## COE 405, Term 152

## Design \& Modeling of Digital Systems

HW\# 7 Solution
Due date: Sunday, May 1
Q.1. Consider the logic network below with inputs $\{a, b, c, d, e\}$ and output $\{k\}$ :


Assume that the delay of the inverter gate is 1 and that the delay of the 2 -input NAND gate is 2 . Also, assume that the input data-ready times are zero except for input $a$, which is equal to 2 .
(i) Compute the data ready times and slacks for all vertices in the network.
(ii) Determine the topological critical path.
(iii) Suggest an implementation of the function $k$ using only inverters and 2-input NAND gates to reduce the delay of the circuit to the minimum possible and determine the maximum propagation delay in the optimized circuit. Has the area been affected?
(i) Data ready time Data required tire slack

| $t_{a}=2$ | $\bar{t}_{a}=4-2=2$ | $s_{a}=2-2=0$ |
| :--- | :--- | :--- |
| $t_{b}=0$ | $\bar{t}_{b}=4-2=2$ | $s_{b}=2-0=2$ |
| $t_{c}=0$ | $\bar{t}_{c}=6-2=4$ | $s_{c}=4-0=4$ |
| $t_{d}=0$ | $\bar{t}_{d}=9-2=7$ | $s_{d}=7-0=7$ |
| $t_{e}=0$ | $\bar{t}_{e}=9-2=7$ | $s_{c}=7-0=7$ |
| $t_{f}=2+2=4$ | $\bar{t}_{f}=6-2=4$ | $s_{f}=4-4=0$ |
| $t_{g}=4+2=6$ | $\bar{t}_{g}=7-1=6$ | $s_{g}=6-6=0$ |
| $t_{h}=6+1=7$ | $\bar{t}_{h}=9-2=7$ | $s_{h}=7-7=0$ |
| $t_{i}=4+2=6$ | $\overline{t_{i}}=11-2=9$ | $s_{i}=9-6=3$ |
| $t_{j}=7+2=9$ | $\overline{t_{j}}=11-2=9$ | $s_{j}=9-9=0$ |
| $t_{k}=9+2=11$ | $\bar{t}_{k}=11$ | $s_{k}=11-11=0$ |

(ii) The topological critical path in the inputloutput path with zero slack which is $\{a, f, g, h, j, k\}$.
(III)

$$
\begin{aligned}
k & =f e+d h=f e+f c d=f(e+c d) \\
& =\overline{(a b)}(e+c d)
\end{aligned}
$$

This leads to the following implementation:


The maximum propagation delay in the optimized circuit is 7 .

The number of literals on the orrgmal circuit is 11 .

The number of literals in the optimized crrcult is 10 .

Thus, the area has also been reduced.
Q.2. It is required to design a circuit to compute the equation $Y=A+B-C-D$, where $A, B, C$ and D are N -bit inputs. Assume that inputs are available only during the first cycle when a START input is asserted. Assume that a DONE signal will be set when the result is ready and the result will remain held until the next Start operation.
(i) Show a schedule of the operations with minimum latency (i.e., clock cycles) assuming that the clock cycle is limited by the time for performing one addition/subtraction operation. Store the output Y in a register.

(ii) Show the Data Path design of your circuit indicating all the control signals and the used adder/subtractor sizes.

(iii) Show the ASMD diagram of your control unit.

(iv) Write the necessary Verilog modules to module the data path unit, control unit and the overall circuit.

```
module HW7 #(parameter N=4) (output [N:0] Y, output DONE,
input [N-1:0] A, B, C, D, input START, RESET, CLK);
HW7_DPU #(N) M1 ( Y, A, B, C, D, SEL, R1IN, R2IN, CIN, CLK);
HW7_CU M2 (SEL, R1IN, R2IN, CIN, DONE, START, RESET, CLK);
endmodule
module HW7_DPU #(parameter N=4) (output [N:0] Y, input [N-1:0]
A, B, C, D, input SEL, R1IN, R2IN, CIN, CLK);
reg [N:0] R1, R2;
wire [N:0] MUX1, MUX2;
assign MUX1 = SEL? R1 : A;
assign MUX2 = SEL? ~{1'b0,R2} : B;
assign Y = R1;
```

```
always @(posedge CLK) begin
if (R1IN)
    R1 = MUX1 + MUX2 + CIN;
if (R2IN)
    R2 = C + D;
end
endmodule;
module HW7 CU (output reg SEL, R1IN, R2IN, CIN, DONE, input
START, RESET, CLK);
parameter S0 = 2'b00, S1=2'b01, S2=2'b10;
reg [2:0] state, next_state;
always @(posedge CLK, posedge RESET)
    if (RESET) state <= SO;
    else state <= next_state;
always @(state, START) begin
    SEL=0; R1IN=0; R2IN=0; CIN=0; DONE=0;
    case (state)
        S0:
            if (START) begin
                    next_state=S1; R1IN=1; R2IN=1; end
                else next_state=S0;
    S1: begin SEL=\overline{1; CIN=1; R1IN=1; next_state=S2; end}
    S2: begin DONE=1;
            if (START) begin
                    next_state=S1; R1IN=1; R2IN=1; end
            else nex}t\mathrm{ state=S2;
            end
            default: next_state='bx;
    endcase
end
endmodule
```

(v) Write a test bench to test the correct operation of your circuit. Include simulation snapshots.

```
module HW7_Test();
wire [4:0] Y;
wire DONE;
```

```
reg [3:0] A, B, C, D;
reg START, RESET, CLK;
```

HW7 \# (4) M1 ( Y, DONE, A, B, C, D, START, RESET, CLK);
initial begin
CLK = 0;
forever
\#50 CLK = ~ CLK;
end
initial begin
RESET=1;
\#100 RESET=0; START=1; $A=15 ; B=15 ; C=1 ; ~ D=2$;
\#100 START=0;
end
endmodule

(vi) Implement your circuit on FPGA assuming $\mathrm{N}=2$ bits. Include a link for a video snapshot to demonstrate correct functionality of your circuit.

