

COE 405, Term 131

Design & Modeling of Digital Systems

Quiz# 1

Date: Tuesday, Oct. 1, 2013

Q.1. Consider the two functions: $F_1(A, B, C, D) = AB + BC + CD + \overline{A}\overline{D}$ and $F_2(A, B, C, D) = ABC + \overline{A}\overline{B}\overline{C} + \overline{A}D + \overline{A}\overline{D}$

(i) Compute the expansion of F_1 and F_2 using the **Orthonormal Basis** $\{\phi_1 = \overline{A}\overline{B}, \phi_2 = \overline{A}B, \phi_3 = A\overline{B}, \phi_4 = AB\}$.

(ii) Compute the function $F_1 \oplus F_2$.

$$\begin{aligned} \text{(i)} \quad F_1 &= \overline{A}\overline{B} [cD + \overline{D}] + \overline{A}B [c + cD + \overline{D}] \\ &\quad + A\overline{B} [cD] + AB [1] \\ &= \overline{A}\overline{B} [c + \overline{D}] + \overline{A}B [c + \overline{D}] \\ &\quad + A\overline{B} [cD] + AB [1] \end{aligned}$$

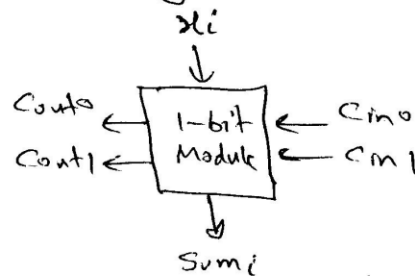
$$\begin{aligned} F_2 &= \overline{A}\overline{B} [\overline{C} + D] + \overline{A}B [D] \\ &\quad + A\overline{B} [\overline{D}] + AB [c + \overline{D}] \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad F_1 \oplus F_2 &= \overline{A}\overline{B} [(c + \overline{D}) \oplus (\overline{C} + D)] \\ &\quad + \overline{A}B [(c + \overline{D}) \oplus D] \\ &\quad + A\overline{B} [cD \oplus \overline{D}] \\ &\quad + AB [1 \oplus (c + \overline{D})] \\ &= \overline{A}\overline{B} [c\overline{D} + \overline{C}D] \\ &\quad + \overline{A}B [\overline{C} + \overline{D}] \\ &\quad + A\overline{B} [c + \overline{D}] \\ &\quad + AB [\overline{C}D] \end{aligned}$$

Q.2. It is required to design a combinational circuit that computes the equation $Y=3*X+2$, where X is an n -bit unsigned number.

- (i) Design the circuit as a modular circuit where each module receives a single bit of the input, X_i .
- (ii) Derive the truth table of your 1-bit module in (i).
- (iii) Derive minimized two-level sum-of-product equations for your 1-bit module circuit.

(i) By analyzing the circuit functionality, we determine that the maximum carry-out is 2. Thus, we need 2 bits for the carry-in and 2 bits for the carry-out between modules.



Note the $+2$ will be achieved by setting $C_{in1} C_{in0} = 10$ in the first module.

(ii) Truth Table:

C_{in1}	C_{in0}	x_i	C_{out1}	C_{out0}	Sum_i
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	1
0	1	1	1	0	0
1	0	0	0	1	0
1	0	1	1	0	1
1	1	0	x	x	x
1	1	1	x	x	x

(iii)

	$C_{in} X_i$			
C_{in}	00	01	11	10
0	0	0	1	0
1	0	1	x	x

$$C_{out1} = C_{in0} X_i' + C_{in1} X_i$$

	$C_{in} X_i'$			
C_{in}	00	01	11	10
0	0	1	0	0
1	1	0	x	x

$$C_{out0} = \overline{C_{in1}} \overline{C_{in0}} X_i' + C_{in1} X_i$$

	$C_{in} X_i'$			
C_{in}	00	01	11	10
0	0	1	0	1
1	0	1	x	x

$$S_{out1} = \overline{C_{in0}} X_i' + C_{in0} X_i$$