Jan. 7, 2010

COMPUTER ENGINEERING DEPARTMENT

COE 561

Digital System Design and Synthesis

MAJOR EXAM II

(Open Book Exam)

First Semester (091)

Time: 3:30-6:00 PM

Student Name : _KEY____

Student ID.   : ____

Question	Max Points	Score
Q1	30	
Q2	15	
Q3	15	
Q4	20	
Q5	20	
Total	100	

[30 Points]

(Q1) Consider the function F(A, B, C, D) with **ON-SET=Σm(5, 6, 7, 13)** and **OFF-SET=Σm(1, 8, 12)**. Note that you do not need to use the positional-cube notation in your solution.

- **(i)** **Expand** the minterm **ABC'D** using ESPRESSO heuristics.
- **(ii)** A cover of the function is given by F = C + BD. **Reduce** the cube **C** using Theorem 7.4.1.
- **(iii)** Use Corollary 7.4.1 to check if the implicant **C** is an **essential** prime implicant.

(i) Expand the minterm $AB\bar{C}D$

Free set = {1, 3, 6, 7}

Column 7 can't be raised as it has distance 1 from the offset.

Examining the off-set, we can see that col. 6 is all 0's in the offset. Thus, col. 6 can always be raised. Thus, free set = {1, 3}.

The overexpanded cube = D

Thus, we need to check the cubes $\bar{A}B\bar{C}D$ and $\bar{A}BCD$ for being feasibly covered.

SC ($AB\bar{C}D$, $\bar{A}B\bar{C}D$) = $B\bar{C}D$ feasible

SC ($AB\bar{C}D$, $\bar{A}BCD$) = BD feasible

Thus, the minterm is expanded to BD as it covers the other feasibly covered cube.

Thus, free set = {3}. This col. can't be raised as it has distance 1 from the offset.

Thus, the expanded cube is BD.

(ii) Reduce the cube c

$$Q = BD + \underbrace{\bar{A}\bar{C}\bar{D} + \bar{B}C + AC + A\bar{B}D}_{DC-set}$$

$$= \bar{B}\bar{D}\,[\bar{A}\bar{C} + C + AC] + \bar{B}D\,[C + AC + A]$$

$$+ B\bar{D}\,[\bar{A}\bar{C} + AC] + BD\,[1]$$

$$= \bar{B}\bar{D}\,[\bar{A} + C] + \bar{B}D\,[A + C] + B\bar{D}\,[\bar{A}\bar{C} + AC]$$

$$+ BD\,[1]$$

$$\bar{Q} = \bar{B}\bar{D}\,[A\bar{C}] + \bar{B}D\,[\bar{A}\bar{C}] + B\bar{D}\,[A\bar{C} + \bar{A}C]$$

$$\bar{Q}_c = B\bar{D}\bar{A}, \quad sc(\bar{Q}_c) = \bar{A}B\bar{D}$$

$$c \cap sc(\bar{Q}_c) = \bar{A}BC\bar{D}$$

Thus, the cube c is reduced to $\bar{A}BC\bar{D}$.

(iii) Check if c is an essential prime implicant

$$\alpha = c$$

$$G = \{BD, \bar{A}\bar{C}\bar{D}, \bar{B}C, AC, A\bar{B}D\}$$

$$G \# \alpha = \{B\bar{C}D, \bar{A}\bar{C}\bar{D}, A\bar{B}\bar{C}D\}$$

$$H = \text{Consensus}(G \# \alpha, \alpha)$$

$$= \{BD, \bar{A}\bar{D}, A\bar{B}D\}$$

$$\{H \cup DC\}_\alpha = \{BD, \bar{A}\bar{D}, A\bar{B}D, \bar{A}\bar{C}\bar{D}, \bar{B}C, AC, A\bar{B}D\}_C$$

$$= \{BD, \bar{A}\bar{D}, A\bar{B}D, \bar{B}, A, A\bar{B}D\}$$

$$= \{BD, \bar{A}\bar{D}, \bar{B}, A\}$$

$$= \{D, \bar{D}, \bar{B}, A\} \Rightarrow \text{Tautology}$$

Thus, the implicant c is not an essential prime implicant.

[15 Points]

(Q2) Consider the logic network defined by the following expression:

$$x = a\,c\,e + a\,c\,f + b\,c\,e + b\,c\,f + d\,e + d\,f$$

Using the recursive procedure **KERNELS**, compute all the kernels and co-kernels of *x*. Show all the steps of the algorithm. Assume the following lexicographic order: {a, b, c, d, e, f}.

