and simplify its function.

(i) $SDC_x = x \oplus (a\bar{b} + \bar{a}b) = xab + x\bar{a}\bar{b} + \bar{x}a\bar{b} + \bar{x}\bar{a}b$

$SDC_y = y \oplus (\bar{a}\bar{c}\bar{x} + bx) = y\bar{b}x + ya\bar{x} + yc\bar{x} + \bar{y}\bar{a}\bar{c}\bar{x} + \bar{y}bx$

(ii)

$y = 0$

ab \ cx	00	01	11	10
00	x	x	x	0
01	x	x	x	x
11	0	x	x	0
10	x	1	1	x

$y = 1$

ab \ cx	00	01	11	10
00	1	x	x	x
01	x	1	1	x
11	x	x	x	x
10	x	x	x	x

$\Rightarrow \quad z = y + x$

(iii) $CDC_y = SDC_x = xab + x\bar{a}\bar{b} + \bar{x}a\bar{b} + \bar{x}\bar{a}b$

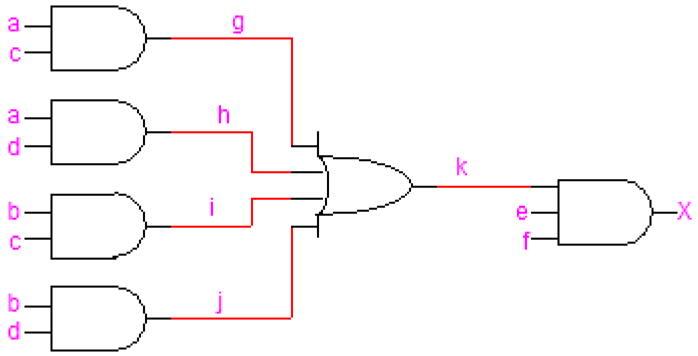
$ODC_y = x$

ab \ cx	00	01	11	10
00	1	x	x	0
01	x	x	x	x
11	0	x	x	0
10	x	x	x	x

$y = \bar{a}\bar{c}$ or $y = \bar{b}\bar{c}$

[18 Points]

(Q6) Consider the logic network below with inputs *{a, b, c, d, e, f}* and output *{X}*:



Assume that the delay of a gate is related to the number of its inputs i.e. the delay of a 2-input AND gate is 2. Also, assume that the input data-ready times are zero for all inputs except input *a*, which has a data-ready time of 2.

(i) Compute the data ready times and slacks for all vertices in the network.

(ii) Determine the topological critical path.

(iii) Suggest an implementation of the function *X* to reduce the delay of the circuit to the minimum possible and determine the maximum propagation delay in the optimized circuit. Has the area been affected?

(i)

Data Ready Time	Required Time	Slack
$t_a = 2$	$\bar{t}_a = \min(4-2, 4-2) = 2$	$S_a = 2-2 = 0$
$t_b = 0$	$\bar{t}_b = \min(4-2, 4-2) = 2$	$S_b = 2-0 = 2$
$t_c = 0$	$\bar{t}_c = \min(4-2, 4-2) = 2$	$S_c = 2-0 = 2$
$t_d = 0$	$\bar{t}_d = \min(4-2, 4-2) = 2$	$S_d = 2-0 = 2$
$t_e = 0$	$\bar{t}_e = 8$	$S_e = 8-0 = 8$
$t_f = 0$	$\bar{t}_f = 8$	$S_f = 8-0 = 8$
$t_g = 4$	$\bar{t}_g = 4$	$S_g = 4-4 = 0$
$t_h = 4$	$\bar{t}_h = 4$	$S_h = 4-4 = 0$
$t_i = 2$	$\bar{t}_i = 4$	$S_i = 4-2 = 2$
$t_j = 2$	$\bar{t}_j = 4$	$S_j = 4-2 = 2$
$t_k = 8$	$\bar{t}_k = 8$	$S_k = 8-8 = 0$
$t_x = 11$	$\bar{t}_x = 11$	$S_x = 11-11 = 0$

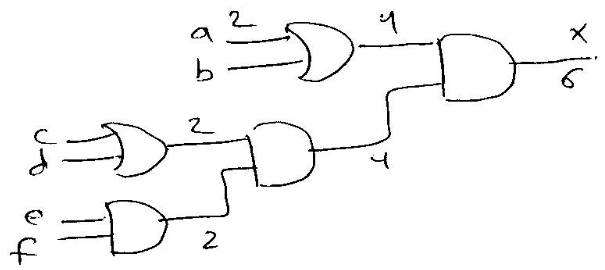
(ii) The topological critical paths are:

$\{a, g, k, x\}$ and $\{a, h, k, x\}$

(iii) To optimize the delay of the network, we need to improve the delay of nodes in the critical paths.

K can be factored into $(a+b)(c+d)$

Then, x can be implemented as follows:



The resulting delay is 6.
Number of literals is 10.

Thus, we have improved the delay from 11 to 6 and area from 15 literals to 10 literals.