$i = 1$ (a):

cubes $(x, a) = \{ace, acf\} \geq 2$

$C = ac$

The kernel $e+f$ will be returned

$i = 2$ (b):

cubes $(x, b) = \{bce, bcf\} \geq 2$

$C = bc$

The kernel $e+f$ will be returned

$i = 3$ (c):

cubes $(x, c) = \{ace, acf, bce, bcf\}$

$C = c$

Kernel found: $ae + af + be + bf$

Recursive call on the kernel with $i = 4$ {d}

- No kernel will be found for literal d
- $i = 5$:
  cubes containing e: $\{ae, be\} \geq 2$
  $C = e$
  The kernel $a+b$ is generated
- $i = 6$:
  cubes containing f: $\{af, bf\}$
  $C = f$
  The kernel $a+b$ is generated

$i = 4$ (d):

cubes $(x, d) = \{de, df\} \geq 2$

$c = d$

The Kernel $e+f$ is generated

$i = 5$ (e):

cubes $(x, e) = \{ace, bce, de\}$

$C = e$

The Kernel $ac + bc + d$ is generated

recursive call with $i = 6$ does not produce any Kernel

$i = 6$ (f):

cubes $(x, f) = \{acf, bcf, df\}$

$c = f$

The Kernel $ac + bc + d$ is generated

recursive call with $i = 7$ does not produce any Kernel.

Thus, the list of Kernels and co-Kernels of x are:

Kernel	co-Kernel
$e+f$	ac, bc, d
$ae + af + be + bf$	c
$a+b$	ce, cf
$ac + bc + d$	e, f
$ace + acf + bce + bcf + de + df$	1

[15 Points]

(Q3) Consider the logic network defined by the following expression:

$$x = a\,b\,e'\,f' + a\,b'\,e + a\,b'\,f + a'\,d\,e'\,f' + a'\,d'\,e + a'\,d'\,f + c\,e'\,f'$$

Compute the weight of the double cube divisors $d_1 = a\,b + a'd$ and $d_2 = e + f$. Extract the double cube divisor with the highest weight and show the resulting network after extraction and the number of literals saved.

Double cube Divisor	Base
$d1 = ab + \bar{a}d$	$\bar{e}\bar{f}$
$\bar{d1} = a\bar{b} + \bar{a}\bar{d}$	e, f
$d2 = e + f$	$a\bar{b}, \bar{a}\bar{d}$

weight (d1) = 3 + 4 − 3 − 4 + 2 + 1 + 1 = 9

weight (d2) = 2 + 2 − 2 − 2 + 2 + 2 + 3 = 7

Since d1 has a higher weight, it will be extracted.

The resulting network after extraction of d1 is:

$[1] = ab + \bar{a}d$

$x = [1]\bar{e}\bar{f} + \overline{[1]}e + \overline{[1]}f + c\bar{e}\bar{f}$

14 literals

original number of literals = 23

Number of literals saved = 9 literals

[20 Points]

(Q4) Consider the logic network defined by the following expressions with inputs *{a, b, c, d, e}* and output {*y*}:

$$x = a\,b + a'\,d$$

$$w = x'\,d' + b\,c'$$

$$y = (w \oplus e)\,a'\,b$$

(i) Simplify the function *w* based on the utilization of don't care conditions.

(ii) Based on perturbation analysis starting with the original network, determine if it is possible to change the implementation of *x* to *x*= *a*.

(i) We need to compute SDC_x and ODC_w to simplify w.

$$SDC_x = x \oplus (ab + \bar{a}d)$$

$$= x(a\bar{b} + \bar{a}\bar{d}) + \bar{x}(ab + \bar{a}d)$$

$$= x a\bar{b} + x\bar{a}\bar{d} + \bar{x}ab + \bar{x}\bar{a}d$$

$$ODC_w = \overline{y_{\bar{w}} \oplus y_w} = \overline{e\bar{a}b \oplus \bar{e}\bar{a}b}$$

$$= \overline{\bar{a}b(e \oplus \bar{e})} = \overline{\bar{a}b} = a + \bar{b}$$

$x=0$

ab \ cd	00	01	11	10
00	x	x	x	x
01	1	x	x	1
11	x	x	x	x
10	x	x	x	x

$x=1$

ab \ cd	00	01	11	10
00	x	x	x	x
01	x	1	0	x
11	x	x	x	x
10	x	x	x	x

$$w = \bar{x} + \bar{c}$$

or $w = \bar{d} + \bar{c}$

(ii) $\delta = (ab + \bar{a}d) \oplus a$

$= (a\bar{b} + \bar{a}\bar{d})a + (ab + \bar{a}d)\bar{a}$

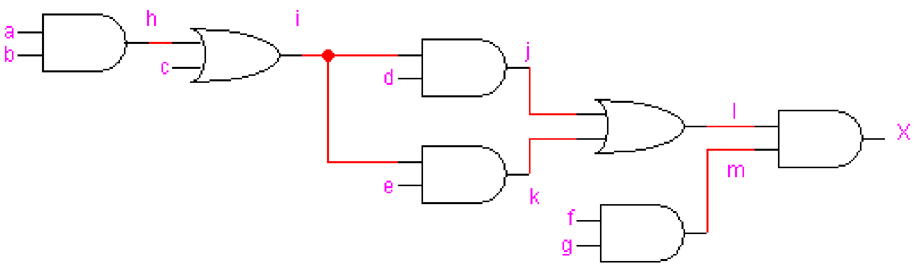
$= a\bar{b} + \bar{a}d$

$ODC_x = d + b\bar{c} + a + \bar{b}$

since $\delta \leq ODC_x$, then it is possible to change the implementation of x to $x = a$

[20 Points]

(Q5) Consider the logic network below with inputs *{a, b, c, d, e, f, g}* and output *{X}*:



Assume that the delay of a gate is related to the number of its inputs i.e. the delay of a 2-input AND gate is 2. Also, assume that the input data-ready times are zero for all inputs.

(i) Compute the data ready times and slacks for all vertices in the network.

(ii) Determine the topological critical path.

(iii) Suggest an implementation of the function *X* using only 2-input gates to reduce the delay of the circuit to the minimum possible and determine the maximum propagation delay in the optimized circuit. Has the area been affected?

(i)

Data ready time	Required time	Slack
$t_a = 0$	$\bar{t}_a = 0$	$S_a = 0$
$t_b = 0$	$\bar{t}_b = 0$	$S_b = 0$
$t_c = 0$	$\bar{t}_c = 2$	$S_c = 2-0=2$
$t_d = 0$	$\bar{t}_d = 4$	$S_d = 4-0=4$
$t_e = 0$	$\bar{t}_e = 4$	$S_e = 4-0=4$
$t_f = 0$	$\bar{t}_f = 6$	$S_f = 6-0=6$
$t_g = 0$	$\bar{t}_g = 6$	$S_g = 6-0=6$
$t_h = 2$	$\bar{t}_h = 2$	$S_h = 2-2=0$
$t_i = 4$	$\bar{t}_i = 4$	$S_i = 4-4=0$
$t_j = 6$	$\bar{t}_j = 6$	$S_j = 6-6=0$
$t_k = 6$	$\bar{t}_k = 6$	$S_k = 6-6=0$
$t_l = 8$	$\bar{t}_l = 8$	$S_l = 8-8=0$
$t_m = 2$	$\bar{t}_m = 8$	$S_m = 8-2=6$
$t_x = 10$	$\bar{t}_x = 10$	$S_x = 10-10=0$

(ii) The topological critical paths are:

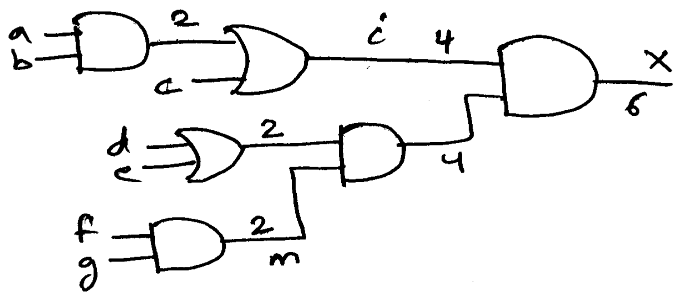
$\{a, h, i, j, l, x\}$
$\{a, h, i, k, l, x\}$
$\{b, h, i, j, l, x\}$
$\{b, h, i, k, l, x\}$

(iii) To optimize the delay of the network, we need to improve the delay of nodes in the critical path.

$l = id + ie \Rightarrow l = i(d+e)$

$x = l \cdot m = i \cdot [(d+e) \cdot m]$

The resulting network:



The maximum propagation delay in the optimized circuit is 6.
Number of literals in the original circuit = 14 literals
Number of literals in optimized circuit = 12 literals
Thus, the area has also been reduced.